

The fictitious control method for the internal controllability of underactuated coupled systems of PDEs

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Studying the controllability for linear or non-linear coupled systems has been an intensive subject of interest these last years. The main issue is to try to control many equations with less controls than equations, with the hope that one can act indirectly on the equations that are not directly controlled thanks to the coupling terms. In the case of internal controls, one way to prove this kind of property is the use of what is called the "fictitious control method", which was first introduced in [1] in the context of PDEs. This method can be roughly decomposed into two steps:

1. Firstly, control the system with a control acting on each equation. This is in general simpler than the original underactuated problem and may be performed by using classical tools.
2. Secondly, try to find a way to get rid of the control that should not appear in order to obtain the desired results. This can be done using the notion of "algebraic solvability", which relies on a particular construction of a solution for a related underdetermined linear partial differential equations with a source term. This strategy has been introduced in [2] in the context of PDEs.

After having explained the method and highlighted its advantages and drawbacks, I will provide some examples of applications, coming from [3, 4, 5].

References

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