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Group 2

Estimating the amount of wood

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Abstract

The amount of firewood is usually measured, sold and delivered according to three different units: solid, stere and loose cubic meter. Unfortunately it is not an easy task for the final consumer to check the effective amount of wood, nor change from one unit of measure to another, and even though some rules of thumb can be found, there is a lack of scientific literature about the existence and the validity of a conversion rate between these units.
Hence we now investigate this question, following two different approaches. We first perform an analytical and a more theoretical analysis, using basic geometry and Kepler's conjecture, in order to get bounds for the estimated conversion rates. Then we carry out image analysis and compute some numerical simulation of wood packing, considering both two and three dimensional case. Thus we obtain an estimate of the conversion rate, that allows us to make a comparison with the existing most common rule of thumb.

2.1 Introduction

Upon delivery of firewood the supplier can give the amount of wood in three different units, solid, stere or loose cubic meter. However for a consumer it's not trivial to check the actual delivered amount towards the amount presumed by the company [1].

Conversion factors between these units exist and this project aims to check to what extent they are accurate. The most common factor is 0.7, i.e. a stere is equal to 0.7 solid cubic meters (scm) and a loose cubic meter (lcm) is equal to 0.7 steres [2][4].

The exact conversion factors are dependant on various properties of the wood and also natural variations in the ordering of the stere or heap [3]. Hence this work will aim to check the suggested rule of thumb factor of 0.7 and determine in which cases it's a good estimate for the true conversion factor.

Definitions

scm Solid cubic meter, the volume of the wood not counting any void space.

stere A stere is a cubic meter of orderly stacked wood including void.

lcm A loose cubic meter is the required space for an unordered heap including void.

Methods

In order to answer the question of estimation, two main methods where used. Firstly, an analytic procedure to retrieve bounds for the estimated values. Secondly, simulations aimed to give estimates of the conversion fractions under certain assumptions.

2.2 Analytics

We looked at the simplest case of a stere, reducing it to the 2D case: the packing of circles of the same radius in a square.

From Kepler's conjecture, reduced to the two-dimensional case, we know that there are only two ways to optimally pack circles; the square packing and the hexagonal packing.

We pack the squares by placing circles of equal radius next to each other, starting from the lower left corner of the square. For the square packing we simply copy the row $n-1$ times, where n is the number of circles in the first row, and stack them on top, giving us $n \times n$ circles of the same radius.

For the hexagonal packing it's a bit more complex. Either the first row is totally (or almost totally) packed leaving only space for $n-1$ circles in the even rows, or the packing leaves a void space big enough to allow the same number of circles in each row. Also there's the possibility for the last row to be either odd or even when stacked. We will take a closer look at these cases in a moment.
All of this was first calculated by hand, and then put into a code in Maple.

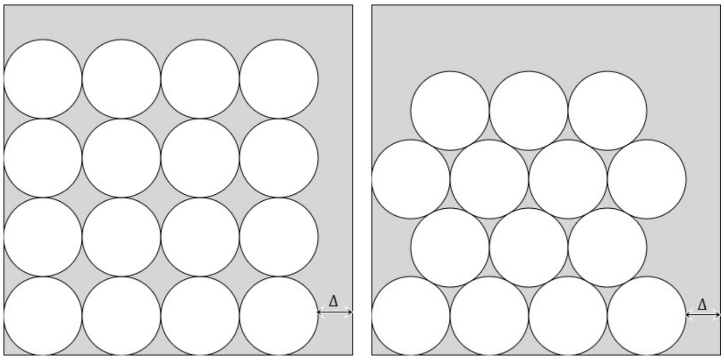

Figure 2.1: Square Packing and Hexagonal Packing with the void space Δ

When placing the circles in the first row, we do not necessarily reach the boundary of the square. If we describe the width of the void from the last circle in the bottom row to the square as Δ, then we have the four scenarios:

case 1: $\Delta = 0$

case 2: $0 < \Delta < r$

case 3: $\Delta = r$

case 4: $\Delta > r$

where r is the fixed radius of the large circles.

