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Tracking and filtering on SE(2)

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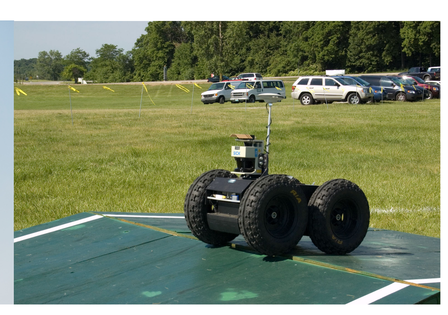
Objective

Attitude estimation is a core problem in many robotic systems, such as unmanned aerial and ground vehicles. The configuration space of these systems is properly modelled exploiting the theory of Lie groups. In this poster we propose a second-order-optimal minimum-energy filter on the matrix Lie group SE(2). The mathematics behind it is quite challenging and is not a simple generalization of previous results on Lie groups.



Introduction

The pose of the robot are crucial for unmanned aerial and ground vehicles (UAV, UGV) that exploit algorithms for computing their position and for planning their trajectory. In the last decades many observers have been proposed in the literature. The most famous approach is based on Kalman filtering and its many extensions.

Other researchers exploit the theory of Lie groups to provide the proper mathematical structure for the attitude of a mechanical system [1]. Recent results on the design of dynamic filters on Lie groups the authors proposed a *second order optimal minimum energy filter*. The filter is based on the results of Mortensen where a methodology of generating progressive realizable approximations of a minimum-energy functional was proposed. The solution is obtained by differentiating the boundary conditions of the associated optimal control problem. The filter takes the form of a gradient observer coupled with a kind of Riccati differential equation that updates its gain.

The theoretical result in [2] is our starting point. We apply it to the the case of a system modelled on the Lie group SE(2). This case turns out to be particularly complicated since we don't have the Lie algebra isomorphism between $\mathfrak{se}(2)$ and $\mathbb{R}^3$. Our main contribution is then to derive a second order optimal minimum energy filter for SE(2) and to provide all the mathematics for the many technical operations needed to compute it.

The Filter

Consider the deterministic system

$$\dot{g}(t) = g(t)(\lambda(g(t), u(t), t) + B\delta(t)),$$
$$g(t_0) = g_0,$$

where $g(t) \in G$ is the state, $u(t) \in \mathbb{R}^m$ is the input, $\delta(t)$ is the unknown model error, $\lambda : G \times \mathbb{R}^m \times \mathbb{R} \to \mathfrak{g}$ the left trivialized dynamics and $B : \mathbb{R}^d \to \mathfrak{g}$ a linear map.

Consider now $y \in \mathbb{R}^p$, a map $h : G \times \mathbb{R} \to \mathbb{R}^p$, the unknown measurement error $\varepsilon \in \mathbb{R}^p$ and an invertible linear map $D : \mathbb{R}^p \to \mathbb{R}^p$, then the measurement equation is

$$y(t) = h(g(t), t) + D\varepsilon(t).$$

The unknown errors δ and ϵ are assumed to be deterministic functions of time.

Given the input u and the measurement output y, the goal is to find the best estimate of the state trajectory $g(\cdot)$ minimizing a cost functional. For the derivation of the exact filter, it would be necessary to solve an infinite dimension Hamilton-Jacobi-Bellman (HJB) equation, but as we said, the filter that we consider comes from a second-order approximation of the value function.

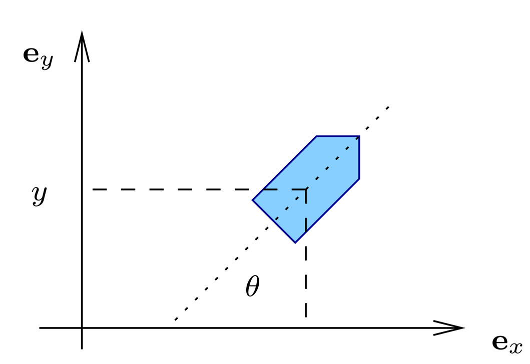


Figure 1: Planar rigid body.

SE(2) Case

Consider the study of a planar rigid body as a simple mechanical control system on the Lie group SE(2) as shown in Figure 1.

Given the Lie algebra $\mathfrak{se}(2)$, we introduce the isomorphisms $\vee : \mathfrak{se}(2) \to \mathbb{R}^3$ and $\wedge : \mathbb{R}^3 \to \mathfrak{se}(2)$ as

$$(v^1, v^2, v^3)^\wedge = \begin{bmatrix} 0 & -v^1 & v^2 \\ v^1 & 0 & v^3 \\ 0 & 0 & 0 \end{bmatrix}.$$

We denote by $\mathbb{I}$ the inertia tensor and u^m the control inputs that are functions of time. f^m are a collection of covectors in $\mathfrak{g}^*$, defining a collection of left-invariant control forces and $(\Omega)^\vee = (\omega, v_1, v_2) \in \mathbb{R}^3$ where ω is the angular velocity and v_1, v_2 the linear velocity of the body written in the body representation.

The dynamics of the body evolves on $G = \mathrm{TSE}(2)$, the tangent bundle of $G := \mathrm{SE}(2)$ that we can identify with $\mathrm{SE}(2)\times\mathfrak{se}(2)$ via left translation, [1].

The idea of the second-order-optimal filter is to reconstruct $G \ni g := (g, \Omega)$, assuming that we have the following measurements corrupted by noise

$$\mathbb{R}^3 \ni y(t) = h(g(t), t) + D\varepsilon(t), \quad (1)$$

where

$$h(g(t), t) = \Omega^\vee(t)$$

is the measurement output model and ε represents the unknown measurement error.

