Robust PMU Placement

Margarida Carvalho [1] Ana Viana [2]

[1] INESC TEC, Portugal

[2] INESC TEC and Instituto Superior de Engenharia do Porto, Portugal

Optimization Challenges in the evolution of energy networks to smart grids
October 27, 2016

Contents

Contents

PMU Placement Problem

Preliminaries

Motivation

- Need to monitor the power system network;
- Guarantee full observability;
- Minimize costs.

Preliminaries

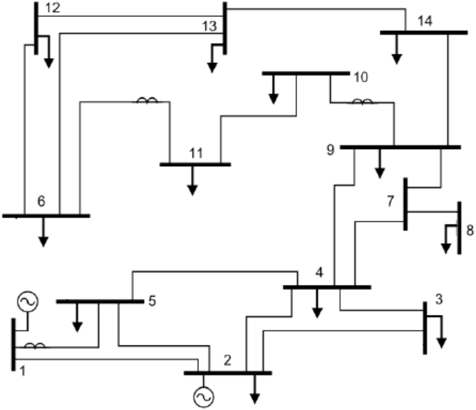
Motivation

- Need to monitor the power system network;
- Guarantee full observability;
- Minimize costs.

Phasor Measurement Units (PMUs)

- Measuring device;
- Provides time synchronized phasor measurements;
- Very expensive.

Example: IEEE 14 Bus



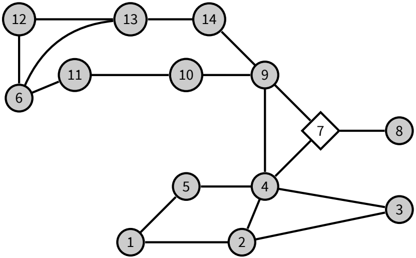
Example: IEEE 14 Bus

Power System Graph (PSG)

$G = (V, E)$: $V = \{\text{set of buses (nodes)}\}$
$E = \{\text{set of transmission lines (edges)}\}$.

$N(i)$: is the neighborhood of node $i \in V$. $N[i] = N(i) \cup \{i\}$.

S: set of zero-injection nodes.



$$S = \{7\}$$

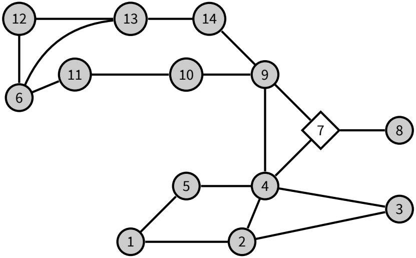
Example: IEEE 14 Bus

A node is **observed** if its voltage is know.

An edge is **observed** if its current is known.

A PSG is **fully observed** if the voltage and current in all its buses and transmission lines are known.

The **PMU placement problem** (PPP) consists in deciding the localization of PMUs such that the PSG is fully observed and the number (or cost) of PMUs is minimized.



$$S = \{7\}$$

Properties to reduce the number of PMUs

Laws

Ohm's law $I_{ij} = \frac{V_i - V_j}{R_{ij}}$, where R_{ij} is the resistance.

Kirchhoff's law $\sum_{j \in N(v_i)} I_{ij} = 0$ for a zero-injection bus i.

Rule 1: A PMU in a bus i measures its voltage V_i and the current I_{ij} for all $j \in N(i)$. By Ohm's law, the voltage for bus j can be computed.

Properties to reduce the number of PMUs

Laws

Ohm's law $I_{ij} = \frac{V_i - V_j}{R_{ij}}$, where R_{ij} is the resistance.

Kirchhoff's law $\sum_{j \in N(v_i)} I_{ij} = 0$ for a zero-injection bus i.

Rule 1: A PMU in a bus i measures its voltage V_i and the current I_{ij} for all $j \in N(i)$. By Ohm's law, the voltage for bus j can be computed.

Rule 2: A zero-injection node i is observed if all of its neighbors are observed.

Properties to reduce the number of PMUs

Laws

Ohm's law $I_{ij} = \frac{V_i - V_j}{R_{ij}}$, where R_{ij} is the resistance.

Kirchhoff's law $\sum_{j \in N(v_i)} I_{ij} = 0$ for a zero-injection bus i.

Rule 1: A PMU in a bus i measures its voltage V_i and the current I_{ij} for all $j \in N(i)$. By Ohm's law, the voltage for bus j can be computed.

Rule 2: A zero-injection node i is observed if all of its neighbors are observed.

Rule 3: If zero-injection node j and all its neighbors, except node i, are observed, then i is observed.

Summary of the Rules

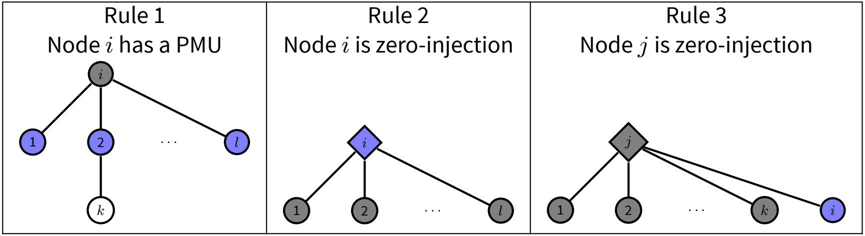


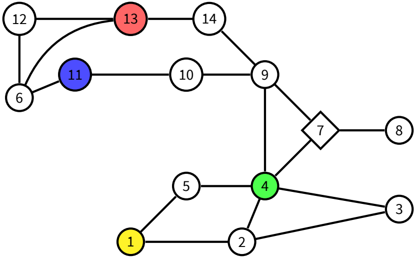
Figure 1: The grey nodes are observed. The blue nodes become observed.

