Rolling Grassmannians – Constructive Proof of Controllability

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The poster will address controllability properties of the kinematic equations describing rolling motions of Grassmann manifolds (Grassmannians) over their affine tangent space at a point. These rolling motions, which are assumed to have nonholonomic constraints of no-slip and no-twist, result from the action of a certain Lie group on the vector space of symmetric matrices where the rolling manifold and the static manifold are embedded.

The basis for this work is the article [1], where the kinematic equations have been derived. These kinematic equations can be rewritten as a nonlinear control system evolving on a particular connected Lie group and it turns out that the system is controllable. This analytical proof is based on showing that the control vector fields have the bracket generating property, but does not give any insight on how to join two different configurations of the Grassmannian using trajectories of the control system only.

The objective of this poster is to highlight a constructive proof of controllability of the rolling system, by showing how the forbidden motions of twisting and slipping can be accomplished by rolling without breaking the nonholonomic constraints of no-slip and no-twist. This geometric construction generalizes a similar approach that was accomplished in [2] for the rolling sphere. Details concerning controllability of the rolling Grassmannians and, in particular, this constructive proof can be found in [3].

The study of these problems was motivated by the potential applications of the Grassmann manifold in several engineering areas that deal with sets of images, such as face recognition problems under varying illumination conditions, or reconstruction of planar scenes from multiple views (see, for instance, [4] and [5] for several interesting real problems about this topic).

References

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[1] Acknowledges support from ISR and DMUC.

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