Since both types of stacking can leave a lot of void spaces, especially in the square packing and for circles with large radius, it makes sense to try and fill these spaces with circles of smaller radii.
In Figure 2.2 the slightly grey circles are the ones with a different radius than the fixed.

If we take a closer look at the hexagonal packing, we see that for the different scenarios of Δ, there will be up to four different big void spaces, which makes sense to try and fill:

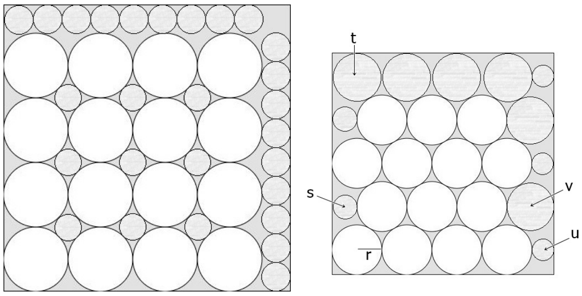

Figure 2.2: The packings with extra circles of different radii

$$\text{case } 1 = \begin{cases} s = \frac{r}{2} \\ t = \texttt{tCr} \end{cases} \qquad \text{case } 3 = \begin{cases} s = \frac{r}{2} \\ t = \texttt{tCr} \end{cases}$$

$$\text{case } 2 = \begin{cases} s = \frac{r}{2} \\ t = \texttt{tCr} \\ u = \frac{\Delta}{2} \\ v = \frac{\Delta + r}{2} \end{cases} \qquad \text{case } 4 = \begin{cases} s = \frac{r}{2} \\ t = \texttt{tCr} \\ v = \frac{\Delta + r}{2} \end{cases}$$

where `tCr` is short for top circle radius. We assume that in the physical world, you won't get too small branches in your stack, so we put in a lower limit of the size of a log, so for t: $\texttt{tCr} > 2$ cm and for u: $r > 2$cm. This is shown in the figure below.

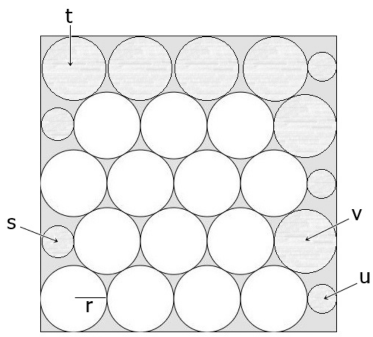

Figure 2.3: Hexagonal Packing that represents Case 2, where it shows s, t, u, v circles

Results

With the geometry in hand, we made a program in Maple to be able to plot the fraction and see the behaviour as we change the radius (between 1 and 16 cm).

In Figure 2.4 we compare the packings with and without extra circles of different radii, and of course for smaller radii the void spaces will be to small to allow new circles. This is seen as the packings are the same for small r's. But otherwise the packings with extra circles of course win!

In Figure 2.5 we compare the square and hexagonal packing, both with and without the extra circles.