Lemma

If the buses of a PSG are all observed, then the PSG is fully observed.

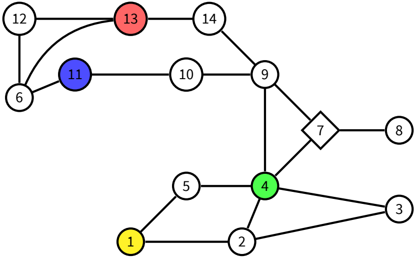
Example

Colored nodes have a PMU.



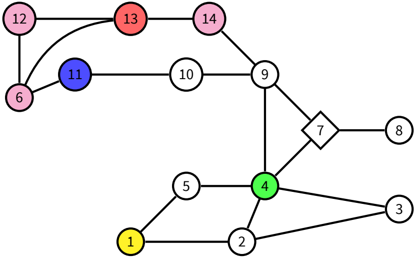
Example

Apply Rule 1.



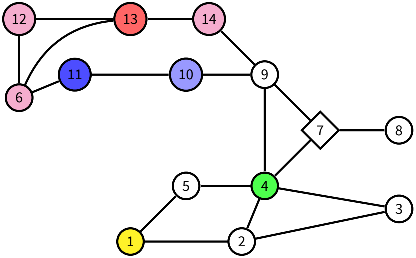
Example

Apply Rule 1.



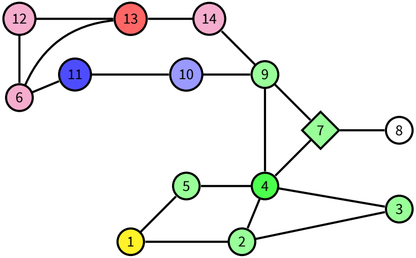
Example

Apply Rule 1.



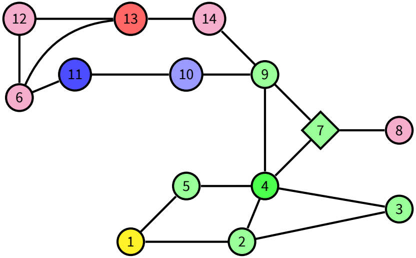
Example

Apply Rule 1.



Example

Apply Rule 1. Apply Rule 3.



Literature Review

State-of-the-art: Solution Techniques

D. Dua, S. Dambhare, R. K. Gajbhiye and S. A. Soman. *Optimal multistage scheduling PMU placement: An ILP approach*. IEEE Transactions on Power Delivery, 2008.

Model the problem with binary linear programming formulation. Consider the possibility of redundancy. **Limited propagation depth of the rules.**

N. Fan and J-P. Watson. *On integer programming models for the multi-channel PMU placement problem and their solution*. Energy Systems, 2006.

Propose a more general model with L-channel PMU's. The model is exponential. **Limited propagation depth of the rules.**

State-of-the-art: Solution Techniques

P-L. Poirion, S. Toubaline, C. D'Ambrosio and L. Liberti. *Observing the State of a Smart Grid Using Bilevel Programming*. Networks, 2016.

Model the 1-channel PMU problem with all nodes being zero-injection. First formulation properly modeling the propagation rules.

N. M. Manousakis, G. N. Korres and P. S. Georgilakis. *Taxonomy of PMU placement methodologies*. IEEE Transactions on Power Systems, 2012.
N. M. Manousakis, G. N. Korres and P. S. Georgilakis. *Optimal Placement of phasor measurement units: A literature review*. In 16th Int. Conference Intelligent System Application to Power Syst., 2011.

Provide a literature review of the formulations and solution techniques that have been presented in the literature.

State-of-the-art: computational complexity

D. J. Brueni and L. S. Heath. *The PMU placement problem*. SIAM Journal on Discrete Mathematics, 2005.

Prove that the PPP without zero-injection nodes is NP-hard even for planar bipartite graphs.

D. Gyllstrom, E. Rosensweig and J. Kurose . *On the impact of PMU placement on observability and cross-validation*. In Proceedings of the 3rd International Conference on Future Energy Systems, 2012.

The PPP is NP-hard as well as some variants of it. The variants are: 1)maximize the number of observed buses for a fixed number of PMUs, 2)minimize the number of PMUs such that full observability is ensured as well as cross validation, and 3)maximize the number of observed buses for a fixed number of PMUs and ensure cross-validation.

Contents

Decision variables

$$\forall i \in V \qquad x_i = \begin{cases} 1 & \text{if a PMU is installed in node } i \\ 0 & \text{otherwise} \end{cases}$$

Iterative Model

$$\forall i \in V, \forall d = 0, \ldots, T-1 \qquad w_{i,d} = \begin{cases} 1 & \text{if node } i \text{ is observed in iteration } d \\ 0 & \text{otherwise} \end{cases}$$

Iterative Model

$$\forall i \in V, \forall d = 0, \ldots, T-1 \qquad w_{i,d} = \begin{cases} 1 & \text{if node } i \text{ is observed in iteration } d \\ 0 & \text{otherwise} \end{cases}$$

$$\forall i \in S, \forall d = 0, \ldots, T-1 \qquad y^2_{i,d} = \begin{cases} 1 & \text{if Rule 2 is applied in iteration } d \\ & \text{to observe node } i \\ 0 & \text{otherwise} \end{cases}$$

