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The Relative Contemporaneous Information Response. A New Cointegration-Based Measure of Price Discovery

ESTUDOS DO GEMF
N.º 04 2012

PUBLICAÇÃO CO-FINANCIADA PELA
FUNDAÇÃO PARA A CIÊNCIA E TECNOLOGIA

Impresso na Secção de Textos da FEUC
COIMBRA 2012

The relative contemporaneous information response.
A new cointegration-based measure of price discovery

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March, 2012

Abstract

This paper describes the cointegration-based technologies commonly used to assess the relative price discovery across markets, namely the Hasbrouck information shares and Gonzalo-Granger long memory common factor weights, and presents a new metric denominated contemporaneous information response. These metrics are compared via simulation experiments. It is shown that, under fairly regular market conditions, the contemporaneous information response is a reliable measure of the relative incorporation of information, and in most cases is more resilient to microstructural noise than the other two metrics.

JEL classification: G13; G14; G15; G21

Keywords: Price discovery; High frequency data; Information shares, Common factors, FTSE 100; Stock index futures; Market microstructure.

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1. Introduction

Price discovery relates to the ability, timing and efficiency of financial markets to incorporate and reveal information about the fundamental value of assets. In perfect and continuous markets, price processes should adapt seamlessly to the stochastic process of the underlying efficient price; market imperfections and noise, however, prevent prices from being fully adjusted.

Although cross-correlation functions and lead-lag regressions can provide a reasonable characterization of the temporal distribution of the interrelationship between markets and can support Granger-causality tests, they are often criticized for not providing an obvious metric for assessing the relative informational importance of each market. This criticism led to the construction of normalized measures of the relative price discovery process taken from vector error correction specifications.

Section 2 presents the most commonly used price discovery metrics: Hasbrouck information shares and Gonzalo-Granger long memory common factor weights. Section 3 presents a permanent-transitory (P-T) decomposition of a cointegrated bivariate system specially designed to analyse the relative price discovery across markets. The P-T decomposition is then used to interpret and discuss the above price discovery metrics and to construct the relative contemporaneous information response metric. Section 4 contains a comparative analysis of the three price discovery measures using simulation methods and discusses their reliability under several market conditions. Section 5 presents the most relevant conclusions.

2. Information shares and common factor weights

As in Lehmann (2002), suppose that an asset is traded on two different venues $i=1,2$. Market fragmentation implies the existence of two prices $p_{i,t}$, which share the same implicit efficient price, m_t. This efficient price is usually assumed to follow a martingale difference. Therefore, no function, linear or nonlinear, of previously released public information can help forecasting the current efficient price. Prices are most probably distinct given the idiosyncrasy of microstructural noise in each market, $s_{i,t} = p_{i,t} - m_t$. The requirement to distinguish the two sources of uncertainty, at least theoretically, is that m_t should permanently affect prices, whilst $s_{i,t}$ is merely transient, although it could be just white noise or have some kind of short-run structure.

Usually some additional assumptions are brought into this basic framework with a view to enhancing its operational features. For example, the efficient price can be assumed to follow a random walk with zero mean and constant variance and serially uncorrelated increments. The transient component can be assumed to follow a covariance stationary process. Hence,

$$\mathbf{p}_t = \boldsymbol{\iota} m_t + \mathbf{s}_t\,,\ \mathbf{p}_t = \begin{bmatrix} p_{1,t} \\ p_{2,t} \end{bmatrix},\ \mathbf{s}_t = \begin{bmatrix} s_{1,t} \\ s_{2,t} \end{bmatrix};$$

$$\Delta m_t = \eta_t^P\,,\ E\left(\eta_t^P\right) = 0\,,\ E\left[\left(\eta_t^P\right)^2\right] = \sigma^2_{\eta^P}\,, E(\eta_t^P \eta_\tau^P) = 0\,,\ \forall \tau \neq t\,; \tag{1}$$

$$\mathbf{s}_t = \boldsymbol{\Gamma}(L)\boldsymbol{\upsilon}_t\text{; and}$$

$$E\left(\eta_t^P\right) = 0\,,\ E\left(\boldsymbol{\upsilon}_t \boldsymbol{\upsilon}_t'\right) = \boldsymbol{\Sigma}_{\boldsymbol{\upsilon}}\,,\ E\left(\boldsymbol{\upsilon}_t \boldsymbol{\upsilon}_\tau'\right) = \mathbf{0}\,,\ \forall \tau \neq t\,.$$

Where $\mathbf{\iota}$ is a column vector of ones, $\mathbf{\Gamma}(L)$ is a matrix polynomial in the lag operator, such that $\mathbf{\Gamma}(L)=\sum_{j=0}^{\infty}\mathbf{\Gamma}_j L^j$, with $\mathbf{\Gamma}_0=\mathbf{I}$ ($\mathbf{I}$ is the identity matrix). This basic model does not exclude the possibility of correlation between η_t^P and $\mathbf{\upsilon}_t$, and therefore the transient component can also be a function of permanent innovations.[2]

In accordance with the structural model (1), the Wold representation of price increments is

$$\Delta\mathbf{p}_t=\mathbf{\iota}\eta_t^P+\Delta\mathbf{s}_t=\mathbf{\iota}\eta_t^P+(1-L)\mathbf{\Gamma}(L)\mathbf{\upsilon}_t . \tag{2}$$

In reduced form, price levels and price increments can also be represented as infinite moving average processes:

$$\mathbf{p}_t=\mathbf{C}(1)\sum_{\tau=0}^{t}\mathbf{\varepsilon}_\tau+\mathbf{C}^*(L)\mathbf{\varepsilon}_t \text{, and} \tag{3}$$

$$\Delta\mathbf{p}_t=\mathbf{C}(L)\mathbf{\varepsilon}_t=\mathbf{C}(1)\mathbf{\varepsilon}_t+(1-L)\mathbf{C}^*(L)\mathbf{\varepsilon}_t , \tag{4}$$

with $\mathbf{C}(0)=\mathbf{I}$, $\mathbf{C}^*(L)=\sum_{j=0}^{\infty}\mathbf{C}^*{}_j L^j$, $\mathbf{C}_j^*=-\sum_{i=j+1}^{\infty}\mathbf{C}_i$.

This representation clearly shows the decomposition of the price vector into a permanent component, integrated of order one, $\mathbf{\iota}m_t=\mathbf{C}(1)\sum_{\tau=0}^{t}\mathbf{\varepsilon}_\tau$, and a transitory component, integrated of order zero, $\mathbf{s}_t=\mathbf{C}^*(L)\mathbf{\varepsilon}_t$ (Stock and Watson, 1988). The matrix $\mathbf{C}(1)$ has rank 1 in the bivariate case and represents the long-run impact of disturbances on the price vector:

[2] For example, bid-ask spreads may reflect the attempt of market makers to offset their expected losses to informed traders with expected gains from liquidity traders. Accordingly spreads should be interpreted as an information-related cost (Copeland and Galai, 1983).

$$\lim_{\tau\to\infty} \frac{\partial E\left[\mathbf{p}_{t+\tau} \mid \boldsymbol{\theta}_t\right]}{\partial \boldsymbol{\varepsilon}_t'} = \lim_{\tau\to\infty} \sum_{\tau=0}^{\infty} \frac{\partial E\left[\Delta\mathbf{p}_{t+\tau} \mid \boldsymbol{\theta}_t\right]}{\partial \boldsymbol{\varepsilon}_t'} = \sum_{k=0}^{\infty} \mathbf{C}_k = \mathbf{C}(1). \quad (5)$$

If the elements of $\mathbf{p}_t$ refer to the same security then these prices are fundamentally equal and therefore share a common trend with a normalized cointegrating vector $\boldsymbol{\alpha} = \left[1 \quad -1\right]'$. The linear combination $z_t = \boldsymbol{\alpha}'\mathbf{p}_t$ is stationary and by definition does not have a stochastic trend, therefore $\boldsymbol{\alpha}'\mathbf{C}(1) = 0$. This implies that the long-run impact matrix has two identical rows $\mathbf{c} = \left[c_1 \quad c_2\right]'$ ($\mathbf{c}$ is sometimes referred to as the coefficients of the efficient price). The permanent structural innovations and the contemporaneous reduced form innovations are related via the long-run impact matrix as $\boldsymbol{\iota}\eta_t^P = \mathbf{C}(1)\boldsymbol{\varepsilon}_t$, which, given the equality between the rows of the long-run impact matrix, implies a perfect correlation between the current efficient price innovation and a linear combination of current disturbances: $\eta_t^P = \mathbf{c}'\boldsymbol{\varepsilon}_t$. The variance of the common trend is independent of the transient parameters $\sigma_{\eta^P}^2 = E(\boldsymbol{\iota}\boldsymbol{\iota}'\eta_t^{P2}) = E\left(\mathbf{C}(1)\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{C}(1)'\right) = \mathbf{c}'\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\mathbf{c}$, where $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$ represents the innovation variance-covariance matrix.

Hasbrouck (1995) proposes a technology to measure relative price discovery based on the variance decomposition of the common trend. In the bivariate case, the common trend variance has three components:

$$\frac{c_1^2}{\mathbf{c}'\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\mathbf{c}}\sigma_{\varepsilon_1}^2 + 2\frac{c_1 c_2}{\mathbf{c}'\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\mathbf{c}}\rho\sigma_{\varepsilon_1}\sigma_{\varepsilon_2} + \frac{c_2^2}{\mathbf{c}'\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\mathbf{c}}\sigma_{\varepsilon_2}^2 = 1. \quad (6)$$

If price innovations are uncorrelated, $\rho = 0$, the procedure provides a clean cut between each market contribution to the long-run volatility, that is, between each market's information share:

$$IS_i = \frac{c_i^2 \sigma_{\varepsilon_i}^2}{c_1^2 \sigma_{\varepsilon_1}^2 + c_2^2 \sigma_{\varepsilon_2}^2}. \tag{7}$$

When price innovations are correlated, allocation of the relative price discovery process is not unique and the best one can do is to establish lower and higher bounds from the reordering and factorization of $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$. For this purpose, Hasbrouck (1995) recommends the Cholesky factorization $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} = \mathbf{FF}'$, where $\mathbf{F}$ is a lower triangular matrix. The information shares' bounds are computed according to $IS_i = \left([\mathbf{cF}]_i\right)^2 / \mathbf{c}'\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\mathbf{c}$, where $[\mathbf{cF}]_i$ refers to the ith element of the row matrix $\mathbf{cF}$. If $\rho > 0$ the maximum (minimum) value occurs when the variable is ordered first (last). In the presence of correlated errors, Baillie et al. (2002) suggest the use of the mid-point between the lower and upper bounds as a reasonable measure of each market contribution to price discovery.

Differing from the Stock-Watson decomposition where the common trend depends on lagged values of $\mathbf{p}_t$, Gonzalo and Granger (1995) propose an alternative decomposition into permanent and temporary components that are linear combinations of contemporaneous prices alone:

$$\mathbf{p}_t = \mathbf{A}_1 f_t + \mathbf{A}_2 z_t,\; f_t = \boldsymbol{\gamma}'_\perp \mathbf{p}_t,\; z_t = \boldsymbol{\alpha}'\mathbf{p}_{t-1}; \tag{8}$$

where $\mathbf{A}_1$ and $\mathbf{A}_2$ are loading matrices ($\perp$ denotes the orthogonal complement). This decomposition is achieved via an additional identifying assumption. It is assumed that the transient term, z_t, does not Granger-causes the common factor f_t, in the long-run. More precisely, this means that the cumulative impact of z_t on f_t is zero.