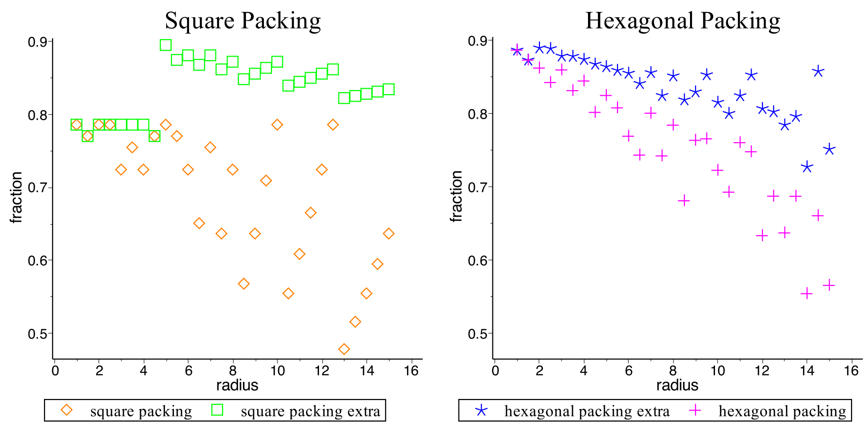

Figure 2.4: The packings with and without extra circles of different radii

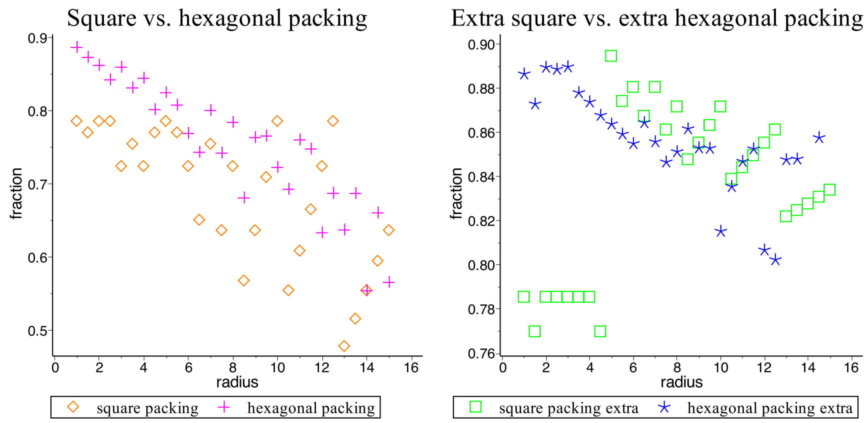

Figure 2.5: Comparisons of the square and hexagonal packings

As seen in the plots above, the most optimal packing vary as we increase the radius on the circles/logs. This packing is not close to reality, this is a perfect circle optimal packing, hence an unrealistic one - logs won't be perfect circles, and their radii will vary more. But we still gain an upper bound for our estimation, and some good prospects of the problem.

2.3 Simulations

The problem of stacking wood was divided into two parts:

1. estimating the amount of wood stacked in a box of unit volume (conversion from a solid cubic meter into a stere) and

2. estimating the amount of wood in a heap (conversion from a solid cubic meter to a loose cubic meter).

This way the problem was separated into 2D and 3D considerations.

The stere was represented by a pack of logs of one meter length (along the z-axis of the Cartesian coordinate system) and we assumed that the cross section remains the same along its length, neglecting any lumps or log curvature. This reduced the 3D problem of a box of stacked logs into a 2D problem of the cross section of the box, in the x-y plane.

For the loose cubic meter estimate a different, more complicated method had to be used.

2.4 Stere of wood

a) The simplified case

First, we started with the cylindrical shape with a fixed radius, so the problem was to stack circles in a 1m×1m square. We represented the circles with their centres and plotted the corresponding circumferences (with the same radii for all the logs) once the stacking simulation was done. The logs were stacked one by one from the bottom left position to the right and up by demanding that:

- the distance between the centre of a log and all of the previously stacked ones is equal to or greater than the log diameter,

- the distance between the centre of a log and the edges of the box is equal to or greater than the log radius.

As an illustration of the method, the simulation result for the radius $r = 5$ cm is shown on Figure 2.6. We used this procedure in a series of simulations with radii starting from $r = 1$ cm up to $r = 20$ cm.

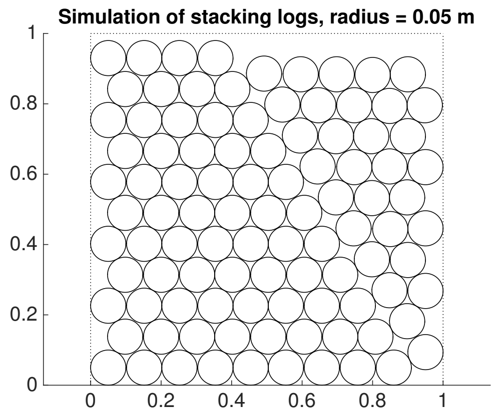

Figure 2.6: Simulation of packing of cylindrical logs

b) The general case

In the next step we used a different algorithm, one that allowed us more complex shapes. The 1m × 1m box was represented by an initially empty grid and the log cross sections were represented by small grids with elements that are either empty or contain a piece of log (matrices with just zeroes and ones). These smaller grids were then stacked into the larger one (the 1m × 1m box) from the bottom left position to the right and up. The algorithm searches for the first bottom left position that is empty, i.e. the position where the large matrix has enough empty cells to fit the small matrix. One possible stack in this simulation is shown on Figure 2.7, where we packed circular, half-circular and quarter-circular logs.