Iterative Model

$$\forall i \in V, \forall d = 0, \ldots, T-1 \qquad w_{i,d} = \begin{cases} 1 & \text{if node } i \text{ is observed in iteration } d \\ 0 & \text{otherwise} \end{cases}$$

$$\forall i \in S, \forall d = 0, \ldots, T-1 \qquad y^2_{i,d} = \begin{cases} 1 & \text{if Rule 2 is applied in iteration } d \\ & \text{to observe node } i \\ 0 & \text{otherwise} \end{cases}$$

$$\forall i \in V, \forall j \in N(i) \cap S, \forall d = 0, \ldots, T-1 \qquad y^3_{i,j,d} = \begin{cases} 1 & \text{if Rule 3 is applied to} \\ & \text{node } j \text{ in iteration } d \\ & \text{to observe node } i \\ 0 & \text{otherwise} \end{cases}$$

Iterative Model

$$\min_{x,w,y^2,y^3 \text{ binary}} \quad \sum_{i \in V} x_i$$

$$\text{s. t} \quad w_{i,T-1} = 1, \quad \forall i \in V \qquad \text{Full observability}$$

$$x_i + \sum_{j \in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V \qquad \text{Rule 1}$$

Iterative Model

$$
\begin{aligned}
\min_{x,w,y^2,y^3 \text{ binary}} \quad & \sum_{i \in V} x_i & \\
\text{s. t} \quad & w_{i,T-1} = 1, \quad \forall i \in V & \text{Full observability} \\
& x_i + \sum_{j \in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V & \text{Rule 1} \\
& w_{i,d+1} \geq w_{i,d}, \quad \forall i \in V, d = 0, \ldots, T-2 & \text{Monotone}
\end{aligned}
$$

Iterative Model

$$\min_{x,w,y^2,y^3 \text{ binary}} \sum_{i \in V} x_i$$

s. t

$$w_{i,T-1} = 1, \quad \forall i \in V \qquad \text{Full observability}$$

$$x_i + \sum_{j \in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V \qquad \text{Rule 1}$$

$$w_{i,d+1} \geq w_{i,d}, \quad \forall i \in V, d = 0, \ldots, T-2 \qquad \text{Monotone}$$

$$w_{i,d+1} - w_{i,d} \leq y^2_{i,d} + \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in S, d = 0, \ldots, T-2 \qquad \text{Active rule}$$

Iterative Model

$$\min_{x,w,y^2,y^3 \text{ binary}} \sum_{i \in V} x_i$$

s. t

$$w_{i,T-1} = 1, \quad \forall i \in V \qquad \text{Full observability}$$

$$x_i + \sum_{j \in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V \qquad \text{Rule 1}$$

$$w_{i,d+1} \geq w_{i,d}, \quad \forall i \in V, d = 0, \ldots, T-2 \qquad \text{Monotone}$$

$$w_{i,d+1} - w_{i,d} \leq y^2_{i,d} + \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in S, d = 0, \ldots, T-2 \qquad \text{Active rule}$$

$$w_{i,d+1} - w_{i,d} \leq \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in V \setminus S, d = 0, \ldots, T-2 \qquad \text{Active rule}$$

Iterative Model

$$
\begin{aligned}
\min_{x,w,y^2,y^3 \text{ binary}} \quad & \sum_{i \in V} x_i \\
\text{s. t} \quad & w_{i,T-1} = 1, \quad \forall i \in V && \text{Full observability} \\
& x_i + \sum_{j \in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V && \text{Rule 1} \\
& w_{i,d+1} \geq w_{i,d}, \quad \forall i \in V, d = 0, \dots, T-2 && \text{Monotone} \\
& w_{i,d+1} - w_{i,d} \leq y^2_{i,d} + \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in S, d = 0, \dots, T-2 && \text{Active rule} \\
& w_{i,d+1} - w_{i,d} \leq \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in V \setminus S, d = 0, \dots, T-2 && \text{Active rule} \\
& y^3_{i,j,d} \leq w_{j,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, d = 0, \dots, T-2 && \text{Rule 3}
\end{aligned}
$$

Iterative Model

$$\min_{x,w,y^2,y^3 \text{ binary}} \quad \sum_{i \in V} x_i$$

s. t

$$w_{i,T-1} = 1, \quad \forall i \in V \qquad \text{Full observability}$$

$$x_i + \sum_{j \in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V \qquad \text{Rule 1}$$

$$w_{i,d+1} \geq w_{i,d}, \quad \forall i \in V, d = 0, \dots, T-2 \qquad \text{Monotone}$$

$$w_{i,d+1} - w_{i,d} \leq y^2_{i,d} + \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in S, d = 0, \dots, T-2 \qquad \text{Active rule}$$

$$w_{i,d+1} - w_{i,d} \leq \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in V \setminus S, d = 0, \dots, T-2 \qquad \text{Active rule}$$

$$y^3_{i,j,d} \leq w_{j,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, d = 0, \dots, T-2 \qquad \text{Rule 3}$$

$$y^3_{i,j,d} \leq w_{k,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, \forall k \in N(j) \setminus \{i\}, d = 0, \dots, T-2 \qquad \text{Rule 3}$$