Considering the error correction representation of the price vector:

$$\Delta \mathbf{p}_t = \boldsymbol{\gamma}\boldsymbol{\alpha}'\mathbf{p}_{t-1} + \boldsymbol{\Phi}(L)\Delta\mathbf{p}_{t-1} + \boldsymbol{\varepsilon}_t ; \tag{9}$$

where $\boldsymbol{\Phi}(L)$ indicates a (2×2) block matrix polynomial in the lag operator *L*, and $\boldsymbol{\alpha}'\mathbf{C}(1) = \mathbf{0}$ and $\mathbf{C}(1)\boldsymbol{\gamma} = \mathbf{0}$. The non-causality assumption implies that $\mathbf{A}_1 = \boldsymbol{\alpha}_\perp(\boldsymbol{\gamma}'_\perp\boldsymbol{\alpha}_\perp)^{-1}$ and $\mathbf{A}_2 = \boldsymbol{\gamma}(\boldsymbol{\alpha}'\boldsymbol{\gamma})^{-1}$ where $\boldsymbol{\alpha}'_\perp\boldsymbol{\alpha} = 0$ and $\boldsymbol{\gamma}'_\perp\boldsymbol{\gamma} = 0$. If $\boldsymbol{\alpha} = [1 \quad -1]'$, then $\boldsymbol{\alpha}_\perp = \boldsymbol{\iota}$ and the (normalized) common factor weights vector is simply given by $\mathbf{A}_1\boldsymbol{\gamma}'_\perp = \boldsymbol{\iota}(\boldsymbol{\gamma}'_\perp\boldsymbol{\iota})^{-1}\boldsymbol{\gamma}'_\perp$. In the bivariate case, the common-factors are just

$$CF_1 = \frac{-\gamma_2}{(\gamma_1 - \gamma_2)} \text{ and } CF_2 = \frac{\gamma_1}{(\gamma_1 - \gamma_2)}. \tag{10}$$

The proof that common factor weights and information shares are closely related can be found in Baillie et al. (2002), Lehmann (2002) and de Jong (2002). Basically, these authors show that if $\boldsymbol{\alpha} = [1 \quad -1]'$ then $CF_i = \kappa c_i$, with $\kappa = (\mathbf{c}'\boldsymbol{\iota})^{-1}$, hence the Gonzalo-Granger common trend weights are simply equal to the normalized long-run impacts. Similarly, Hasbrouck's information shares, with uncorrelated errors, are given by $IS_i = \kappa\sigma_i^2 c_i^2$, where $\kappa = \mathbf{c}'\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\mathbf{c}$ is now the normalizing constant.

3. The relative contemporaneous information response

King et al. (1991) and Gonzalo and Ng (2001) propose a systematic framework for analyzing the dynamic effects of permanent and transitory shocks to a system of cointegrated variables. The identification problem is solved, not by imposing restrictions on the initial structural impacts to the system, as in the conventional VAR analysis, but instead by using the cointegration restrictions to impose constraints on the long-run multipliers. In other words, shocks are identified not by their origin but by their degree of persistence. The two approaches differ essentially in the treatment given to the cointegration restrictions: while King et al. (1991) derive the cointegrating vector from theoretical considerations and then superimpose it into the reduced form model, in Granger and Ng (2001) the cointegrating vector is estimated jointly with the other parameters. The P-T decomposition applied here to study the price discovery process in fundamentally related markets is based on those two papers.

Let $\mathbf{p}_t$ be a bivariate price process, with $\mathbf{p}_t \sim I(1)$; such that the bivariate return process $\mathbf{\Delta p}_t$ has a moving average representation:

$$\mathbf{\Delta p}_t = \mathbf{C}(L)\boldsymbol{\varepsilon}_t = \mathbf{C}(1)\boldsymbol{\varepsilon}_t + (1-L)\mathbf{C}^*(L)\boldsymbol{\varepsilon}_t, \tag{11}$$

where $\boldsymbol{\varepsilon}_t$ is i.i.d. with zero mean and variance $\mathbf{\Sigma}_{\boldsymbol{\varepsilon}}$. Let $\mathbf{p}_t$ be cointegrated, which implies that $\mathbf{C}(1)$ has rank 1, meaning that there exist two non-null (2×1) vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\gamma}$ such that $\boldsymbol{\alpha}'\mathbf{C}(1) = \mathbf{0}$ and $\mathbf{C}(1)\boldsymbol{\gamma} = \mathbf{0}$. Additionally, let $\mathbf{C}(z)$ be 1-summable and $\mathbf{C}^*(z)$ be full rank everywhere on $|z| \leq 1$.

Since $\mathbf{p}_t$ is integrated of order one, some of its innovations must have permanent effects, hence price increments have a structural bivariate moving average representation:

$$\Delta\mathbf{p}_t = \mathbf{D}(L)\boldsymbol{\eta}_t = \mathbf{D}^P(L)\boldsymbol{\eta}_t^P + \mathbf{D}^T(L)\boldsymbol{\eta}_t^T = \sum_{k=0}^{\infty}\mathbf{D}_k^P\boldsymbol{\eta}_{t-k}^P + \sum_{k=0}^{\infty}\mathbf{D}_k^T\boldsymbol{\eta}_{t-k}^T\,,\ \mathbf{D}_0 \neq \mathbf{I}\,, \tag{12}$$

where $\boldsymbol{\eta}_t = \begin{bmatrix}\boldsymbol{\eta}_t^P & \boldsymbol{\eta}_t^T\end{bmatrix}'$ represent the column vector of shocks. In order to guarantee the invertibility of $\mathbf{D}(L)$, it is assumed that the number of shocks equals the number of variables in the system, but only a subset has permanent effects. Thus in the bivariate case there is only one permanent shock η_t^P and one transitory shock η_t^T defined respectively by their long-run persistence: $\lim_{\tau\to\infty}\dfrac{\partial E[\mathbf{p}_{t+\tau} \mid \theta_t]}{\partial\eta_t^P} = \lim_{\tau\to\infty}\sum_{k=0}^{\tau}\mathbf{D}_k^P = \mathbf{D}^P(1) \neq \mathbf{0}$, $\lim_{\tau\to\infty}\dfrac{\partial E[\mathbf{p}_{t+\tau} \mid \theta_t]}{\partial\eta_t^T} = \lim_{\tau\to\infty}\sum_{k=0}^{\tau}\mathbf{D}_k^T = \mathbf{D}^T(1) = \mathbf{0}$. These structural shocks in $\boldsymbol{\eta}_t$ are assumed to be serially and mutually uncorrelated and to have equal variances, i.e. $E(\boldsymbol{\eta}_t\boldsymbol{\eta}_\tau') = \mathbf{0}\ \ \forall\tau \neq t$ and $E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') = \sigma\mathbf{I}$.[3]

The P-T decomposition, specially designed for analysing the relative price discovery process is accomplished, in practice, through several steps: (a) A VECM with a superimposed cointegrating vector $\boldsymbol{\alpha} = [1 \quad -1]'$ is estimated; (b) the moving average representation $\Delta\mathbf{p}_t = \hat{\mathbf{C}}(L)\boldsymbol{\varepsilon}_t$ is numerically computed by iterating forward the restricted VAR in first differences; (c) the VECM estimated errors, $\hat{\boldsymbol{\varepsilon}}_t$, are rotated in order to obtain the "unorthogonalized" permanent and transitory shocks, $\hat{\mathbf{u}}_t = \begin{bmatrix}\hat{u}_t^P & \hat{u}_t^T\end{bmatrix}'$, which is

[3] Gonzalo and Ng (2001) assume that structural shocks have unit variance. We relax that assumption, but still impose one restriction on the diagonal elements of the variance-covariance matrix of structural shocks.

achieved via $\hat{\mathbf{u}}_t = \hat{\mathbf{G}}\hat{\boldsymbol{\varepsilon}}_t = \begin{bmatrix} \hat{\mathbf{c}}' & \boldsymbol{\alpha}' \end{bmatrix}' \hat{\boldsymbol{\varepsilon}}_t$; (d) the orthogonalized shocks are obtained as $\hat{\boldsymbol{\eta}}_t = \hat{\mathbf{H}}^{-1}\hat{\mathbf{G}}\hat{\boldsymbol{\varepsilon}}_t$ and the structural moving average parameters are estimated by $\hat{\mathbf{D}}(L) = \hat{\mathbf{C}}(L)\hat{\mathbf{G}}^{-1}\hat{\mathbf{H}}$, where $\hat{\mathbf{H}}$ is the lower triangular matrix obtained from the Cholesky factorization of the variance-covariance matrix of $\mathbf{u}_t$; and, (e) finally, the short-run dynamics and the structural shocks are rescaled by a constant κ that guarantees a long-run impact of a permanent innovation on both prices equal to the magnitude of that innovation, i.e. $\mathbf{D}^P(1) = \boldsymbol{\iota}$.

The above procedure can be summarized as:

$$\Delta\mathbf{p}_t = \mathbf{C}(L)\mathbf{G}^{-1}\mathbf{H}\kappa\kappa^{-1}\mathbf{H}^{-1}\mathbf{G}\boldsymbol{\varepsilon}_t = \mathbf{D}(L)\overline{\mathbf{H}}\overline{\mathbf{H}}^{-1}\mathbf{u}_t = \mathbf{D}(L)\boldsymbol{\eta}_t . \quad (13)$$

Some assumptions underlying this P-T decomposition are now discussed in detail. Although the assumption that $\mathbf{C}(z)$ is 1-summable guarantees that $\mathbf{C}^*(z)$ is absolutely summable, this does not imply that $\mathbf{C}^*(L)$ is invertible or even fundamental. If just one zero of the moving average representation has a modulus greater than unity then that representation is "nonfundamental" in the sense that the original (structural disturbances) are not recoverable from the reduced form disturbances by any "orthogonalization" (Lippi and Reichlin, 1993). Blanchard and Quah (1993) observe that "nonfundamentalness" can be a recurring issue in common trend representations. In fact the representation (18) for $\mathbf{p}_t \sim I(1)$ implies that $\mathbf{C}^*(1)$ is full rank, but no reasonable regularity conditions a priori sustain the invertibility of $\mathbf{C}^*(L)$.

If there is only one permanent shock and one transitory shock, under the above assumptions, the P-T decomposition exactly identifies these shocks, because the usual imprecision associated with the Cholesky factorization does not apply in the bivariate case (Gonzalo and Ng, 2001). If there is more than one transient shock, the P-T decomposition is subject to an estimation error resulting from the non-invertibility of $\mathbf{D}(L)$. While it seems reasonable to expect that there is only one permanent shock because the two prices share the same common stochastic trend it is more difficult to accept that there is only one transient shock. Transient shocks in each market are driven by noise and there are no reasons to defend its uniqueness. Moreover, the impact of noise probably differs in the two markets because of their different trading protocols. Nevertheless, the unique permanent shock is still well defined by its long-run multiplier in the presence of several transient shocks. If there is more than one transient shock and permanent and transitory shocks are correlated, the estimation error is intensified in a non-trivial manner and there is no guarantee whatsoever that the dynamics obtained by the P-T decomposition resemble the original ones.

