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Group 3

Optimal solution for a KAMIKAZE RANGER

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Abstract

Amusement rides such as the Kamikaze Ranger are very popular around the world and give joy to many. One issue is that they can be heavy on energy consumption.

In this project optimisation of energy required by the ride is studied. It is shown that if the ride is inefficient enough it is possible to make large reductions of energy consumption without greatly changing the experience. Further studies are suggested.

3.1 Introduction

Kamikaze Ranger

The Kamikaze Ranger is a pendulum amusement ride first produced by Far Fabbri & Sartori in 1984. It has two 16 man gondolas and has been sold in more than 150 copies over several different versions.[2]

Figure 3.1: Image of the Kamikaze Ranger

Task

The assigned task was inspired by the needs of an owner of the Kamikaze Ranger who wants to adjust the current trajectory of the attraction such that it consumes less energy. The task was thus reduced to the following:

- Derive a model for the Kamikaze Ranger.
- Simulate a path for the Kamikaze Ranger utilising the model.
- Optimise a path that is close enough to the previous path but that minimises energy consumption.
- Check if the path is reasonable, i.e. are the g-forces survivable.

3.2 Model

To model the motion of the Kamikaze Ranger we started considering a mathematical pendulum with friction, using the following assumptions:

- The rod on which the bob swings is massless, inextensible and always remains taut. It has length l.
- The bob (that represents the gondola) is a point mass, of mass m.
- Motion occurs only in two dimensions, i.e. the bob does not trace an ellipse but an arc.

The differential equation which represents the motion of our model is

$$\frac{d^2\varphi}{dt^2} + \frac{g}{l}\sin\varphi + \frac{k}{m}\frac{d\varphi}{dt} - \frac{M}{l^2 m} = 0\,, \tag{3.1}$$

where g is the gravitational acceleration, φ is the angular displacement and M represents the angular momentum. We obtained some real measurements of the attraction. The parameters were set into the values from Table 3.1. These values where simplifications and estimates made from the data available from the producers. [1]

	Value
gravity g	9.81 m/s^2
mass m	6500 kg
length l	20 m

Table 3.1: Simulation parameters

Assumptions

Apart from what is implied by modelling the Kamikaze Ranger as a mathematical pendulum we added a few assumptions to simplify our model. Firstly we assume that the added angular momentum is proportional to the energy consumption. This means we ignore the low efficiency of electrical motors on low output. We can then optimise the added angular momentum instead of energy consumption since the optimisations will be equivalent. Secondly we assume that the engine can switch energy output instantaneously allowing us to allow the solutions to have discontinuous functions. The last assumption was that the amusement ride stops with a negligible force, so the path is created only for riding time but not for the stopping.

Desired path

We obtained a desired path by applying force to pendulum during the whole ride (see Figure 3.2). The path was chosen as an example, so we cannot proclaim that the ride is really amusing.

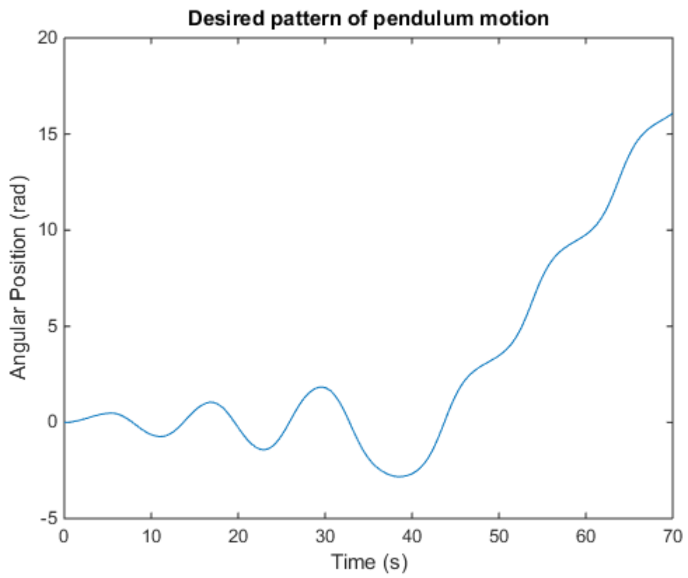


Figure 3.2: Desired path

After choosing the path we calculated the acceleration to test the survivability of the ride (see Figure 3.3).