Iterative Model

$$\min_{x,w,y^2,y^3 \text{ binary}} \sum_{i \in V} x_i$$

s. t

$$w_{i,T-1} = 1, \quad \forall i \in V \qquad \text{Full observability}$$

$$x_i + \sum_{j \in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V \qquad \text{Rule 1}$$

$$w_{i,d+1} \geq w_{i,d}, \quad \forall i \in V, d = 0, \dots, T-2 \qquad \text{Monotone}$$

$$w_{i,d+1} - w_{i,d} \leq y^2_{i,d} + \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in S, d = 0, \dots, T-2 \qquad \text{Active rule}$$

$$w_{i,d+1} - w_{i,d} \leq \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in V \setminus S, d = 0, \dots, T-2 \qquad \text{Active rule}$$

$$y^3_{i,j,d} \leq w_{j,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, d = 0, \dots, T-2 \qquad \text{Rule 3}$$

$$y^3_{i,j,d} \leq w_{k,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, \forall k \in N(j) \setminus \{i\}, d = 0, \dots, T-2 \qquad \text{Rule 3}$$

$$y^2_{i,d} \leq w_{j,d}, \quad \forall i \in S, \forall j \in N(i), d = 0, \dots, T-2 \qquad \text{Rule 2}$$

Iterative Model

Lemma

The binary requirement on the decision variables w can be relaxed to $[0, 1]$.

Iterative Model

Lemma

The binary requirement on the decision variables w can be relaxed to $[0, 1]$.

Lemma

It is enough to consider $T = |S| + 1$ for the binary programming formulation to be correct.

Bilevel Model

$$\min_{x \in \{0,1\}^{|V|}} \quad \sum_{i \in V} x_i$$

$$\text{subject to} \quad \sum_{i \in V} w_i \geq |V| \qquad \text{Full observability}$$

Bilevel Model

$$\min_{x \in \{0,1\}^{|V|}} \quad \sum_{i \in V} x_i$$

$$\text{subject to} \quad \sum_{i \in V} w_i \geq |V| \qquad \text{Full observability}$$

$$\min_{w \in \{0,1\}^{|V|}} \quad \sum_{i \in V} w_i$$

$$\text{s.t.} \quad w_i \geq x_i, \quad \forall i \in V \qquad \text{Rule 1}$$

$$w_i \geq x_j, \quad \forall i \in V, \forall j \in N(i) \qquad \text{Rule 1}$$

Bilevel Model

$$\min_{x \in \{0,1\}^{|V|}} \quad \sum_{i \in V} x_i$$

$$\text{subject to} \quad \sum_{i \in V} w_i \geq |V| \qquad \text{Full observability}$$

$$\min_{w \in \{0,1\}^{|V|}} \quad \sum_{i \in V} w_i$$

$$\text{s.t.} \quad w_i \geq x_i, \quad \forall i \in V \qquad \text{Rule 1}$$

$$w_i \geq x_j, \quad \forall i \in V, \forall j \in N(i) \qquad \text{Rule 1}$$

$$w_i \geq 1 - |N(i)| + \sum_{j \in N(i)} w_j, \quad \forall i \in S \qquad \text{Rule 2}$$

Bilevel Model

$$\min_{x \in \{0,1\}^{|V|}} \quad \sum_{i \in V} x_i$$

subject to $\sum_{i \in V} w_i \geq |V|$ Full observability

$$\min_{w \in \{0,1\}^{|V|}} \quad \sum_{i \in V} w_i$$

s.t.

$$w_i \geq x_i, \quad \forall i \in V \qquad \text{Rule 1}$$

$$w_i \geq x_j, \quad \forall i \in V, \forall j \in N(i) \qquad \text{Rule 1}$$

$$w_i \geq 1 - |N(i)| + \sum_{j \in N(i)} w_j, \quad \forall i \in S \qquad \text{Rule 2}$$

$$w_i \geq 1 - |N(j)| + w_j + \sum_{k \in N(j) \setminus \{i\}} w_k, \quad \forall i \in V, \forall j \in N(i) \cap S \qquad \text{Rule 3}$$

Bilevel Model: solution approach

Possible directions to solve the bilevel model:

1. There is a MILP which is equivalent to the bilevel model. However, it demands the use of big M values.
2. Solve

$$\min_{x \in \{0,1\}^{|V|}} \sum_{i \in V} x_i \tag{1}$$

$$\text{s.t.} \quad \sum_{i \in V : \bar{x}_i = 0} x_i \geq 1, \qquad \forall \bar{x} \text{ infeasible.} \tag{2}$$

Bilevel Model: solution approach

Possible directions to solve the bilevel model:

1. There is a MILP which is equivalent to the bilevel model. However, it demands the use of big M values.
2. Solve

$$\min_{x \in \{0,1\}^{|V|}} \quad \sum_{i \in V} x_i \tag{1}$$

$$\text{s.t.} \quad \sum_{i \in V : \bar{x}_i = 0} x_i \geq 1, \qquad \forall \bar{x} \text{ infeasible.} \tag{2}$$

Improvements: 1) $\bar{x}$ must be maximal.

Bilevel Model: solution approach

Possible directions to solve the bilevel model:

1. There is a MILP which is equivalent to the bilevel model. However, it demands the use of big M values.
2. Solve

$$\min_{x \in \{0,1\}^{|V|}} \quad \sum_{i \in V} x_i \tag{1}$$

$$\text{s.t.} \quad \sum_{i \in V : \bar{x}_i = 0} x_i \geq 1, \qquad \forall \bar{x} \text{ infeasible.} \tag{2}$$

Improvements: 1) $\bar{x}$ must be maximal.