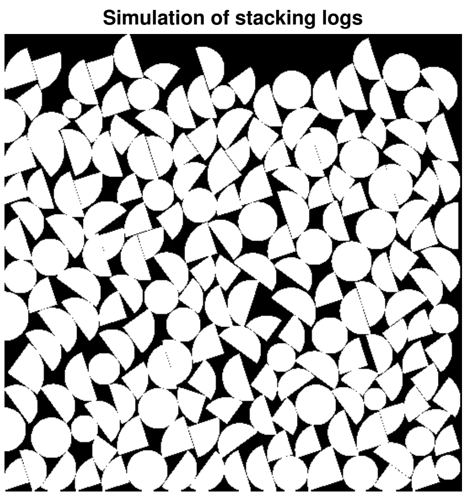

Figure 2.7: Simulation of packing of logs of three different shapes

Once we had generated a set of logs of desired shape and radii, we made multiple simulations, permuting the order in which logs were placed into the box to get statistically significant results. We used logs of circular, half-circular and quarter-circular shape, with radii drawn from a uniform, a Poisson and a normal distribution.
We choose not to use a fixed radius in order to get a realistic model and tried these distributions with the following motivation:

- the uniform distribution is the simplest possible model,
- the normal distribution is commonly found in nature,
- the Poisson distribution has positive values and thin tails (we don't expect to have many logs with very large/small radii).

c) Image Analysis

Looking at Figure 2.6 and Figure 2.7, the next method comes to mind naturally: as a substitute for measurement, we performed an image analysis of a photograph of stacked wood logs.

We extracted a black and white version of the photograph by only considering the red channel with a threshold, which proved to be superior to blue or green as discriminator between wood and void, see Figure 2.8. The threshold in the red channel separates the wood logs (now kept as white) from the void spaces (black), see Figure 2.8. The number of white pixels relative to the total number gave us the desired coverage fraction.

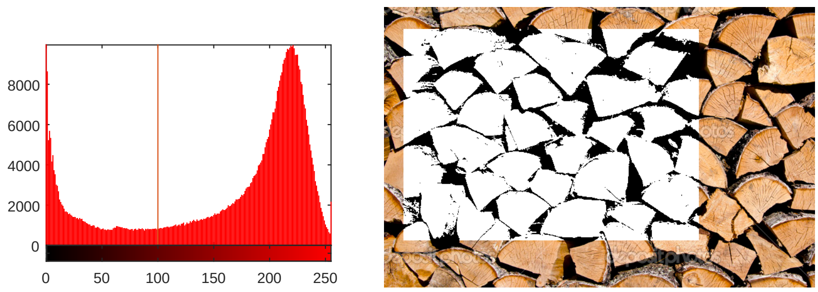

Figure 2.8: Illustration of a threshold in the red channel used to separate wood from empty space and the resulting black and white image on the right.

Results

a) The simplified case

In these first simulations we only used circular logs and in a particular stacking all were of the same radii. We ran a series of simulations, with radii starting from $r = 1$ cm up to $r = 20$ cm. The coverage fractions obtained are shown on Figure 2.9.