Using the P-T decomposition, Yan and Zivot (2010) provide an interesting and remarkably simple interpretation of Hasbrouck's information shares and Gonzalo-Granger's common factor weights.

The Wold representation (11) of the cointegrated system and the structural representation (12) imply that reduced form forecasting errors, $\boldsymbol{\varepsilon}_t$, and structural innovations, $\boldsymbol{\eta}_t$, are contemporaneously related via:

$$\boldsymbol{\varepsilon}_t = \mathbf{D}_0 \boldsymbol{\eta}_t = \begin{bmatrix} d_1^P & d_1^T \\ d_2^P & d_2^T \end{bmatrix} \begin{bmatrix} \eta_t^P \\ \eta_t^T \end{bmatrix} = \begin{bmatrix} d_1^P \eta_t^P + d_1^T \eta_t^T \\ d_2^P \eta_t^P + d_2^T \eta_t^T \end{bmatrix}. \tag{14}$$

Where d_i^P and d_i^T ($i=1,2$) are the contemporaneous responses of the price system to permanent and transient innovations respectively. Under the assumptions stated earlier, $\mathbf{D}_0$ is invertible; and thus

$$\boldsymbol{\eta}_t = \begin{bmatrix} \eta_t^P \\ \eta_t^T \end{bmatrix} = \mathbf{D}_0^{-1}\boldsymbol{\varepsilon}_t = \begin{bmatrix} \frac{\left(d_2^T \varepsilon_{1t} - d_1^T \varepsilon_{2t}\right)}{|\mathbf{D}_0|} & \frac{-d_2^P \varepsilon_{1t} + d_1^P \varepsilon_{2t}}{|\mathbf{D}_0|} \end{bmatrix}'. \tag{15}$$

Considering that $\eta_t^P = \mathbf{c}'\boldsymbol{\varepsilon}_t$ and (15) then

$$\frac{\left(d_2^T \varepsilon_{1t} - d_1^T \varepsilon_{2t}\right)}{|\mathbf{D}_0|} = c_1\varepsilon_{1t} + c_2\varepsilon_{2t} \Rightarrow \begin{bmatrix} c_1 & c_2 \end{bmatrix}' = \begin{bmatrix} \frac{d_2^T}{|\mathbf{D}_0|} & -\frac{d_1^T}{|\mathbf{D}_0|} \end{bmatrix}'. \tag{16}$$

The result (16) seems contradictory because permanent innovations and efficient price are related to the reduced form errors not only via the permanent innovation structural parameter, but also via the transient parameters. This paradox is explained by the fact that permanent innovations are isolated by purging for the transitory effects present in the reduced form errors. Hence, transient impacts are the mirror image of permanent impacts.

Given (16), the Gonzalo-Granger common factor weights (*CF*) have the following structural interpretation:

$$CF_1 = \frac{d_2^T}{(d_2^T - d_1^T)} \text{ and } CF_2 = \frac{-d_1^T}{(d_2^T - d_1^T)}. \tag{17}$$

Hence, common factor weights do not measure price discovery directly from the incorporation of fundamental information, instead they measure the relative response to transitory shocks. This methodology can conceivably give a completely misleading inference about the price discovery process. For instance, suppose that market 1 incorporates immediately all information and market 2 incorporates just a negligible fraction. If noise in market 1 is relatively higher, then the metric attributes the bulk of price discovery to market 2, and the bias increases as the transient impact in market 2 tends to an arbitrarily small number.

Harris et al. (2002) claim that their application of the Gonzalo-Granger common factor weights is able to measure the long-run impact of current price innovations on futures prices and therefore it can uncover the relative speed by which markets incorporate permanent innovations, that is information. However, it appears that the authors have relied on the relative noise and market frictions to explain the relative price discovery across markets. Quoting the authors:

> "Our work focuses on the adjustment of cointegrated trading prices to any event that causes a divergence from the law of one price. Suppose that there is a trade at the same price on each of three informationally-linked exchanges. Then as a result of an information event, there is a new trade on each exchange, but at different prices. Sometimes this divergence reflects differences of opinion in assessing the same information, sometimes differential market conditions and sometimes differences in depth at the prior quotes." (Harris et al., 2002, p. 278)

Under general conditions, the *CF* metric only answers the following question: What is the relative long-run impact on the price vector of a current price innovation? And additional information is needed to relate the long-run multipliers to the information arrival process, and consequently to the relative price discovery (Lehmann, 2002).

In order to give a structural interpretation to the information shares (*IS*) methodology, Yan and Zivot (2010), consider the simplest case of uncorrelated reduced form errors. They show that in this situation

$$IS_1 = \frac{d_1^P d_2^T}{|\mathbf{D}_0|} \text{ and } IS_2 = \frac{-d_1^T d_2^P}{|\mathbf{D}_0|}. \tag{18}$$

Expressions (18) show that information shares consist of responses to both permanent and transitory shocks, and so, even with uncorrelated reduced form errors its interpretation is not completely straightforward. Heuristically, in the *IS* methodology the long-run multipliers are calibrated by the variance-covariance matrix as a "proxy" for the intensity and timing of the relative information arrival to each market. However, this matrix is a function not both information arrival and noise, and so the difference in the disturbances volatility across markets can be the observable result of the two processes. In other words, one market can present a higher information share not because information is impounded quickly into prices but because the cost of trading and noise trading are lower.

Matrix $\mathbf{D}_0 \neq \mathbf{I}$ obtained from the P-T decomposition (13) can resolve the ambiguity introduced by the fact that the *IS* measure is computed from a reduced form model. The matrix $\mathbf{D}_0$ contains the initial impacts of structural shocks and is a function of the error correction coefficients, the cointegrating vector and the variance-covariance matrix of the reduced form disturbances. A relevant property of $\mathbf{D}_0$ is that it does not depend on the short-run dynamics and therefore does not suffer from possible biases introduced by "nonfundamentalness". This information can be used naturally by computing the relative contemporaneous information response (denoted hereafter by *IR*). It is easily shown (see Appendix) that in the bivariate case IR_i (with $i, j = 1,2$ and $i \neq j$) is computed, unambiguously, as follows:

$$IR_i = \frac{d_i^P}{d_i^P + d_j^P} = \frac{c_i \sigma_{\varepsilon_i}^2 + c_j \sigma_{\varepsilon_i,\varepsilon_j}}{c_i \sigma_{\varepsilon_i}^2 + \left(c_i + c_j\right) \sigma_{\varepsilon_i,\varepsilon_j} + c_j \sigma_{\varepsilon_j}^2} . \tag{19}$$

The next section provides some simulation results that intend to show that the *IR* is a good measure of the relative price discovery under fairly regular market conditions.[4]

[4] It is worth noticing that we do not claim that the contemporaneous permanent impacts d_i^P are efficiently estimated in absolute terms, our focus is only on their relative values.

4. Comparative analysis of price discovery metrics

This section presents a sensitivity analysis of the common-factor weights (*CF*), information shares (*IS*) and contemporaneous information response (*IR*) measures to different market variables. More specifically, simulation methods are used to study how these measures behave in the presence of different information-signal to noise ratios, unequal trading costs, different correlation structures, "stale prices", overreaction and asymmetric information.

Sensitivity analysis of price discovery metrics to all variables except asymmetric information is performed via simulations of the "partial adjustment with noise" model, with some modifications concerning the possibility of lagged initial reaction to information and correlation between the noise processes.

The model is described as follows:

$$\begin{aligned}
p_{1,t} &= p_{1,t-1} + \delta_1 \left(m_t - p_{1,t-1} \right) + b_1 q_{1,t} \\
p_{2,t} &= p_{2,t-1} + \delta_2 \left(L^{\varsigma} m_t - p_{2,t-1} \right) + b_2 q_{2,t} \\
m_t &= m_{t-1} + \eta_t^P \\
\eta_t^P &\sim N\left(0, \sigma_{\eta^P}^2\right) \\
\mathbf{q}_t &= \begin{bmatrix} q_{1,t} \\ q_{2,t} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_q\right) \\
\boldsymbol{\Sigma}_q &= \begin{bmatrix} 1 & \rho_{q_1,q_2} \\ \rho_{q_1,q_2} & 1 \end{bmatrix}
\end{aligned} \tag{20}$$

In this dynamic structural model, the price process $\{\mathbf{p}_t\}$ depends on information, specified by the latent fundamental price m_t, and on an unobservable bivariate noise process $\{\mathbf{q}_t\}$ that follows a bivariate normal distribution with normalized variance and correlation $-1 \leq \rho_{q_1,q_2} \leq 0$. Thus the model does not preclude the existence of contemporaneous linear dependence between the noise processes in both markets, nor does it eliminate the possibility of two completely idiosyncratic noise processes. The economic reason why contemporaneous cross-correlation is assumed to be non-positive is that, on average, noise should have a divergent impact on prices, i.e., prices should be related by information and distanced by noise.[5] Occasionally, price movements originated by noise or liquidity trading in one market can be transmitted to the other market via arbitrage, or, for example, large liquidity traders can induce a positive cross-correlation between returns when they fragment their order flow through both markets (Chowdhry and Nanda, 1991). But, on average, this linkage should be negligible in light of the connexion caused by fundamental information. Given the normalized noise volatility, parameters b_i, with $b_i > 0$, capture all microstructural frictions indistinctly.

The model also allows the possibility of "stale prices" in market 2. This is achieved through parameter ς. If $\varsigma = 0$, $L^0 m_t = m_t$ and both markets react immediately to new information. If $\varsigma = 1$, $L^1 m_t = m_{t-1}$, and market 1 reacts immediately to information while market 2 reacts with a lag. The existence of "stale prices" may be an economic phenomenon motivated by private information - implying that trade-durations are informative (Easley and O'Hara, 1992) - or simply the result of any microstructural friction that delays the immediate realization and/or observation of the "true price

[5] This price disintegration force could be also modelled by a positive correlation between the transient innovations, but with an impact of a different sign on prices (Yan and Zivot, 2010).

process". In the latter event, "stale prices" are just a purely spurious statistical artefact.[6] Although the two effects are economically distinct they are observationally equivalent, in the sense that observed prices "represent the most recent datum available to market participants" (Hasbrouck, 2003).[7] If $\varsigma = 1$, *ceteris paribus*, the observed returns in both markets present the same returns variance, but the contemporaneous cross-correlation diminishes as a result of the lagged reaction to information. When market 2 has a lagged reaction to information and $\delta_1 = 1$, the observed contemporaneous cross-correlation is equal to zero and all the cross-correlation structure resumes to $Cov(\Delta p_{1,t-1}, \Delta p_{2,t}) = \delta_2 \sigma^2_{\eta^P_t}$. Purging for the "stale price" effect approximates the error contemporaneous cross-correlation to $Corr(\Delta p_{1,t-1}, \Delta p_{2,t})$.