2) $$\sum_{i \in V : \bar{x}_i = 0 \wedge \exists j \in N[i] : \bar{w}_j = 0} x_i \geq 1$$

Computational Results

IEEE System	$\lvert S \rvert$	Dua et al. # PMUs	It. Model # PMUs	It. Model time	Bi. Model # PMUs	Bi. Model iter	Bi. Model time	Imp. Bi. Model # PMUs	Imp. Bi. Model iter	Imp. Bi. Model time
14 Bus	1	3	3	0.01	3	216	7.08	3	11	0.01
30 Bus	0	-	10	0.00			tl	10	19	0.02
57 Bus	15	14	11	0.97			tl	11	41	0.08
118 Bus	10	29	29	0.24			tl	29	86	0.30

Table 1: Comparison with Dua et al. of the iterative and bilevel models.

Contents

Robust PMU Placement Problem

Robust and Redundancy

The redundancy of a node i reflects the number of PMUs that observe i.

Robust and Redundancy

The redundancy of a node i reflects the number of PMUs that observe i.

The robustness of a node i measures the number of PMUs that can be attacked and i is still observed.

Robust and Redundancy

The redundancy of a node i reflects the number of PMUs that observe i.

The robustness of a node i measures the number of PMUs that can be attacked and i is still observed.

Motivation: PMUs can fail and full observability must still be guaranteed.

Robust Vs Redundancy

Redundancy = 1, i.e., each node must be observed twice.

Robust Vs Redundancy

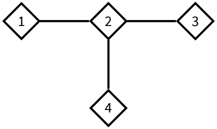
Redundancy = 1, i.e., each node must be observed twice.
Robustness = 1, i.e., at most one PMU will be attacked.

Robust Vs Redundancy

Redundancy = 1, i.e., each node must be observed twice.
Robustness = 1, i.e., at most one PMU will be attacked.

Instance

$S = \{1, 2, 3, 4\}$



Redundant Sol.

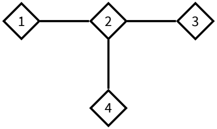
Robust Sol.

Robust Vs Redundancy

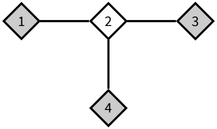
Redundancy = 1, i.e., each node must be observed twice.
Robustness = 1, i.e., at most one PMU will be attacked.

Instance

$S = \{1, 2, 3, 4\}$



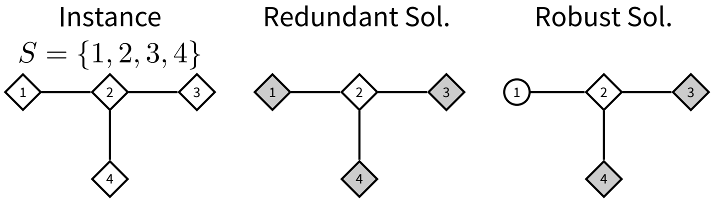
Redundant Sol.



Robust Sol.

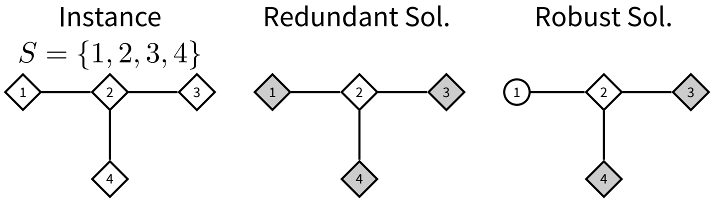
Robust Vs Redundancy

Redundancy = 1, i.e., each node must be observed twice.
Robustness = 1, i.e., at most one PMU will be attacked.



Robust Vs Redundancy

Redundancy = 1, i.e., each node must be observed twice.
Robustness = 1, i.e., at most one PMU will be attacked.



The robust solution is less expensive.

Attacker’s Problem: unable full observability

Attacker's Problem

Given a PMU placement strategy x,

$$\min_{w,\ \in\{0,1\}^{|V|}} \quad \sum_{i \in V} w_i$$

s.t.

$$\begin{aligned}
w_i &\geq x_i , && \forall i \in V \\
w_i &\geq x_j , && \forall i \in V, \forall j \in N(i) \\
w_i &\geq 1 - |N(i)| + \sum_{j \in N(i)} w_j, && \forall i \in S \\
w_i &\geq 1 - |N(j)| + w_j + \sum_{k \in N(j) \setminus \{i\}} w_k, && \forall i \in V, \forall j \in N(i) \cap S
\end{aligned}$$

Attacker's Problem

Given a PMU placement strategy x, the attacker solves:

$$
\begin{aligned}
\min_{w,z\in\{0,1\}^{|V|}} \quad & \sum_{i\in V} w_i \\
\text{s.t.} \quad & \sum_{i\in V} z_i \leq \rho \\
& w_i \geq x_i(1 - z_i), \quad \forall i \in V \\
& w_i \geq x_j(1 - z_j), \quad \forall i \in V, \forall j \in N(i) \\
& w_i \geq 1 - |N(i)| + \sum_{j\in N(i)} w_j, \quad \forall i \in S \\
& w_i \geq 1 - |N(j)| + w_j + \sum_{k\in N(j)\setminus\{i\}} w_k, \quad \forall i \in V, \forall j \in N(i) \cap S
\end{aligned}
$$

Attacker's decision problem

Problem: ATTACK

Instance: A graph $G = (V, E)$, a set of zero-injection nodes $S \subseteq V$, a set of nodes $X \subseteq V$ with PMUs and two positive integers ρ and K.

Question: Does there exist a set $Z \subseteq X$ such that $|Z| \leq \rho$ and the set of observed nodes when the PMUs are in $X \setminus Z$ decreases by K?