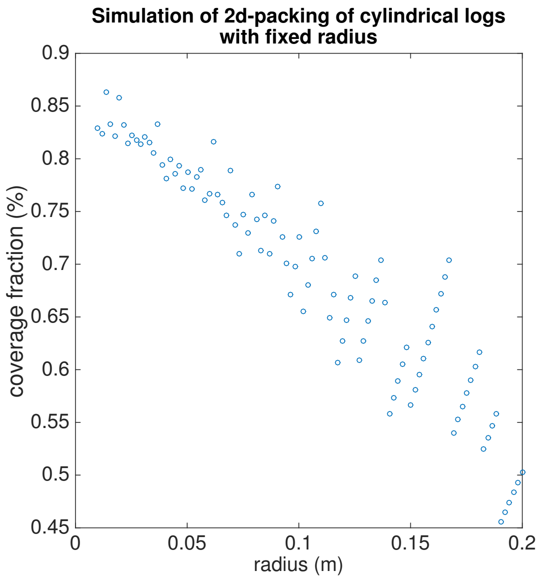

Figure 2.9: Results of different simulations of 2d-packing of cylindrical logs

We can identify a decreasing trend in the coverage fraction of the box with increasing log radii.
Furthermore, there is significant noise for small values of radii and for log radius greater than 0.1 m, the coverage fraction grows locally with the radius and exhibits a jump every time the stacked number of log rows decreases (the almost-vertical sets of points on the right). In this limit, the logs are no longer nicely stacked (there is a lot of empty space on right ends of the rows) so the results for these large radii are probably not realistic. If we only take radii between around 5 and 10 cm we have the coverage fraction of around 70%. Of course, a realistic stack of wood would contain larger radii logs, but they wouldn't all be circular so this simulation is to be viewed as a rough estimate.

b) The general case

The results of 2D stacking simulations for non-circular shaped logs are shown on Figure 2.10 and Figure 2.11. The first three graphs were obtained with logs of circular, half-circular and quarter-circular shape with radii drawn from a uniform, a Poisson and a normal distribution, respectively. The fourth case is with radii drawn from a normal distribution but only with quarter-circular shaped logs.

Overall, all the results exhibit the same decreasing pattern like the ones in the simplified case (Figure 2.9). We can observe some fluctuation in the

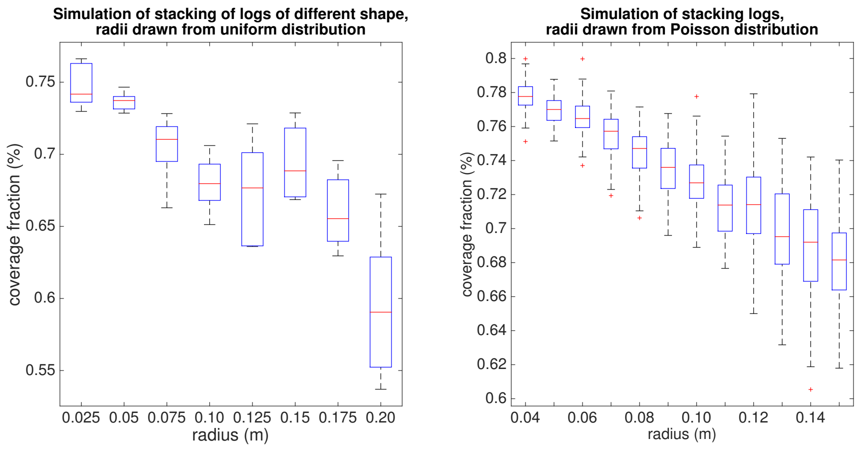

Figure 2.10: Results of different simulations of 2D-packing of logs of different shapes and with radii from different distributions

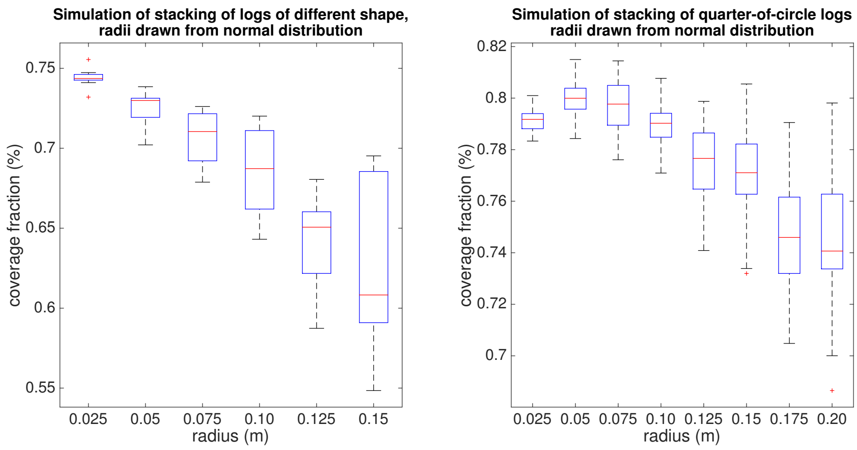

Figure 2.11: Results of different simulations of 2D-packing of logs of different shapes and with radii from different distributions

data, but we believe these can have the same explanation as given in the previous case.

