Groupoids and Distributions: A new way to deal with non-uniform material bodies

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A groupoid, called *material groupoid*, is associated in a natural way over a general non uniform body (see [1, 2, 3]). The *material distribution* is introduced due to the (possible) lack of differentiability of the material groupoid (see [4, 5]). Thus, the inclusion of these new objects in the theory of material bodies opens the possibily of studying non-uniform bodies. As an example, the material distribution and its associated singular foliation result in a rigorous and unique subdivision of the material body into strictly smoothly uniform sub-bodies, laminates, filaments and isolated points. Furthermore, the material distribution permits us to present a "measure" of uniformity of a simple body as well as more general definitions of homogeneity for non-uniform bodies.

References

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[1]This work has been partially supported by MINECO Grants MTM2016-76-072-P and the ICMAT Severo Ochoa project SEV-2015-0554. V.M. Jimnez wishes to thank MINECO for a FPI-PhD Position