Attacker's problem complexity

Theorem

The ATTACK *is NP-complete.*

Proof: Knapsack is contained in ATTACK.

Problem: KNAPSACK

Instance: A sequence of positive integers $p_1, \ldots, p_n, w_1, \ldots, w_n$ and two positive integers C and P.

Question: Does there exist a set $J \subseteq \{1, \ldots, n\}$ such that

$$\sum_{i \in J} w_i \leq C \text{ and } \sum_{i \in J} p_i \geq P \text{ ?}$$

Attacker's problem complexity

Figure 2: The blue nodes have a PMU. The blue and grey nodes are observed. The red nodes are attacked.

Attacker's problem complexity

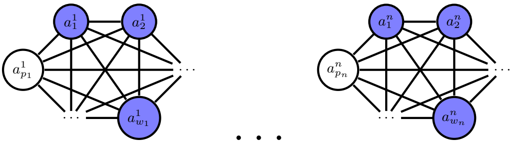


Figure 2: The blue nodes have a PMU. The blue and grey nodes are observed. The red nodes are attacked.

Attacker's problem complexity

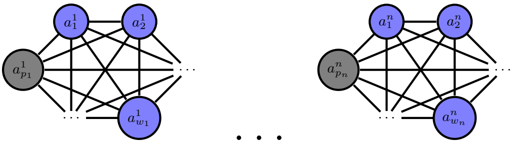


Figure 2: The blue nodes have a PMU. The blue and grey nodes are observed. The red nodes are attacked.

Attacker's problem complexity

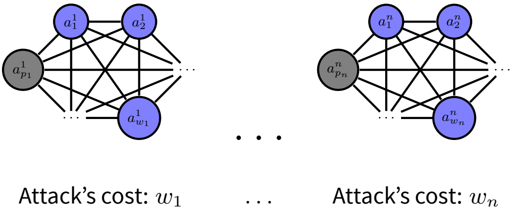


Figure 2: The blue nodes have a PMU. The blue and grey nodes are observed. The red nodes are attacked.

Attacker's problem complexity

Attack's cost: w_1 . . . Attack's cost: w_n

Figure 2: The blue nodes have a PMU. The blue and grey nodes are observed. The red nodes are attacked.

Attacker's problem complexity

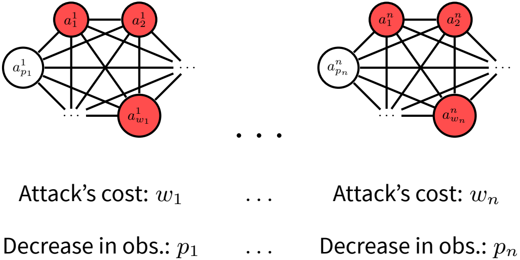


Figure 2: The blue nodes have a PMU. The blue and grey nodes are observed. The red nodes are attacked.

Solving the Robust PPP: Iterative and Bilevel Models

Iterative Model

$$
\begin{aligned}
\min_{x,w,y^2,y^3 \text{ binary}} \quad & \sum_{i \in V} x_i \\
\text{subject to} \quad & w_{i,T-1} = 1, \quad \forall i \in V, \\
& x_i + \sum_{j \in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V, \\
& w_{i,d+1} \geq w_{i,d}, \quad \forall i \in V, d = 0, \ldots, T-2, \\
& w_{i,d+1} - w_{i,d} \leq y^2_{i,d} + \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in S, d = 0, \ldots, T-2, \\
& w_{i,d+1} - w_{i,d} \leq \sum_{j \in N(i) \cap S} y^3_{i,j,d}, \quad \forall i \in V \setminus S, d = 0, \ldots, T-2, \\
& y^3_{i,j,d} \leq w_{j,d}, \quad \forall i \notin V, \forall j \in N(i) \cap S, d = 0, \ldots, T-2, \\
& y^3_{i,j,d} \leq w_{k,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, \forall k \in N(j) \setminus \{i\}, d = 0, \ldots, T-2, \\
& y^2_{i,d} \leq w_{j,d}, \quad \forall i \in S, \forall j \in N(i), d = 0, \ldots, T-2,
\end{aligned}
$$

Iterative Model

$$
\begin{aligned}
\min_{x,w,y^2,y^3 \text{ binary}} \quad & \sum_{i\in V} x_i \\
\text{subject to} \quad & w^u_{i,T-1} = 1, \quad \forall i \in V, \forall u \in \mathcal{U} \\
& x_i + \sum_{j\in N(i)} x_j \geq w_{i,0}, \quad \forall i \in V, \\
& w_{i,d+1} \geq w_{i,d}, \quad \forall i \in V, d = 0,\ldots,T-2, \\
& w_{i,d+1} - w_{i,d} \leq y^2_{i,d} + \sum_{j\in N(i)\cap S} y^3_{i,j,d}, \quad \forall i \in S, d = 0,\ldots,T-2, \\
& w_{i,d+1} - w_{i,d} \leq \sum_{j\in N(i)\cap S} y^3_{i,j,d}, \quad \forall i \in V \setminus S, d = 0,\ldots,T-2, \\
& y^3_{i,j,d} \leq w_{j,d}, \quad \forall i \notin V, \forall j \in N(i) \cap S, d = 0,\ldots,T-2, \\
& y^3_{i,j,d} \leq w_{k,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, \forall k \in N(j) \setminus \{i\}, d = 0,\ldots,T-2, \\
& y^2_{i,d} \leq w_{j,d}, \quad \forall i \in S, \forall j \in N(i), d = 0,\ldots,T-2,
\end{aligned}
$$

where $\mathcal{U}$ are all possible attack scenarios.

