Poisson–Hopf algebras in Lie–Hamilton systems

Eduardo Fernández-Saiz
Department of Algebra, Geometry and Topology, Complutense University of Madrid
eduardfe@ucm.es

Ángel Ballesteros
Departament of Physics, University of Burgos
angelb@ubu.es

Rutwig Campoamor-Stursberg
Department of Algebra, Geometry and Topology, Complutense University of Madrid
rutwig@ucm.es

Francisco J. Herranz
Departament of Physics, University of Burgos
fjherranz@ubu.es

Javier de Lucas
Department of Mathematical Methods in Physics, University of Warsaw
javier.de.lucas@fuw.edu.pl

We merge quantum algebras with Lie systems in order to establish a new formalism, say Poisson–Hopf algebra deformations of Lie systems. Our procedure can be applied to those Lie systems endowed with a symplectic structure, the so called Lie–Hamilton systems. This is a general approach since it can be applied to any quantum deformation, any underlying manifold and any dimension. One of its main features is that, under quantum deformation, Lie systems are promoted to involutive distributions. Thus a quantum deformed Lie system has no longer an underlying Vessiot–Guldberg Lie algebra nor a quantum algebra one. However, it keeps a (deformed) Poisson–Hopf algebra structure which enables one to obtain, in an explicit way, the t-independent constants of motion from quantum deformed Casimir invariants which can be useful in a further construction of the corresponding deformed superposition rules. Moreover, we illustrate our general approach by considering the non-standard quantum deformation of sl(2) applied to well-known Lie systems, such as the oscillator problem or Milne–Pinney equation and several types of Riccati equations.

References

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