The reveal of minimal ruled surfaces using control theory

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We seek minimal surfaces among the family of ruled surfaces in the Euclidean space. Such surfaces are generated by a straight line, called generatrix, that moves along a curve, the directrix. We approach this problem using the techniques from control theory. In the literature the minimal revolution surface is often obtained as an optimal control problem because of the peculiarity of the parameterization that reduces the problem to an ordinary differential equation. Nothing similar has been published for other families of minimal surfaces because partial differential equations cannot be avoided. This is why we use the k-symplectic formalism, which allows us to solve the general optimal control problem by means of the Hamilton-De Donder-Weyl equation. Applying it to minimize the area of a ruled surface subject to a dynamics associated with the surface we recover the plane and the helicoid as candidates to be minimizers. These two surfaces are the only minimal ruled surfaces, as known from the geometrical characterization of zero mean curvature for the minimal surfaces.

References

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