Iterative Model

$$
\begin{aligned}
\min_{x,w,y^2,y^3 \text{ binary}} \quad & \sum_{i \in V} x_i \\
\text{subject to} \quad & w^u_{i,T-1} = 1, \quad \forall i \in V, \forall u \in \mathcal{U} \\
& x_i (1 - z^u_i) + \sum_{j \in N(i)} x_j (1 - z^u_j) \geq w^u_{i,0}, \quad \forall i \in V, \forall u \in \mathcal{U} \\
& w^u_{i,d+1} \geq w^u_{i,d}, \quad \forall i \in V, d = 0, \dots, T-2, \forall u \in \mathcal{U} \\
& w^u_{i,d+1} - w^u_{i,d} \leq y^{2,u}_{i,d} + \sum_{j \in N(i) \cap S} y^{3,u}_{i,j,d}, \quad \forall i \in S, d = 0, \dots, T-2, \forall u \in \mathcal{U} \\
& w^u_{i,d+1} - w^u_{i,d} \leq \sum_{j \in N(i) \cap S} y^{3,u}_{i,j,d}, \quad \forall i \in V \setminus S, d = 0, \dots, T-2, \forall u \in \mathcal{U} \\
& y^{3,u}_{i,j,d} \leq w^u_{j,d}, \quad \forall i \notin V, \forall j \in N(i) \cap S, d = 0, \dots, T-2, \forall u \in \mathcal{U} \\
& y^{3,u}_{i,j,d} \leq w^u_{k,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, \forall k \in N(j) \setminus \{i\}, d = 0, \dots, T-2, \forall u \in \mathcal{U} \\
& y^{2,u}_{i,d} \leq w^u_{j,d}, \quad \forall i \in S, \forall j \in N(i), d = 0, \dots, T-2, \forall u \in \mathcal{U}
\end{aligned}
$$

where $\mathcal{U}$ are all possible attack scenarios.

Iterative Model

$$
\begin{aligned}
\min_{x,w,y^2,y^3 \text{ binary}} \quad & \sum_{i \in V} x_i \\
\text{subject to} \quad & w^u_{i,T-1} = 1, \quad \forall i \in V, \forall u \in \mathcal{U} \\
& x_i (1 - z^u_i) + \sum_{j \in N(i)} x_j (1 - z^u_j) \geq w^u_{i,0}, \quad \forall i \in V, \forall u \in \mathcal{U} \\
& w^u_{i,d+1} \geq w^u_{i,d}, \quad \forall i \in V, d = 0, \ldots, T-2, \forall u \in \mathcal{U} \\
& w^u_{i,d+1} - w^u_{i,d} \leq y^{2,u}_{i,d} + \sum_{j \in N(i) \cap S} y^{3,u}_{i,j,d}, \quad \forall i \in S, d = 0, \ldots, T-2, \forall u \in \mathcal{U} \\
& w^u_{i,d+1} - w^u_{i,d} \leq \sum_{j \in N(i) \cap S} y^{3,u}_{i,j,d}, \quad \forall i \in V \setminus S, d = 0, \ldots, T-2, \forall u \in \mathcal{U} \\
& y^{3,u}_{i,j,d} \leq w^u_{j,d}, \quad \forall i \notin V, \forall j \in N(i) \cap S, d = 0, \ldots, T-2, \forall u \in \mathcal{U} \\
& y^{3,u}_{i,j,d} \leq w^u_{k,d}, \quad \forall i \in V, \forall j \in N(i) \cap S, \forall k \in N(j) \setminus \{i\}, d = 0, \ldots, T-2, \forall u \in \mathcal{U} \\
& y^{2,u}_{i,d} \leq w^u_{j,d}, \quad \forall i \in S, \forall j \in N(i), d = 0, \ldots, T-2, \forall u \in \mathcal{U}
\end{aligned}
$$

where $\mathcal{U}$ are all possible attack scenarios. The size of $\mathcal{U}$ can be exponential...

Algorithm

Algorithm I

Input: A Robust PPP instance.
Output: A robust optimal solution or proves infeasibility

1. $\bar{\mathcal{U}} \leftarrow \emptyset$
2. $x \leftarrow$ solve the Robust Iterative Model with $\bar{\mathcal{U}}$.
   If infeasible, **return** infeasible.
3. $z, w \leftarrow$ solve the attacker's problem for x.
4. **If** $\sum_{i \in V} w_i = |V|$, **return** x.
   Else $\bar{\mathcal{U}} \leftarrow \bar{\mathcal{U}} \cup \{z\}$, and go to step 2.

Bilevel Model

$$\min_{x \in \{0,1\}^{|V|}} \quad \sum_{i \in V} x_i$$
$$\text{s. t.} \quad \sum_{i \in V} w_i \geq |V|$$

Bilevel Model

$$
\begin{aligned}
\min_{x\in\{0,1\}^{|V|}} \quad & \sum_{i\in V} x_i \\
\text{s. t.} \quad & \sum_{i\in V} w_i \geq |V| \\
& \min_{z,w\in\{0,1\}^{|V|}} \quad \sum_{i\in V} w_i \\
& \text{s.t.} \quad \sum_{i\in V} z_i \leq \rho \\
& \qquad w_i \geq x_i(1-z_i), \quad \forall i \in V \\
& \qquad w_i \geq x_j(1-z_j), \quad \forall i \in V, \forall j \in N(i) \\
& \qquad w_i \geq 1 - |N(i)| + \sum_{j\in N(i)} w_j, \quad \forall i \in S \\
& \qquad w_i \geq 1 - |N(j)| + w_j + \sum_{k\in N(j)\setminus\{i\}} w_k, \quad \forall i \in V, \forall j \in N(i) \cap S
\end{aligned}
$$

