Dynamics of pendula hanging from a string

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We present the dynamics of a hybrid mechanical system composed of a homogeneous flexible and elastic string, with fixed extremities, on which are suspended two identical pendula. In our model we include a dissipative contribution due to internal damping within the string, modelling it as viscoelastic friction.

The aim of this study is double: we want on one side to analyse how the continuous nature of the coupling between the pendula affects their small oscillations, and on the other to investigate the emergence of synchronization patterns and to provide an interpretation of synchronization phenomena in mechanical systems as an outcome of damping contributions affecting the normal modes of oscillation.

The work, making use of the Lagrangian formalism, provides a linear analysis based on the determination of the damped normal modes of oscillation. Hence, we investigate beats and synchronization by means of a numerical study as the parameters that characterize the system change.

The study of this model leaves open future analyses:

- further results on the structure of the phase space;
- an adequate weak formulation;
- a generalization to n pendula.

References

[1] A. L. Kimball, D. E. Lovell (1927). Internal Friction in Solids. In: *Phys. Rev.* 30, 948–959.

[2] D. J. Korteweg (1906). *Les horloges sympathiques de Huygens*. Archives Neelandaises, sér. II, tome XI. The Hague: Martinus Nijhoff, 273–295.

[3] P. Lancaster (2014). *Lambda-Matrices and Vibrating Systems*. International Series in Pure and Applied Mathematics. Elsevier Science.

[4] D. L. Russell (2992). On Mathematical Models for the Elastic Beam with Frequency-Proportional Damping. In: *Control and Estimation in Distributed Parameter Systems*. SIAM, 125–169.