Numerical optimal control of nonholonomic systems

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In order to solve optimal control problems, standard calculus of variations can be applied to a Hamilton-Pontryagin action [1] to obtain a set of equations conforming, together with initial and terminal conditions, the necessary conditions for optimality [2]. Discretization of this principle allows us to obtain numerical methods naturally suited for these problems with good behaviour and geometric properties [3] [4].

Nonholonomic mechanical systems are a type of constrained systems whose equations of motion cannot be derived from a standard variational principle [5] [6]. These equations can be thought of as a special case of forced system and it is very common to default to this interpretation when dealing with optimal control problems for these kinds of systems. Unfortunately when we discretize using this interpretation, the constraint is generally not preserved.

In this talk I will show our latest strides in achieving a correct discretization of the Hamilton-Pontryagin action for the optimal control problem of nonholonomic mechanical systems with exact constraint preservation.

References

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