The procedure used here to compare the three price discovery measures is the following: (a) for a given set of structural parameters $\varpi = \left\{ \varsigma, \sigma^2_{\eta^P}, b_1, b_2, \delta_1, \delta_2, \rho_{q_1,q_2} \right\}$, the model is simulated and 20,000 pseudo-observations are recorded for $\{\mathbf{p}_t\}$; (b) a VECM

[6] If there are multiple informed traders, it seems logical to argue that the individual decision about which market to trade depends not only on the relative noise in both markets, but also on the number and intensity of trading by other informed traders already in place in each market. Because, informed traders will compete among themselves to profit on information, they will tend to fragment their order flow between markets. This will reduce the "stale price effect" caused by private information. This reasoning led Chowdhry and Nanda (1991) to conclude that: "the informativeness of the price system is a direct function of market fragmentation, if the number of markets increases their informativeness also increases." Therefore, the existence of "stale prices" in one market and not in the other is better explained not by some strategic idiosyncrasy of local informed traders, but more reasonably by microstructure trading mechanisms that obstruct prices from revealing the "true" price.

[7] If the data capturing system does not work properly, what is observed by the direct market participant is not the same as is observed by the researcher when looking at recorded time series data. When the precision of the time stamps is of crucial relevance, the research results can be substantially biased and therefore completely spurious.

with 20 lags is fitted to the bivariate return process,[8] and the disturbances' variance-covariance matrix and the error correction terms estimates are used to compute the three measures according to (7), (10) and (19).[9] (c) Results for market 1 are presented graphically (the corresponding values for market 2 are just the unity complement); for the *IS* methodology the reported *IS*, *ISmin* and *ISmax* series represent the information share's mid-point and the lower and higher bounds respectively.

The benchmark model is defined as follows: prices react immediately to new information, i.e. $\varsigma = 0$; markets have identical trading costs and noise effects, i.e. $b_1 = b_2 = 1$; information volatility is superior to the noise volatility, with an information-signal to noise ratio of $\sigma^2_{\eta^P} / b^2 = 4$; and the noise processes are idiosyncratic to each market, i.e. $\rho_{q_1,q_2} = 0$. Markets are distinct in the way they process information, which is captured by the structural adjustment coefficients, restricted to undershooting with $0 < \delta_i < 1$. In sum:

$$\varpi = \left\{ \varsigma = 0, \sigma^2_{\eta^P} = 4, b_1 = 1, b_2 = 1, \delta_1(i), \delta_2(i) = 1 - \delta_1(i), \rho_{q_1,q_2} = 0 \right\}, \text{ with}$$

$$\delta_2(i) = 1 - \delta_1(i) \text{ and } \delta_1(i) = 0.01i, \quad i = 5, 6, 7, ..., 95.$$

The two-market partial adjustment model has the nice property that the long-run impact on prices of a permanent innovation is equal to its magnitude. If, there are only undershooting effects, information-induced price movements are informative, and the

[8] All the structural models are conformable with a moving average representation. The partial adjustment model with $\delta \neq 1$ is conformable with an ARMA(1,1) (Theobald and Yallup, 2004). When $\delta = 1$, the model is fully-adjusted to information and is conformable with a MA(1) (Lehmann, 2002). The MA process is invertible and has an autoregressive representation with infinite order. Hence the estimation of a VECM(20) involves some lag truncation effects.

[9] The computation of the information shares and relative contemporaneous information response is therefore performed by replacing the efficient prices with the orthogonal complement of the parameter estimates of the error correction terms.

dynamic adjustment process is completely characterized by the adjustment rate. Thus the structural relative contribution of each market to price discovery, examined from the perspective of information incorporation, is unambiguously determined by the normalized value of the adjustment coefficient, i.e. price discovery in market 1 can be summarized by $\delta_1(i)/(\delta_1(i)+1-\delta_2(i))=\delta_1(i)$, $\forall i \in \left]0,1\right[$, which corresponds graphically to a straight line with a 45^{o} inclination.[10]

[INSERT FIGURE 1 ABOUT HERE]

Several statements can be made from inspection of Figure 1, which shows the estimates of market 1 price discovery metrics for the structural parameters discussed earlier: On average, the three methodologies correctly measure the true ordering of markets according to the relative price discovery;[11] the measuring precision of the relative information incorporation given by the *IR* metric is remarkable and it is almost indistinguishable from the true information incorporation line; the metric *CF* is indisputably the least efficient, especially in extreme cases (see also Hasbrouck, 2002); there is a widening of the *IS* range when markets converge to a situation of equal contribution to price discovery and the correlation between disturbances increases (see

[10] One special case of the former adjustment model is the symmetrical model with full adjustment and independent noise, characterized by $\delta_1=\delta_2=1$, $b_1=b_2=1$, $\rho_{q_1,q_2}=0$, and $\sigma^2_{\eta^P}=\sigma^2_{q_1}=\sigma^2_{q_2}=1$. In this model, prices are identical in incorporating and revealing information. Hasbrouck (2002) uses a similar model, designated by "two-market "Roll" model". The principal difference between Hasbrouck's model and the full-adjustment model outlined here is that, in the former, noise is characterized solely by the bid-ask bounce and so is modelled as following a binomial distribution.

[11] Yan and Zivot (2010) conclude that "a high *IS* accompanied with a low *CF* provides some evidence that a market has a relatively strong response to information innovation. A high *IS* together with a high *CF* would suggest otherwise". As one can see from Figure 1, this is a quite misleading interpretation of the two measures. Other examples (below) are also symptomatic of that inaccuracy.

also Baillie et al., 2002); there is a tendency for the *IS* underestimate (overestimate) the incorporation of information when there is a smaller (larger) adjustment rate.

This last statement is quite interesting, as it seems that even in a hypothetical situation where the only distinctive feature is the information adjustment process, the *IS* metric does not give an accurate estimate of the relative incorporation of information. Notice however that with $\sigma^2_{\eta^P} \big/ b^2 = 4$, information does not dominate noise because the relation between information-induced volatility and noise-induced volatility depends on the adjustment rate. In fact information only dominates noise if $\sigma^2_{\eta^P} \big/ b^2 > 2/\delta_i$. This effect is captured by the *IS* but not by the other two measures. For example, in the benchmark model, with $\delta_1 = 0.25$ and $\delta_2 = 0.75$, the estimates of *IR*, *IS* and *CF* are respectively 26.54%, 20.33% and 30.40%; thus *IR* is equal to δ_1, up to an estimation error, but because $\left(\delta_2 \sigma^2_{\eta^P} / 2b_2^2\right) \big/ \left(\delta_1 \sigma^2_{\eta^P} / 2b_1^2\right) = 1.5/0.5 = 3$, price movements in market 2 are relatively better explained by information than noise, and therefore market 1 reveals relatively less information than is incorporated into prices. In sum, one should conclude that price discovery in market 1, measured by the revelation of information, is lower than the 25% predicted by the *IS*.

Different information-signal to noise ratios

We now turn to the existence of different information-signal to noise ratios, i.e. to the impact of varying $\sigma^2_{\eta^P} \big/ b_i^2$, with equal costs ($b_1 = b_2 = 1$). Figures 2 and 3 show the estimates for the same parameters as Figure 1 except that $\sigma^2_{\eta^P} = 1$ and $\sigma^2_{\eta^P} = 16$ respectively. That is

$$\varpi = \left\{ \varsigma = 0, \sigma^2_{\eta^P} = 1, b_1 = 1, b_2 = 1, \delta_1(i), \delta_2(i) = 1 - \delta_1(i), \rho_{q_1,q_2} = 0 \right\},$$

$$\varpi = \left\{ \varsigma = 0, \sigma^2_{\eta^P} = 16, b_1 = 1, b_2 = 1, \delta_1(i), \delta_2(i) = 1 - \delta_1(i), \rho_{q_1,q_2} = 0 \right\}, \text{ with}$$

$$\delta_1(i) = 0.01i,\ i = 5, 6, 7, ..., 95.$$

The two experiments can be interpreted as a "calm market" situation, where information trading is dominated by noise (liquidity) and a "fast market" event where the information arrival process is the predominant trading incentive.

[INSERT FIGURE 2 ABOUT HERE]

[INSERT FIGURE 3 ABOUT HERE]

Comparison between Figures 2 and 3 provides the following insights: Increasing information-induced volatility increases the efficiency of *IS* and *IR* (although it drives the *IS* higher and lower bounds apart) but decreases the efficiency of *CF*; and increasing information-induced volatility reduces the distinction between information incorporation and information revelation and therefore reduces the gap between *IS* and *IR* estimates.

This last proposition states a paradoxical characteristic of *CF*: when information trading increases, the *CF* measure captures the relative price discovery across markets less accurately. This happens because the *CF* methodology does not allocate price discovery among markets via the information trading but instead via the relative noise.

Unequal Trading Costs

Probably the most discussed issue when comparing different price discovery metrics is the effect of unequal trading costs. This effect is shown in Figure 4, where the partial adjustment model is simulated using the initial parameters of the benchmark model, except that the impact of noise on market 1 is 4 times less than its impact on market 2, namely $b_1 = 0.25$ and $b_2 = 1$. That is, the experiment is conducted on:

$$\varpi = \left\{ \varsigma = 0, \sigma^2_{\eta^P} = 4, b_1 = 0.25, b_2 = 1, \delta_1(i), \delta_2(i) = 1 - \delta_1(i), \rho_{q_1,q_2} = 0 \right\}, \text{ with}$$

$$\delta_1(i) = 0.01i,\ i = 5, 6, 7, ..., 95.$$

[INSERT FIGURE 4 ABOUT HERE]

The main conclusions are: The *IR* measure remains typically accurate, while reducing the "noise-to-noise ratio" increases the *IS* and more specifically the *CF* estimates, and the existence of unequal costs drives away the *IS* bounds, and approximates the lower bound to the estimates of *IR* for the market with lower trading costs.

In this particular model, the *CF* is dominated by the dissimilar relationship between the noise impacts in the two markets. Consequently it attributes an informational superiority to market 1, regardless of the relative informational dynamics. Just to further highlight this issue, let us explore the situation where $\delta_1 = \delta_2 = 0.5$ in greater detail. The *IR* estimates a contribution of 48.72% for market 1, while the *IS* mid-point estimate is 69.76%. Therefore markets are similar in terms of incorporation of information, but market 1 is informationally superior because it provides a better information signal, given that $\left(\delta_2\sigma^2_{\eta^P} \Big/ 2b_2^2\right) \Big/ \left(\delta_1\sigma^2_{\eta^P} \Big/ 2b_1^2\right) = 1/4$.

Although one can theoretically accept the informational superiority of market 1 provided by the *IS* metric given the relatively lower trading costs in this market it is more difficult to defend an allocation of 88.17% to market 1, as is estimated by the *CF*.

Correlated noise processes

Baillie et al. (2002) and de Jong (2002) provide some simulation results for a VECM, assuming for simplicity that the autoregressive polynomial is zero. Keeping the long-run impact multipliers constant while changing the error contemporaneous covariance they show that *IS* is compatible with any value of *CF*. However, it is worth noticing that what is measured in that framework is the relative convergence of prices, not the convergence of prices to a structural common factor. The relative convergence process is mainly executed via arbitrage, but it can be initially caused by information or noise. So it is at least conceptually dubious to analyse the ability to assess the relative price discovery from the starting point of a reduced form model.

In order to examine the price discovery metrics sensitivity to the noise correlation structure, the partial adjustment model is simulated for a situation with fully-adjusted prices, unequal costs and negative correlation between the noise processes:

$\varpi = \left\{\varsigma = 0, \sigma^2_{\eta^P} = 4, b_1 = 0.25, b_2 = 1, \delta_1 = 1, \delta_2 = 1, \rho_{q_1,q_2} = \rho(i)\right\}$, with

$\rho(i) = -1 + 0.01i$, $i = 1, 2, ..., 100$.