Algorithm

Algorithm II

Input: A Robust PPP instance and a lower bound $LB \geq 1$ on the number of PMUs.
Output: A robust optimal solution or proves infeasibility

1. $\bar{\mathcal{U}} \leftarrow \{x \in \{0,1\}^{|V|} : \sum_{i \in V} x_i \geq \rho + LB\}$
2. $\bar{x} \leftarrow argmin_{x \in \bar{\mathcal{U}}} \sum_{i \in V} x_i$.
   If infeasible, **return** infeasible.
3. $\bar{z}, \bar{w} \leftarrow$ solve attacker's problem for $\bar{x}$.
   If $\sum_{i \in V} w_i = |V|$, **return** $\bar{x}$.
4. Make $\bar{x}$ maximal.
   If $\bar{x}$ is not capable of guaranteeing full observability, **return** infeasible.
   $\bar{\mathcal{U}} = \leftarrow \bar{\mathcal{U}} \cap \{x : \sum_{i \in V : \bar{x}=0 \wedge \exists j \in N[i]; \bar{w}_j = 0} x_i \geq 1\}$
   Go to step 2.

Computational Results

IEEE System	$\rho = 0$		Robust Solution $\rho = 1$						Robust Solution $\rho = 5$					
	num. PMUs	$\lvert S \rvert$	num. PMUs	time	iter	R1+R2 UB	R1+R2 time	R1+R2 iter	num. PMUs	time	iter	R1+R2 UB	R1+R2 time	R1+R2 iter
14 Bus	3	1	7	0.92	12	7	0.19	12	inf	0.80	11	inf	0.22	10
30 Bus	10	0	21	0.43	24	-	-	-	inf	0.17	8	-	-	-
57 Bus	11	15		tl		27	24.12	43	inf*			inf	4.44	16
118 Bus	29	10				62	34.01	80	inf*			inf	5.68	26

Table 2: Algorithm I. The value inf* means that even if there is a PMU in all nodes, there is an attack that strictly decreases observability.

IEEE System	Our Model		Robust Solution $\rho = 1$			Robust Solution $\rho = 5$		
	num. PMUs	$\lvert S \rvert$	num. PMUs	time	iter	num. PMUs	time	iter
14 Bus	3	1	7	0.80	12	inf	0.04	1
30 Bus	10	0	21	0.39	24	inf	0.13	6
57 Bus	11	15		tl		inf	tl	
118 Bus	29	10						

Table 3: Algorithm I with valid cuts of Algorithm II.

Computational Results

IEEE System	Our Model num. PMUs	Our Model $\|S\|$	Robust Solution $\rho = 1$ num. PMUs	Robust Solution $\rho = 1$ time	Robust Solution $\rho = 1$ iter	Robust Solution $\rho = 5$ num. PMUs	Robust Solution $\rho = 5$ time	Robust Solution $\rho = 5$ iter
14 Bus	3	1	7	0.15	25	inf	0.02	1
30 Bus	10	0	21	0.61	68	inf	0.02	2
57 Bus	11	15	23	4.20	163	inf	0.12	5
118 Bus	29	10	62	9.55	254	inf	0.13	3

Table 4: Algorithm II.

Contents

Conclusions

For the first time a robust approach was analyzed, instead of a redundant one.

Conclusions

For the first time a robust approach was analyzed, instead of a redundant one.

The mathematical formulations properly reflect propagation rules. For PPP and robust PPP two formulations were proposed: an iterative and a bilevel programming program.

Conclusions

For the first time a robust approach was analyzed, instead of a redundant one.

The mathematical formulations properly reflect propagation rules. For PPP and robust PPP two formulations were proposed: an iterative and a bilevel programming program.

We proved that the attacker's problem is NP-complete.

Conclusions

For the first time a robust approach was analyzed, instead of a redundant one.

The mathematical formulations properly reflect propagation rules. For PPP and robust PPP two formulations were proposed: an iterative and a bilevel programming program.

We proved that the attacker's problem is NP-complete.

We were able to solve PPP and robust PPP for the 4 IEEE instances generally used in the literature.

On going work

More computational tests to validate our solution techniques.

On going work

More computational tests to validate our solution techniques.

Is the Robust PPP decision problem Σ_p^2-complete?

On going work

More computational tests to validate our solution techniques.

Is the Robust PPP decision problem Σ_p^2-complete?

Study of alternative formulations, new valid cuts, lower bounds, upper bounds...

On going work

More computational tests to validate our solution techniques.

Is the Robust PPP decision problem Σ_p^2-complete?

Study of alternative formulations, new valid cuts, lower bounds, upper bounds...

Consider the possibility of L-channel PMUs and their combination with other measuring devices.

Thank you for your attention

Acknowledgements This work was partially supported by national funds through FCT (Portuguese Foundation for Science and Technology) within projects PTDC/IIMGES/2830/ 2014 (mKEP - Models and optimisation algorithms for multi-country kidney exchange programs). The first author acknowledges the ERDF European Regional Development Fund through the Operational Programme for Competitiveness and Internationalisation - COMPETE 2020 Programme within project POCI-01-0145-FEDER-006961, and National Funds through the FCT (Portuguese Foundation for Science and Technology) as part of project UID/EEA/50014/2013.