Clearly, the general case is an improvement of the simplified one since all of the obtained coverage fractions range between 0.6 and 0.8%, just as expected.

c) Image Analysis

The image analysis of the chosen photograph of regularly stacked wood gave us a coverage fraction of 76%. This result falls into the previously obtained coverage intervals so we see it as a confirmation of our earlier findings.

Further analysis could involve a measurement of the average radius of the photographed logs, in order to better compare our previous simulations with real stacking.

2.5 Loose cubic meter

a) 2D model: rectangular coverage

As a way of representing a 3D pile of wood with a 2D image (this time from the aerial view), we used again a 1m $\times$ 1m box (large grid) and represented the logs with matrices with zeros and ones (small grids). Since one meter long logs are usually sawn in three pieces before being delivered as a heap of wood, we chose rectangular shapes of length 0.3 m and width 0.1 m. This is how a 0.3 m long cylindrical log with a radius of 0.05 m, lying on the ground, would look like from above. We then gave these rectangular logs a random orientation in their grids (corresponding to a random orientation of a log lying on the ground) and placed them into the 1m $\times$ 1m box, from the bottom left position to the upper right as before, with the sole condition that they can only touch but not overlap. The absence of any optimisation can be seen as a way of simulating the loose positioning of logs on the ground layer of a real pile. We calculated the percentage of volume such a layer of randomly positioned logs occupies and assumed that the whole 3D box is filled with repetition of the bottom layer, which allowed us to calculate the volume coverage fraction of this 1m $\times$ 1m $\times$ 1m box.
The resulting layer image of such a stacking is shown on Figure 2.12. To obtain the coverage fractions, we performed multiple simulations with log orientations randomised in every run, for four different radii (half widths of the rectangles): 0.025, 0.05, 0.075, and 0.1 m.

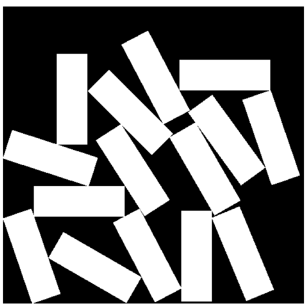

Figure 2.12: Image of stacking of the 3D-logs (from a top view), radius = 0.05 m, length = 0.3 m

b) Full 3D simulation

As a final estimate of the loose cubic meter a 3D simulation was conducted. Again the 1m $\times$ 1m $\times$ 1m box was discretized into a grid, where each cell contains either wood or void, the cells where cubes with $\frac{1}{3}$cm side length. A set of logs where generated, where all logs have cylinder shapes, are 30cm long and have a radius between 3 and 10 cm, drawn from a uniform distribution. The set of generated logs had a total volume of 1 scm, which ensured the existence of a sufficiently large subset (the loose cubic meter).

The simulation then used the following logic. Starting with the empty container it randomly picks a log from the set of logs and simulates a drop by starting at a random xy-coordinate and then finding the lowest possible z-coordinate with respect to any rotation. To simulate a bounce it checks if a lower z-value can be attained in the four neighbouring positions 1cm away. If this is the case, the drop continues from this xy-coordinate. If no lower position is found, a new log is drawn from the set and the process repeats.

When a log doesn't fit within the container, a neighbourhood of it's initial xy-position, from which it was dropped, is considered depleted (unusable for a drop). A new starting point is then randomly chosen from the xy-positions which are not depleted. If this log doesn't fit anywhere, i.e. all xy-positions are considered depleted, the simulation for it terminates and a new log is randomly picked. Every time a new log is drawn all previous informations about search locations are discarded, so again all of the xy-positions are (initially) considered usable for a log drop.

This procedure guarantees that each log will be placed in some local minimum. The choice of this procedure instead of an physically correct simulation was motivated both by a need to simplify the model and an imperative of being able to make an implementation in the given time frame. There are some simplifications; we do not properly account for friction or gravity, e.g. in our simulation the log will not roll but only slide. Hence we do not need to regard any particular bark or surface structure. Figure 2.13 illustrates the result from a simulation. The result looks rather stable - it should not move significantly if subjected to gravity, which somewhat validates the described procedure.