[INSERT FIGURE 5 ABOUT HERE]

Results plotted in Figure 5 show that negative correlation between the noise processes reduces the robustness of *IS* and most particularly of *CF* but increases the efficiency of *IR*.[12]

If the structural disturbances' covariance matrix is non-singular it is impossible to isolate information from noise, because the returns covariance is a function not only of information but of noise, too. Hence the *CF* and the *IS* measures have some difficulty in allocating the relative price discovery between the two markets, but the *IR* metric nevertheless gives more precise results because in these simulations the increased correlation approximates the model to the assumption on the existence of only one transient shock.

Overshooting effects

It has already been shown that on quite general assumptions, and most importantly assuming that prices immediately undershoot the efficient price, the contemporaneous impact of new information is unequivocally related to the dynamic adjustment process. Accordingly, the *IR* and *IS* measures condense in a normalized number the information-induced dynamics, measured in terms of incorporation and revelation of information respectively. Implicit in those simulations is the assumption that the adjustment speed is constant; however, even in this hypothetical situation, the previous relationship only holds if there are no overshooting effects. Suppose that the markets experience the same degree of noise and have the same transaction costs, and that market

[12] We have conducted several experiments involving different market situations with non-zero noise correlation. The resulting pattern was that the ability of the three metrics to assess the price discovery process is increasingly disrupted as the noise processes became more positively correlated; this is particularly true for the *CF* methodology.

1 adjusts immediately and completely to information while market 2 overshoots the efficient price. What is the relative contribution of each market to price discovery? From an economic point of view, market 1 and market 2 are equivalent in incorporating information but market 1 has the "best" prices as they converge immediately to a situation of full-adjustment. It could therefore be said that the contribution of market 1 to price discovery is greater than the contribution of market 2.

Figure 6 presents the simulation results for the parameter set

$\varpi = \left\{ \varsigma = 0, \sigma^2_{\eta^P} = 4, b_1(i), b_2(i) = b_1(i), \delta_1 = 1, \delta_2 = 1.5, \rho_{q_1,q_2} = 0 \right\}$, with

$\delta_1(i) = 0.02i$, $i = 10, 11, 12, \ldots, 100$.

[INSERT FIGURE 6 ABOUT HERE]

The *IR* metric treats all the information-induced price movements as information although part of it is noise, and estimates a contribution of market 1 equal to $\delta_1 / (\delta_1 + \delta_2) = 1/2.5 = 0.4$. Therefore, if overshooting effects are expected some caution must be exercised when drawing inferences from the *IR* measure. The *IS* measure however reverts to its usual interpretation and gives a better assessment of how much information is in fact carried into the price system. This is particularly true if the information-signal to noise ratio is substantial. For example, when $b_1 = b_2 = 0.5$, the *IR*, *IS* and *CF* estimates are 39.98%, 48.42% and 39.34% respectively. The *IS* measure is nearer to the true information incorporation value of 50%, which means that it does not consider the higher overall noise in market 1. Notice however, that the higher and lower bounds of the *IS* metric are so far apart that it would seem meaningless to draw any inference from them.

In sum, from Figure 6, one can conclude that in the presence of overshooting effects: The *IR* measure is biased in favour of the market with relatively higher overreaction, the *IS* is a better measure of the contemporaneous incorporation of information, with better results when information-signal to noise ratio is relatively high and markets have equal trading costs, the *IS* bounds are driven apart as a result of the increased cross-correlation, and the *IS* converges to *IR* when the information-signal to noise ratio decreases.

Stale prices

Originally the information shares were designed to capture the timing, and not the relative contemporaneous magnitude or the multi-period dynamics of information incorporation. From this perspective, it could be said that a market has a higher price discovery contribution if it just begins impounding economically significant information in the price system before the coexisting markets. Figure 7 plots the estimates for

$\varpi = \left\{ \varsigma = 1, \sigma^2_{\eta^P} = 4, b_1 = 1, b_2 = 1, \delta_1(i), \delta_2(i) = \delta_1(i), \rho_{q_1,q_2} = 0 \right\}$, with

$\delta_1(i) = 0.01i$, $i = 5, 6, 7, ..., 100$.

In this situation, markets adjust to information at the same rate, noise is idiosyncratic to each market and noise impacts are equal, the information-signal is superior to noise and, most particularly, market 2 reacts with a lag to new information, i.e. $\varsigma = 1$.

[INSERT FIGURE 7 ABOUT HERE]

According to Hasbrouck's (1995) interpretation, the "true" contribution of market 1 is 100% and is independent of the adjustment coefficient. However, as we can see from Figure 7, the *IS* attributes the bulk of price discovery to market 1, but the estimates range from 50% to 96%. The *IR* and *CF* measures underestimate the relative price discovery more than the *IS*. The average difference between the *IS* and *IR* is about 11% while the average difference between *IS* and *CF* is 7%. The problem with the *IR* measure is that, with stale prices, permanent shocks in the two markets are serially correlated which, in conjunction with the fact that there is not one but two noise shocks, results in a non-trivial estimation error. These insights are summarized by the following statement: The three measures capture the effect of "stale prices" and attribute the bulk of price discovery to the market that observationally reacts first to new information; however there is a clear superiority of the *IS* measure over the *CF*, and especially over the *IR* measure.

Asymmetric information

Model (20) does not allow the distinction between public information and private information. In order to analyse the impact of different sources of information on the price discovery measures the "two markets with public and private information" model of Hasbrouck (2002) is used:

$$
\begin{aligned}
&p_{1,t} = m_t + b_1 q_{1,t} \\
&p_{2,t} = m_{t-1} + b_2 q_{2,t} \\
&m_t = m_{t-1} + u_t + \lambda q_{1,t} \\
&b_1, b_2, \lambda \geq 0 \\
&\boldsymbol{\eta}_t = \begin{bmatrix} u_t \\ q_{1,t} \\ q_{2,t} \end{bmatrix} \sim N\left(\mathbf{0}, \mathbf{I}_3\right)
\end{aligned}
\tag{21}
$$

This model allows the coexistence of public information, u_t, which is disclosed and incorporated into prices exogenously, and private information, $q_{1,t}$, which is related to the trading process itself. Prices are fully-adjusted to both types of information, meaning that the adjustment process is completed in just one period. However market 1 provides an opportune information signal while market 2 reacts with a lag. Private information is revealed from the market 1 trading process and market 2 incorporates the trade-related information of market 1 from the observation of its price process; therefore this type of information is also integrated, although exogenously, with a lag, into the price process of market 2. The informational superiority of market 1 is absolute (pure dominant market), not only due to the inertia of market 2 but also due to the inexistence of a "pure" noise shocks in the former market. In fact, "noise" in market 1 is originated solely by overreactions to the private information signal. Hence, in market 1, the permanent innovation $\eta_t^P = u_t + \lambda q_{1,t}$ is correlated with the transient component $s_{1,t} = b_1 q_{1,t}$. Considering the normalized variance matrix, it is easily shown that $Corr\left(\eta_t^P, s_{1,t}\right) = \lambda \big/ \left(\sqrt{1+\lambda^2}\right)$. The returns' variances and contemporaneous cross-correlation are described by $Var(\Delta p_{1,t}) = 1 + \lambda^2 + 2\lambda b_1 + 2b_1^2$, $Var\left(\Delta p_{2,t}\right) = 1 + \lambda^2 + 2b_2^2$, and $Cov(\Delta p_{1,t}, \Delta p_{2,t}) = -\lambda b_1$.

Accordingly, market 1 has a higher information-induced volatility given the interaction between information and the trading process, captured by the term $2\lambda b_1$. Returns have negative contemporaneous cross-correlation due to the "stale price effect".[13]

Figure 8 shows the measures' estimates for the "two markets with public and private information" model with varying trading costs in market 2; i.e., the experiment is conducted with the following parameters:[14]

$$\varpi = \left\{\sigma_{\eta^P}^2 = 1, b_1 = 1, b_2(i), \lambda = 1\right\},\ \delta_1(i) = 0.02i,\ i = 1,2,3,...,99.$$

[INSERT FIGURE 8 ABOUT HERE]

The informational superiority of market 1 is revealed by the three measures; however when noise in market 2 is low, the *CF* metric nontrivially underestimates the contribution of market 1 to price discovery. This is the principal evidence produced by Hasbrouck (2002) in order to show the inaccuracy of the *CF* measure when prices are driven by both public and private information. In fact, this result is the natural outcome of the combination of three effects: stale prices in market 2, initial overreaction in market 1 and inexistence of a "pure noise process" in market 1, i.e. the non-existence of a process

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[13] The simulations presented hereafter show similar results to those obtained by Lehmann (2002). The author provides (in Table 1) the population values for *CF* and *IS* in some occasional cases. With respect to these measures, the only advantage of our methodology is the visualization of the patterns reported by the author.

[14] This Figure and those following do not show the *IS* bounds, simply because it is more difficult to visualize all the series.

that is not related to fundamental information. If only a relatively small amount of idiosyncratic noise is introduced in market 1, the discrepancies between the *IS* and *CF* measures are substantially reduced, because the prevailing stale price effect is also captured, to great extent, by the *CF* measure.

Figure 9 shows the impact of increasing the overreaction (trading costs, as it is called by Hasbrouck, 2002) in market 1 caused by the trade-related information. The simulation is performed with:

$$\varpi = \left\{ \sigma^2_{\eta^P} = 1, b_1(i), b_2 = 1, \lambda = 1 \right\},\ \delta_1(i) = 0.02i,\ i = 1,2,3,...,99\,.$$

[INSERT FIGURE 9 ABOUT HERE]

A consistent pattern to that exhibit by Figure 8 emerges as the result of the increasing asymmetry between markets due to private information. As the price overestimation in market 1 increases, all the measures allocate a lower contribution to market 1, with particular emphasis on the *CF* methodology.

Finally, one could also look at the impact of varying the importance of public information in relation to private information, i.e. by varying $\sigma^2_{\eta^P}$. This is shown in Figure 10 for:

$$\varpi = \left\{ \sigma^2_{\eta^P} = \sigma^2(i), b_1 = 1, b_2 = 1, \lambda = 1 \right\},\ \sigma^2(i) = \left(0.02i\right)^2,\ i = 1,2,3,...,99\,.$$

[INSERT FIGURE 10 ABOUT HERE]

As public information begins to dominate private information, more weight is given to the stale price effect and so the pattern is similar to the values obtained in Figure 7 for $\delta_1 = \delta_2 \simeq 1$.

Basically, the differences in the estimates of *IS* and *CF* can be intuitively explained by the fact that *CF* does not consider the relationship between the transient component in market 1 and information. In other words, the *CF* metric does not capture the market differences originated by the volatility induced by private information. Consequently, in this framework, *CF* is the least precise price discovery measure.

The *IR* metric also attributes the bulk of price discovery to market 1, but because the informational superiority of market 1 is mostly due to the inertia of market 2, *IR* estimates are lower than those of *IS* for "normal" market conditions. The *IR* bias is on average less than that of the corresponding *CF* estimate.

Basically, one can conclude that the *IR* measure can cope with market asymmetry resulting from the existence of private information. Moreover if overreaction is associated with the existence of private information, then the *IR* captures the existence of that type of information even better than the *IS*.