The error model is

$g^{-1}\dot{g} = \Omega$

$\dot{\Omega} = \mathbb{I}^\sharp \mathrm{ad}^*_\Omega \mathbb{I}^\flat \Omega + (\sum_{m=1}^{2} u^m \mathbb{I}^\sharp f^m)^\wedge + (B_2\delta)^\wedge$

with $(g, \Omega) \in G$.

Proposition

Consider the connection function $\omega^{(0)} = \frac{1}{2}\mathrm{ad}$, the second-order optimal filter is given by

$$\hat{g}^{-1}\dot{\hat{g}} = \hat{\Omega} + (K_{11}r^g + K_{12}r^\Omega)^\wedge$$
$$\dot{\hat{\Omega}} = \mathbb{I}^\sharp \mathrm{ad}^*_{\hat{\Omega}} \mathbb{I}^\flat \hat{\Omega} + (\sum_{m=1}^{2} u^m \mathbb{I}^\sharp f^m)^\wedge + (K_{21}r^g + K_{22}r^\Omega)^\wedge.$$

Let

$$\hat{y} = (\hat{\Omega})^\vee,$$

then the residual r_t is given by

$$r_t = \begin{bmatrix} r^g \\ r^\Omega \end{bmatrix} = \begin{bmatrix} 0_{3\times 1} \\ \mathrm{diag}\left\{\frac{q_1}{d_1^2}, \frac{q_2}{d_2^2}, \frac{q_2}{d_2^2}\right\}(y - \hat{y}) \end{bmatrix},$$

where r^g, $r^\Omega \in \mathbb{R}^3$. The second order optimal gain $K = (K_{11}, K_{12}; K_{21}, K_{22})$ is the solution of the perturbed matrix Riccati differential equation

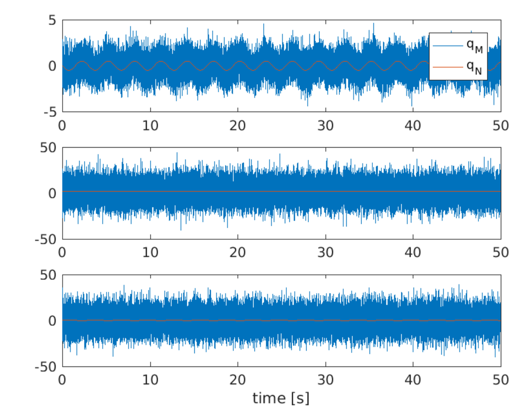
$$\dot{K} = -\alpha K + AK + KA^T - KEK + BR^{-1}B^T$$
$$-W(K, r_t)K - KW(K, r_t)^T.$$

Numerical Experiments

Input

$q_N = \sum_{m=1}^{2} u^m \mathbb{I}^\sharp f^m$

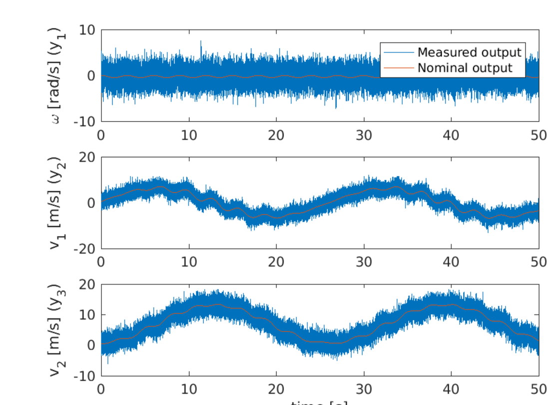
$q_M = q_N + B_2\delta$



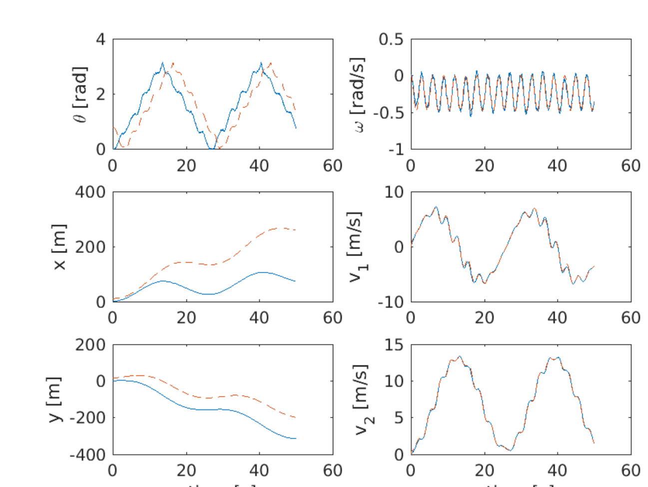
Output

$y_N(t) = h(\bar{g}(t), t)$

$y_M(t) = h(g(t), t) + D\varepsilon(t)$



Nominal (blue) and measured (red) trajectories and velocities



Future Work

1. Validation of the filter on an UGV we have in our lab;
2. include within the measurement equation also the Cartesian position of the vehicle;
3. consider multiplicative noise;
4. extension to the SE(3) case.

References

[1] F. Bullo and A.D. Lewis, "Geometric Control of Mechanical Systems".

[2] A. Saccon, J. Trumpf, R. Mahony and A.P. Aguiar, "Second-Order-Optimal Minimum-Energy Filters on Lie Groups".

[3] C. Segala, N. Sansonetto and R. Muradore, "Second-Order-Optimal Filter on TSE(2) and Applications". Under submission.