Results

a) 2D model: rectangular coverage

For the heap of wood in 2D approach, we ran the simulations for cylindrical log radii of 2.5, 5, 7.5 and 10 cm; the results are shown on Figure 2.14.

These simulations give us a lower bound of the true percentage of coverage, since it is still possible to place more logs in vertical or oblique position. But, as we can see from Figure 2.14, this lower bound is not far from the

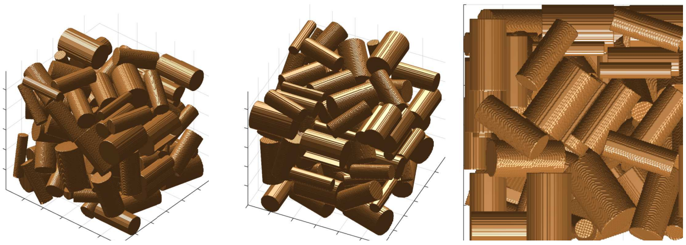

Figure 2.13: Illustrates the result of an 3d simulation, wood fraction is 36%.

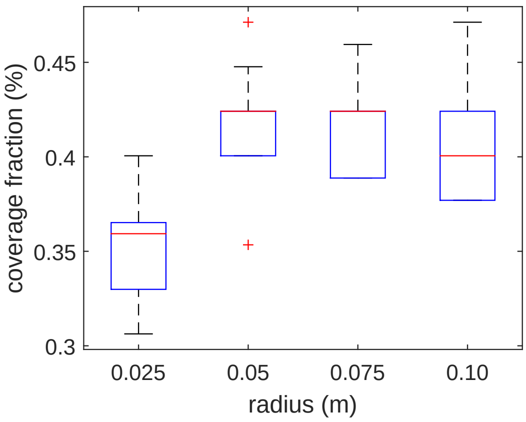

Figure 2.14: Simulation results for a heap of logs of different radii (2D approach) as in figure 2.12.

coverage fraction 0.4% suggested by the literature, especially for bigger log radii.

b) Full 3D simulation

Figure 2.15 shows the results from 50 different 3D simulations, all using the same set of logs. From the figure we see that the simulation gives a mean value of 0.36 as a conversion factor between scm and lcm. This deviation from the 0.49 (as suggested by industry e.g.[4]) can mainly be explained by two factors, firstly the logs are usually not circular cylinders but are generally cloven, e.g. photograph in Figure 2.8, which expands the set of possible configurations and hence might decrease the void fraction in the loose stacking. And secondly upon inspection of Figure 2.13 we can conclude that if this configuration would been subjected to gravity it would collapse somewhat hence give a more dense packing.

However both these simulations and the previous 2D model results indi-

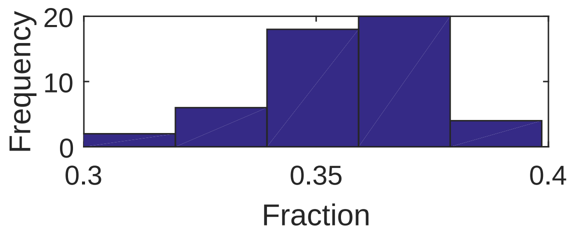

Figure 2.15: Conversion fraction for the loose cubic meter for 50 different simulations using the same set of logs.

cate that, in the case of a heap of circular cylinders, the suggested fraction of 0.49 found in the literature might be too optimistic and that 0.4 might be more reasonable.

2.6 Conclusions

Both the analytic and simulation approach confirm that 0.7scm $\approx$ 1 stere. But if a more precise measure is to be used, more properties of the logs need to be accounted for, e.g. non-cylindrical log shapes, curvature of logs along their length, lumps and other shape deformities. For the loose cubic meter our estimate of 0.4 scm differs from the suggested 0.49 scm and our simulations showed a large variance in the estimate between different permutations of the logs. From this we conclude that a proper conversion rate might not exist, but the suggested 0.49 scm might serve as a decent rule of thumb.

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