Suppose that markets react contemporaneously to public information. This is quite a reasonable assumption. Nowadays, market participants can observe firm-specific, sector-based or macroeconomic public news in real time. Let us assume that investors endowed with private information can choose which market to trade. Because noise can camouflage their informed trades and consequently increase their individual profits (Kyle, 1985; Admati and Pfleiderer, 1988) it seems reasonable, without further strategic considerations, that informed traders will choose the noisiest market or at least that they will trade more aggressively in that market (Chowdhry and Nanda, 1991). Let this be market 1. Without fragmentation of the information-based order flow, participants in market 2 only have access to private information as it becomes public through the reporting of market 1 transaction prices; consequently market 2 reacts to private information with a lag. Price discovery, measured by the timing of incorporation of information, should all be allocated to market 1.

This hypothetical situation, widely supported by theoretical microstructure models, can be simulated in a modified "two markets with public and private information" model, such that:

$$\begin{aligned}
&p_{1,t} = m_t + b_1 q_{1,t} \\
&p_{2,t} = m_{t-1} + u_t + \lambda q_{1,t-1} + b_2 q_{2,t} \\
&m_t = m_{t-1} + u_t + \lambda q_{1,t} \\
&b_1, b_2, \lambda \geq 0, b_1 > b_2 \\
&\boldsymbol{\eta}_t = \begin{bmatrix} u_t \\ q_{1,t} \\ q_{2,t} \end{bmatrix} \sim N\left(\mathbf{0}, \mathbf{I}_3\right)
\end{aligned} \tag{22}$$

We have obtained 100,000 pseudo-observations from model (22), using the following parameters set $\varpi = \left\{\sigma^2_{\eta^P} = 1, b_1 = 1, b_2 = 0.25, \lambda = 1\right\}$. The estimates for the three measures are: $IR_1 = 74.93\%$, $IS_1 = 71.65\%$ and $CF_1 = 52.38\%$. This illustration shows that when causation runs from noise to information, in the sense that differences in information revelation across markets are the result of the behaviour of local liquidity traders, the *IR* metric can provide even better results than the *IS* methodology.

4.1 Small sample properties

Some empirical studies that analyse the relative price discovery across markets, especially when they apply the information shares technology, use the average of daily estimates (Hasbrouck, 2003; Kurov and Lasser, 2004; Ates and Wang, 2005). We conducted a Monte Carlo experiment in order to assess the small sample properties of the three price discovery measures. The detailed description of this experiment and results are presented in Table 1. The main conclusions are: (a) The *CF* is often negative or greater than one. This raises some doubts about its economic interpretation as a price discovery measure. Hasbrouck (2002) notes that there is no theoretical impediment for the long-memory common-factors to be outside the interval $[0,1]$. Accordingly, Theissen (2002) suggests that more meaningful results can be obtained if, instead of using the estimates of the error correction coefficients directly, their absolute value is used. (b) The dispersion of estimates, measured by its standard-deviation, is substantially higher for the *CF*, especially if there is contemporaneous overreaction (0.4316) or positive correlation between the noise processes (0.3150). In all the studied cases the *IR* exhibit the lowest standard deviation, ranging from 0.0267, when there is contemporaneous overreaction, to 0.0945, when there are stale price effects. The same conclusion can be withdrawn from the inter-percentile range. On average (for all the cases) these figures are about 0.77, 0.26 and 0.19 for the *CF*, *IS* and *IR* respectively. (c) The *IR* reported bias in the presence of overreaction (Case 3), and stale prices (Case 3, Case 7 and Case 8) are also noticed in small samples, however minimum values and 5% percentile values show that this metric almost always ascribes the right ordering in accordance with the theoretical relative contribution to price discovery.

5. Conclusion

This paper describes the cointegration-based technologies commonly used to assess the relative price discovery across markets, namely the Hasbrouck information shares and Gonzalo-Granger long memory common factor weights, and presents a new metric denominated contemporaneous information response. This new metric is motivated from the analysis of the dynamic effects of permanent and transitory shocks to a system of cointegrated variables.

The three metrics are then compared from the simulation of structural models for the dynamics of two fundamentally related markets, namely the a generalization of the “partial adjustment with noise” model, the “two markets with public and private information” model of Hasbrouck (2002), and a modified “two markets with public and private information” model.

Basically, the simulations prove that the proposed metric is a reliable measure of the relative incorporation of information, and in most cases is more resilient to microstructural noise than the other two metrics.

Appendix: Derivation of the relative contemporaneous information response (IR)

Let $\mathbf{p}_t$ be a vector (2×1) of cointegrated variables with cointegrating vector $\boldsymbol{\alpha}=\begin{bmatrix}1 & -1\end{bmatrix}'$. The long-run impact matrix has two identical rows $\mathbf{C}(1)=\boldsymbol{\iota}\mathbf{c}'$, with $\mathbf{c}=\begin{bmatrix}c_1 & c_2\end{bmatrix}'$. Assuming that there is only one permanent component, η_t^P, and one transitory component, η_t^T, price increments have a structural bivariate moving average representation

$$\Delta\mathbf{p}_t=\mathbf{D}(L)\boldsymbol{\eta}_t=\mathbf{D}^P(L)\eta_t^P+\mathbf{D}^T(L)\eta_t^T=\sum_{k=0}^{\infty}\mathbf{D}_k^P\eta_{t-k}^P+\sum_{k=0}^{\infty}\mathbf{D}_k^T\eta_{t-k}^T . \qquad \text{(A.1)}$$

According to the P-T decomposition

$$\Delta\mathbf{p}_t=\mathbf{C}(L)\mathbf{G}^{-1}\mathbf{H}\kappa\kappa^{-1}\mathbf{H}^{-1}\mathbf{G}\boldsymbol{\varepsilon}_t=\mathbf{D}(L)\bar{\mathbf{H}}\bar{\mathbf{H}}^{-1}\mathbf{u}_t=\mathbf{D}(L)\boldsymbol{\eta}_t, \qquad \text{(A.2)}$$

the initial impacts of the permanent shock are given as

$$d^P=\begin{bmatrix}d_1^P\\ d_2^P\end{bmatrix}=\mathbf{C}_0\mathbf{G}^{-1}\mathbf{H}\kappa=\mathbf{I}\mathbf{G}^{-1}\mathbf{H}\kappa=\kappa\mathbf{G}^{-1}\mathbf{H}, \qquad \text{(A.3)}$$

where $\mathbf{G}=\begin{bmatrix}\mathbf{c}' & \boldsymbol{\alpha}'\end{bmatrix}'$ and therefore

$$\mathbf{G}^{-1}=\left(\mathbf{c}'\boldsymbol{\iota}\right)^{-1}\begin{bmatrix}1 & c_2{}^{-1}\\ 1 & -c_1{}^{-1}\end{bmatrix}. \qquad \text{(A.4)}$$

and $\mathbf{H}$ is the lower triangular matrix obtained from the Cholesky factorization of the variance-covariance matrix of $\mathbf{u}_t=\mathbf{G}\boldsymbol{\varepsilon}_t$, such that

$$\mathbf{H}=\begin{bmatrix}\sqrt{Var\left(\mathbf{c}'\boldsymbol{\varepsilon}_t\right)} & 0\\ \dfrac{Cov(\mathbf{c}'\boldsymbol{\varepsilon}_t,\boldsymbol{\alpha}'\boldsymbol{\varepsilon}_t)}{\sqrt{Var\left(\mathbf{c}'\boldsymbol{\varepsilon}_t\right)}} & \sqrt{Var\left(\boldsymbol{\alpha}'\boldsymbol{\varepsilon}_t\right)}\end{bmatrix}. \tag{A.5}$$

From (A.3), (A.4) and (A.5)

$$d_1^P=k\left(\mathbf{c}'\boldsymbol{\iota}\right)^{-1}\left(\sqrt{Var\left(\mathbf{c}'\boldsymbol{\varepsilon}_t\right)}\right)^{-1}\left(Var\left(\mathbf{c}'\boldsymbol{\varepsilon}_t\right)+c_2Cov(\mathbf{c}'\boldsymbol{\varepsilon}_t,\boldsymbol{\alpha}'\boldsymbol{\varepsilon}_t)\right),\text{ and} \tag{A.6}$$

$$d_2^P=k\left(\mathbf{c}'\boldsymbol{\iota}\right)^{-1}\left(\sqrt{Var\left(\mathbf{c}'\boldsymbol{\varepsilon}_t\right)}\right)^{-1}\left(Var\left(\mathbf{c}'\boldsymbol{\varepsilon}_t\right)-c_1Cov(\mathbf{c}'\boldsymbol{\varepsilon}_t,\boldsymbol{\alpha}'\boldsymbol{\varepsilon}_t)\right). \tag{A.7}$$

The relative impact of a permanent innovation is given by

$$IR_1=\frac{d_1^P}{d_1^P+d_2^P}=\frac{\left(Var\left(\mathbf{c}'\boldsymbol{\varepsilon}_t\right)+c_2Cov(\mathbf{c}'\boldsymbol{\varepsilon}_t,\boldsymbol{\alpha}'\boldsymbol{\varepsilon}_t)\right)}{\left(2Var\left(\mathbf{c}'\boldsymbol{\varepsilon}_t\right)+\left(c_2-c_1\right)Cov(\mathbf{c}'\boldsymbol{\varepsilon}_t,\boldsymbol{\alpha}'\boldsymbol{\varepsilon}_t)\right)}, \tag{A.8}$$

with

$$Var\left(c'\varepsilon_t\right)=Var\left(c_1\varepsilon_{1,t}+c_2\varepsilon_{2,t}\right)=c_1^2\sigma_{\varepsilon_1}^2+2c_1c_2\sigma_{\varepsilon_1\varepsilon_2}+c_2^2\sigma_{\varepsilon_2}^2,\text{ and} \tag{A.9}$$

$$Cov(c'\varepsilon_t,\alpha'\varepsilon_t)=Cov(c_1\varepsilon_{1,t}+c_2\varepsilon_{2,t},\varepsilon_{1,t}-\varepsilon_{2,t})=c_1\sigma_{\varepsilon_1}^2+\left(c_2-c_1\right)\sigma_{\varepsilon_1\varepsilon_2}-c_2\sigma_{\varepsilon_2}^2 \tag{A.10}$$

Substituting (A.9) and (A.10) into (A.8), and after some manipulation, the final expression of the relative contemporaneous information response of market 1 is given as

$$IR_1=\frac{c_1\sigma_{\varepsilon_1}^2+c_2\sigma_{\varepsilon_1,\varepsilon_2}}{c_1\sigma_{\varepsilon_1}^2+\left(c_1+c_2\right)\sigma_{\varepsilon_1,\varepsilon_2}+c_2\sigma_{\varepsilon_2}^2}. \tag{A.11}$$

The relative initial impact of an information shock in market 2 is the complement of IR_1:

$$IR_2=1-IR_1=\frac{c_2\sigma_{\varepsilon_2}^2+c_2\sigma_{\varepsilon_1,\varepsilon_2}}{c_1\sigma_{\varepsilon_1}^2+\left(c_1+c_2\right)\sigma_{\varepsilon_1,\varepsilon_2}+c_2\sigma_{\varepsilon_2}^2}. \tag{A.12}$$

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Figure 1 "Partial adjustment with noise model". The benchmark situation

