Exact discrete lagrangian for constrained mechanics: an open problem

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The existence of an exact discrete lagrangian function for constrained systems (nonholonomic and vakonomic) is still an open problem in the field of geometric integration (see [1] for the unconstrained case). In the last few decades, an effort has been made to introduce geometric numerical methods, such as variational integrators, which preserve geometric structure. In the case of variational integrators, we discretize the lagrangian function to which we apply a discrete variational principle to obtain the discrete-time equations of motion, whose solutions are sequences of points which approximate the solution for the continuous-time problem.

In this talk we discuss constrained mechanics. After exposing the corresponding discrete descriptions (cf. [6], [8] or [7] for nonholonomic, [2] for vakonomic and [3], [5] for an introduction to constrained systems), we introduce the problem of finding an exact discrete lagrangian function for constrained mechanical systems. We will unveil the exact discrete space where nonholonomic dynamics takes place and explicitly define the corresponding exact retraction, using a nonholonomic connection defined in [4].

The discovery of the nonholonomic and vakonomic discrete exact lagrangian function will make an advance to the study of error analysis of numerical methods. For instance, an exact discrete constrained lagrangian function would have many applications on optimal control.

References

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