Tracking and filtering on SE(2)

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Attitude estimation is a core problem in many robotic systems, such as unmanned aerial and ground vehicles. The configuration space of these systems is properly modelled exploiting the theory of Lie groups. In this paper we propose a second-order-optimal minimum-energy filter on the matrix Lie group SE(2). The mathematics behind it is quite challenging and is not a simple generalization of previous results on Lie groups.

In the last decades many linear and nonlinear, deterministic and stochastic, observers have been proposed in the literature, some of them exploiting the theory of Lie groups to provide the proper mathematical structure for the attitude of a mechanical system [1].

We propose a *second order optimal minimum energy filter*. The filter is based on the results of Mortensen [2] where a methodology of generating progressive realizable approximations of a minimum-energy functional was proposed. The solution is obtained by differentiating the boundary conditions of the associated optimal control problem. It is called *second order* optimal in the sense that it is a truncation of the exact solution that would be an infinite dimensional system. The filter takes the form of a gradient observer coupled with a kind of Riccati differential equation that updates its gain (similarly to the standard Kalman filter).

The theoretical result in [3] is our starting point for the under submission paper [4]. We apply the theorem for a general Lie group to the case of a system modelled on the Lie group SE(2). Such group is very important to model vehicles moving on a plane and so, coupled with the dynamical part, to estimate their pose. However it is worth highlighting that this work is not just a 'straightforward' extension of the result in [3]. The SE(2) case turns out to be particularly complicated since we don't have the Lie algebra isomorphism between $\mathfrak{se}(2)$ and $\mathbb{R}^3$. Contrary to the SO(3) case, we can't identify the bracket operation of the Lie algebra $\mathfrak{se}(2)$ with the classical cross product in $\mathbb{R}^3$.

Our main contribution is then to derive a second order optimal minimum energy filter for SE(2) and to provide all the mathematics for the many technical operations needed to compute it.

We formulate the problem and we recall the explicit formula for the second-order-optimal minimum-energy filter on a general Lie group as stated in [3]. We described the SE(2) case, where we provide the main contribution in a Proposition with proof and all the mathematical details.

References

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[3] A. Saccon, J. Trumpf, R. Mahony and A.P. Aguiar, "Second-Order-Optimal Minimum-Energy Filters on Lie Groups", *IEEE Transactions on Automatic Control*, vol. 61, no. 10, 2016, pp 29062919.

[4] C. Segala, N. Sansonetto and R. Muradore, Second-Order-Optimal Filter on TSE(2) and Applications. Under submission.