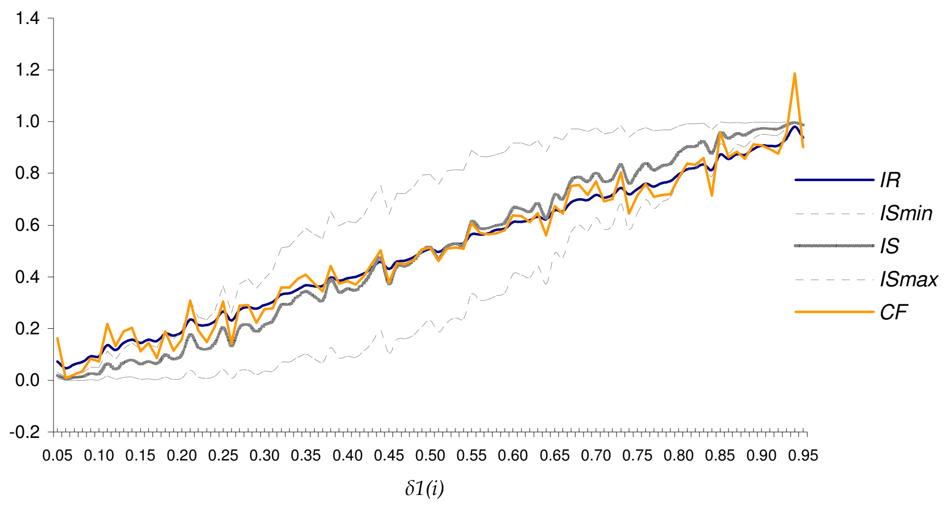

Figure 2 "Partial adjustment with noise model" in a "calm markets" situation

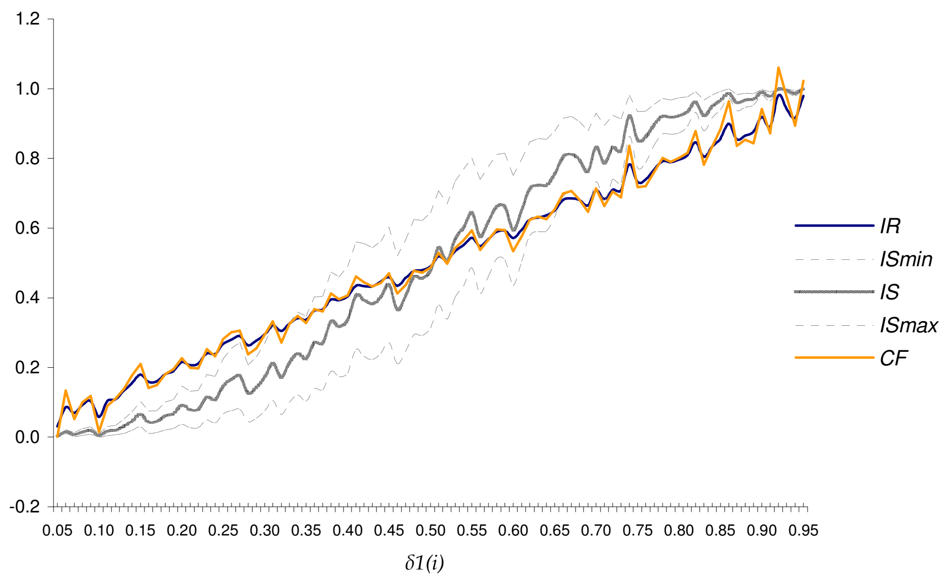

Figure 3 “Partial adjustment with noise model” in a “fast markets” situation

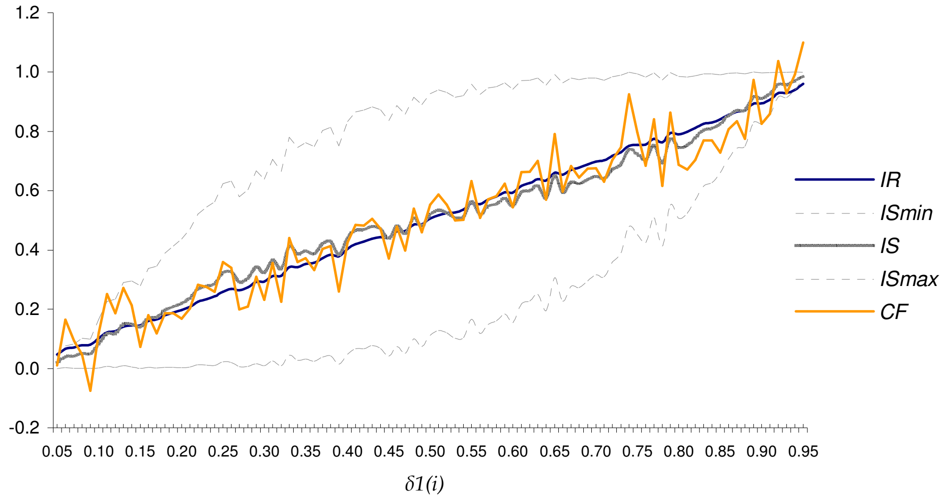

Figure 4 “Partial adjustment with noise model” with unequal trading costs

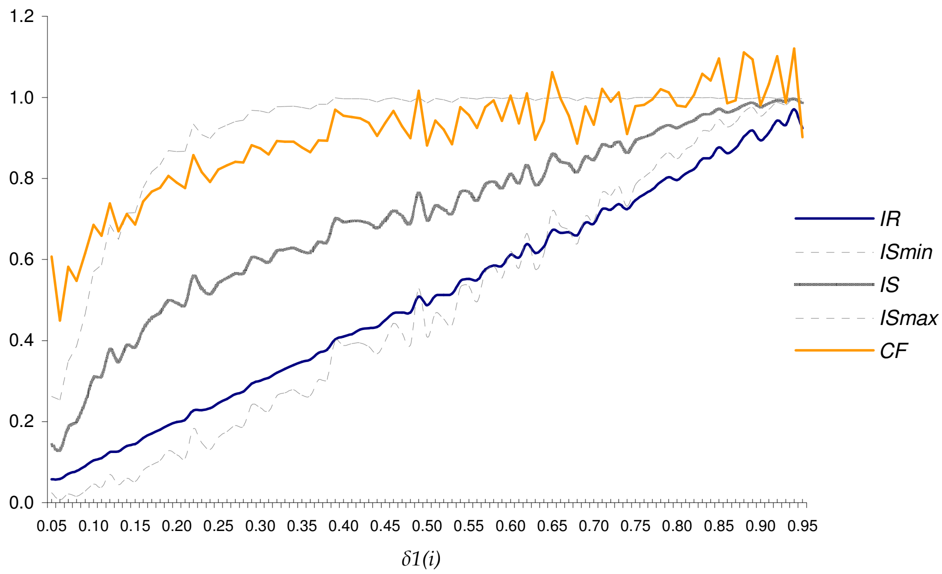

Figure 5 "Partial adjustment with noise model" with correlated noise processes

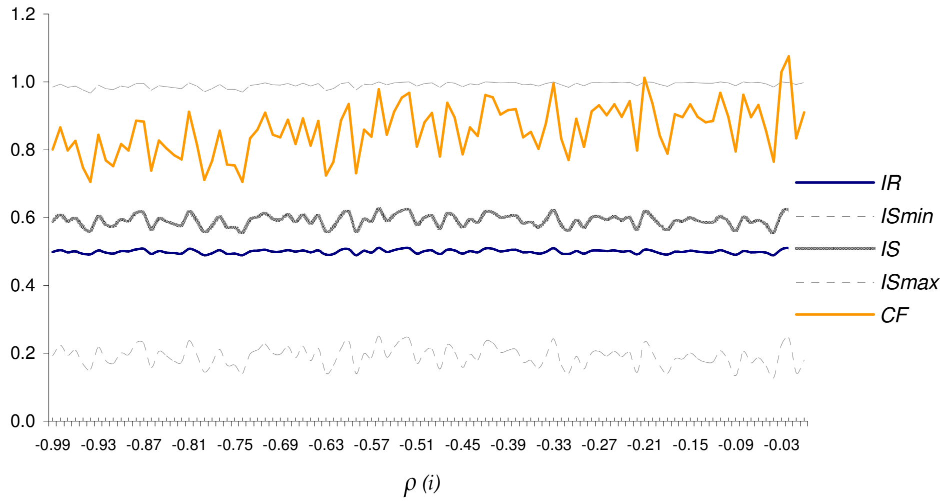

Figure 6 "Partial adjustment with noise model" with price overreaction in market 2

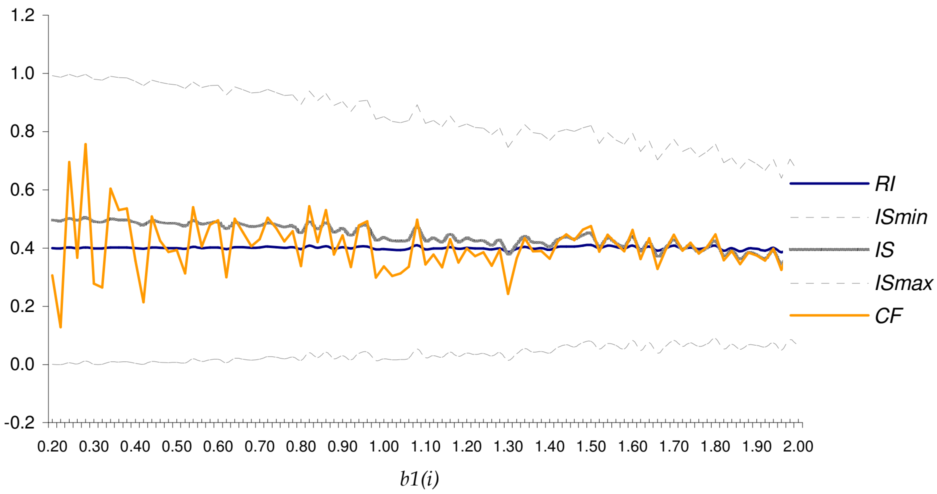

Figure 7 "Partial adjustment with noise model" with stale prices in market 2

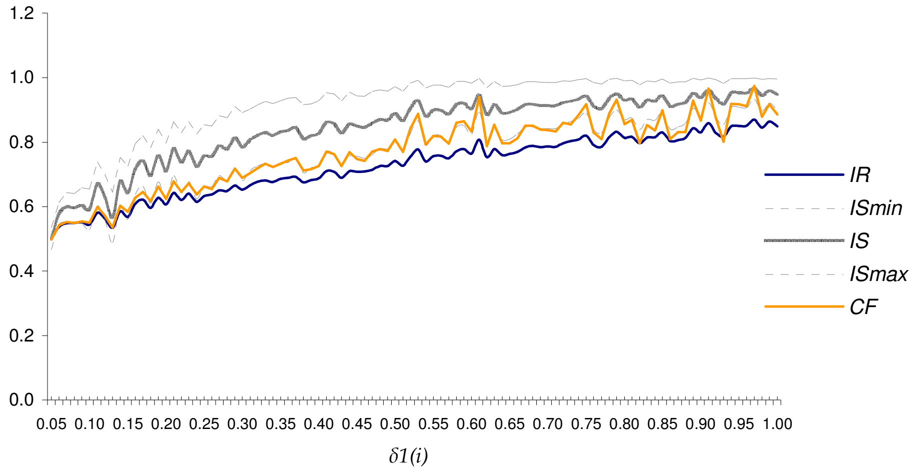

Figure 8 The coexistence of public and private information with varying trading costs in market 2

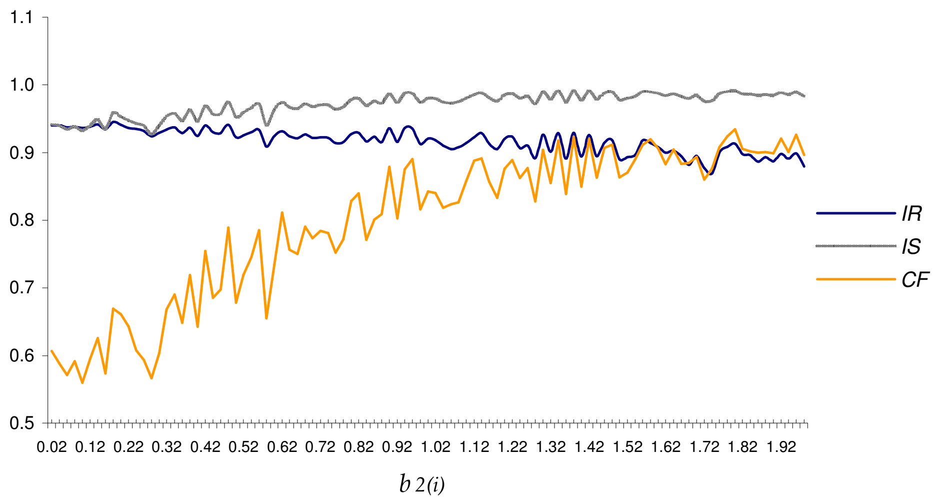

Figure 9 The coexistence of public and private information with varying trading costs in market 1

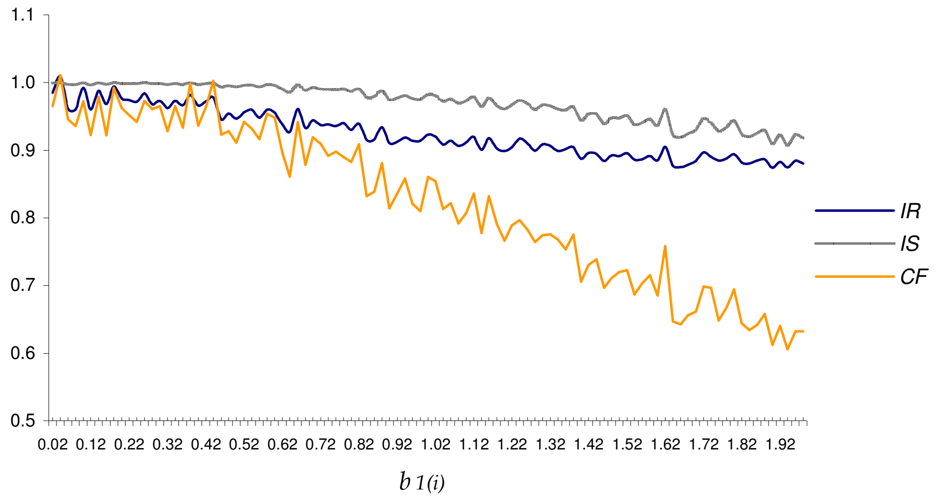

Figure 10 The coexistence of public and private information with varying public information signal

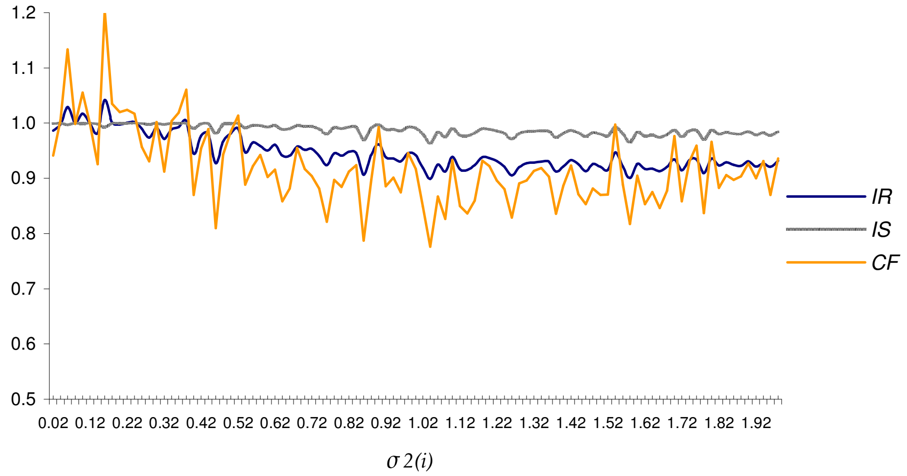

Table 1 Monte Carlo simulations for price discovery measures

This Table reports some sample statistics for the Hasbrouck information share (*IS*), Gonzalo-Granger common factor weights (*CF*) and relative contemporaneous information response (*IR*) measures estimated from a simulated bivariate series of related prices. The specific Monte Carlo method applied here is as follows: (1) a "partial adjustment model with noise" is simulated in order to obtain 600 observations of $\left\{p_{1,t}, p_{2,t}\right\}$, let us say that these observations represent minute-by-minute prices on a specific trading day, (2) the three measures are computed from a VECM(20) and then are saved. The experiment is replicated 1,000 times (which can be interpreted as trading days) for each set of structural parameters $\varpi = \left\{\varsigma, \sigma^2_{\eta^P}, b_1, b_2, \delta_1, \delta_2, \rho_{q_1,q_2}\right\}$.

PARTIAL ADJUSTMENT WITH NOISE MODEL

$$p_{1,t} = p_{1,t-1} + \delta_1\left(m_t - p_{1,t-1}\right) + b_1 q_{1,t}$$

$$p_{2,t} = p_{2,t-1} + \delta_2\left(L^{\varsigma} m_t - p_{2,t-1}\right) + b_2 q_{2,t}$$

$$m_t = m_{t-1} + \eta^P_t$$

$$\eta^P_t \sim N\left(0, \sigma^2_{\eta^P}\right)$$

$$\mathbf{q} = \begin{bmatrix} q_{1,t} \\ q_{2,t} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma}_q\right)$$

$$\boldsymbol{\Sigma}_q = \begin{bmatrix} 1 & \rho_{q_1,q_2} \\ \rho_{q_1,q_2} & 1 \end{bmatrix}$$

The partial adjustment with noise model was estimated for eight cases:

Case 1: $\varpi = \left\{\varsigma = 0, \sigma^2_{\eta^P} = 1, b_1 = 1, b_2 = 1, \delta_1 = 1, \delta_2 = 1, \rho_{q_1,q_2} = 0\right\}$;

Case 2: $\varpi = \left\{\varsigma = 0, \sigma^2_{\eta^P} = 1, b_1 = 1, b_2 = 1, \delta_1 = 0.5, \delta_2 = 1, \rho_{q_1,q_2} = 0\right\}$;

Case 3: $\varpi = \left\{\varsigma = 0, \sigma^2_{\eta^P} = 1, b_1 = 0.5, b_2 = 0.5, \delta_1 = 1.5, \delta_2 = 1, \rho_{q_1,q_2} = 0\right\}$;

Case 4: $\varpi = \left\{\varsigma = 0, \sigma^2_{\eta^P} = 1, b_1 = 0.25, b_2 = 1, \delta_1 = 1, \delta_2 = 1, \rho_{q_1,q_2} = 0\right\}$;

Case 5: $\varpi = \left\{\varsigma = 0, \sigma^2_{\eta^P} = 1, b_1 = 1, b_2 = 1, \delta_1 = 1, \delta_2 = 1, \rho_{q_1,q_2} = 0.5\right\}$;

Case 6: $\varpi = \left\{\varsigma = 0, \sigma^2_{\eta^P} = 1, b_1 = 1, b_2 = 0.25, \delta_1 = 1, \delta_2 = 1, \rho_{q_1,q_2} = -0.5\right\}$;

Case 7: $\varpi = \left\{\varsigma = 1, \sigma^2_{\eta^P} = 1, b_1 = 1, b_2 = 1, \delta_1 = 1, \delta_2 = 1, \rho_{q_1,q_2} = 0\right\}$;

Case 8: $\varpi = \left\{\varsigma = 1, \sigma^2_{\eta^P} = 1, b_1 = 1, b_2 = 1, \delta_1 = 1, \delta_2 = 1.5, \rho_{q_1,q_2} = 0\right\}$.

The sample statistics of the estimated measures for market 1 are presented hereafter. For each case the theoretical value of the relative immediate information incorporation is shown in parenthesis.

	Mean	Minimum	Percentile 5%	Median	Percentile 95%	Maximum	Standard-deviation
Case 1 (0.5000)							
CF	0.5087	-0.2429	0.1937	0.5042	0.8102	1.2380	0.1937
IS	0.5064	0.0951	0.2589	0.5037	0.7430	0.8937	0.1468
IR	0.5029	0.2825	0.4117	0.5023	0.5850	0.6837	0.0533
Case 2 (0.3333)							
CF	0.3184	-0.7057	-0.0312	0.3366	0.5919	0.7845	0.1899
IS	0.2713	0.0403	0.0738	0.2557	0.5184	0.7545	0.1341
IR	0.3336	0.0830	0.2178	0.3364	0.4323	0.5368	0.0637
Case 3 (0.5000)							
IS	0.5442	0.2589	0.3932	0.5463	0.6853	0.7562	0.0886
CF	0.6497	-0.4072	0.0824	0.5812	1.4645	2.8926	0.4316
IR	0.5991	0.4920	0.5547	0.5986	0.6424	0.6906	0.0267
Case 4 (0.5000)							
CF	0.9420	-0.0373	0.6078	0.9431	1.2825	1.5909	0.2116
IS	0.7112	0.2360	0.5180	0.7267	0.8495	0.8918	0.1057
IR	0.5048	0.3314	0.4283	0.5007	0.5966	0.7303	0.0528
Case 5 (0.5000)							
CF	0.4824	-0.8624	-0.0270	0.4832	0.9856	1.9042	0.3150
IS	0.4936	0.2102	0.3106	0.4944	0.6707	0.8061	0.1109
IR	0.4978	0.3635	0.4380	0.4982	0.5536	0.6643	0.0357
Case 6 (0.5000)							
CF	0.8599	0.2505	0.5786	0.8567	1.1583	1.5518	0.1806
IS	0.7103	0.2673	0.5007	0.7217	0.8764	0.8292	0.1185
IR	0.5059	0.3496	0.4194	0.5006	0.6073	0.9132	0.0600
Case 7 (1.0000)							
CF	0.8285	0.3924	0.5250	0.7865	1.2250	5.9343	0.2838
IS	0.8500	0.4127	0.6252	0.8744	0.9774	0.9888	0.1105
IR	0.7409	0.5120	0.6087	0.7302	0.8981	1.6064	0.0945
Case 8 (1.0000)							
CF	0.7804	0.2337	0.5116	0.7647	1.0899	1.6391	0.1794
IS	0.7912	0.2323	0.5390	0.8129	0.9604	0.9794	0.1316
IR	0.6553	0.4278	0.5265	0.6481	0.8035	1.0121	0.0831

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