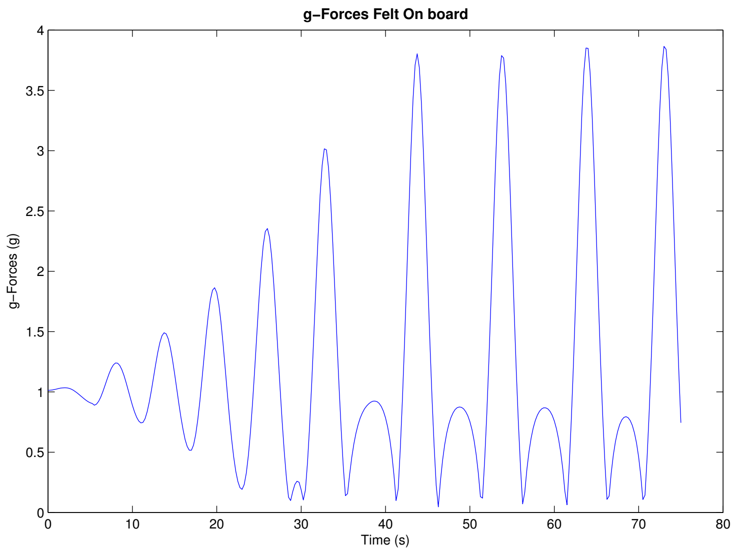


Figure 3.3: g-Forces of desired path

3.3 Approach

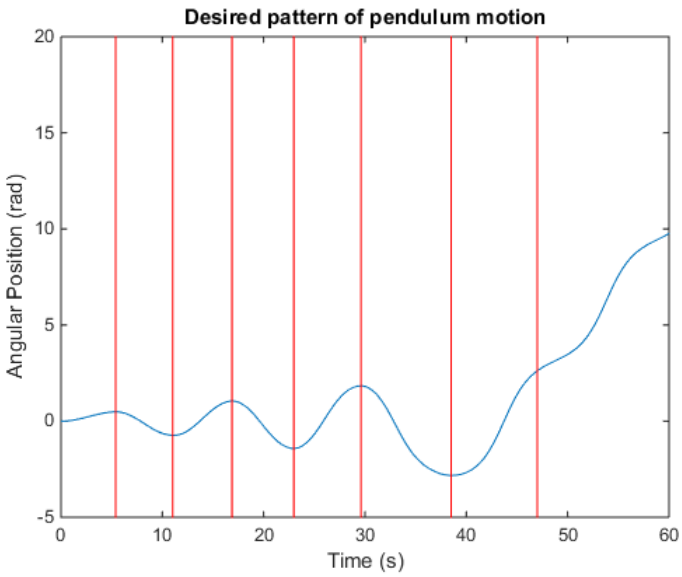


Figure 3.4: Division into sub-intervals

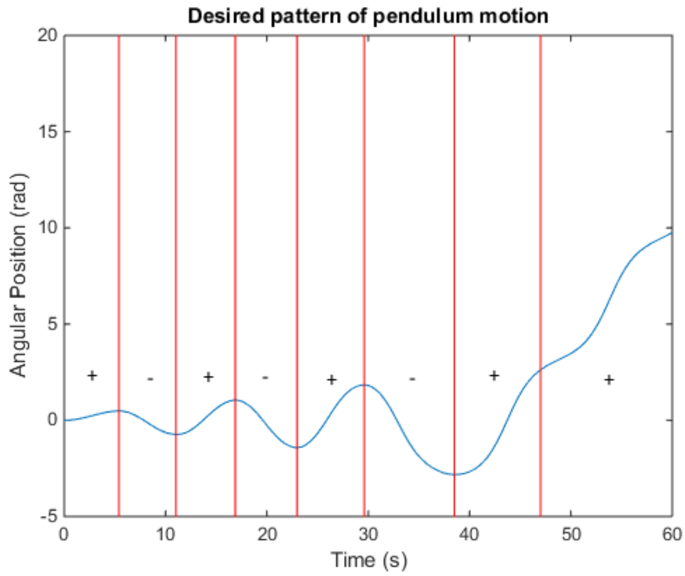


Figure 3.5: Deduction of force's sign

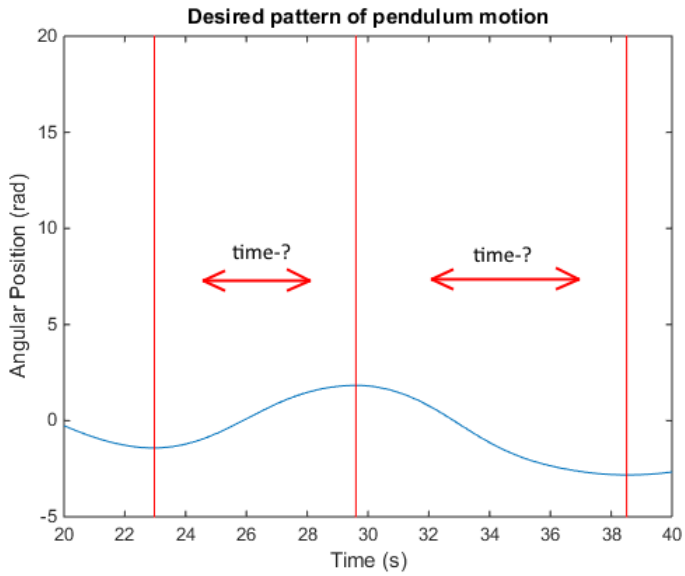


Figure 3.6: Estimating time, so that path is approximated

We decided to go for two different approaches for the optimisation.
First approach is the simplest one: since the engine is electric its power can be considered constant, so the energy only depends on time. Therefore the algorithm goes like this:

1. Divide the desired path into sub-intervals (see Figure 3.4).

2. Deduce the sign of force from the path (see Figure 3.5).

3. Estimate the force applying time in every sub-interval (see Figure 3.6).

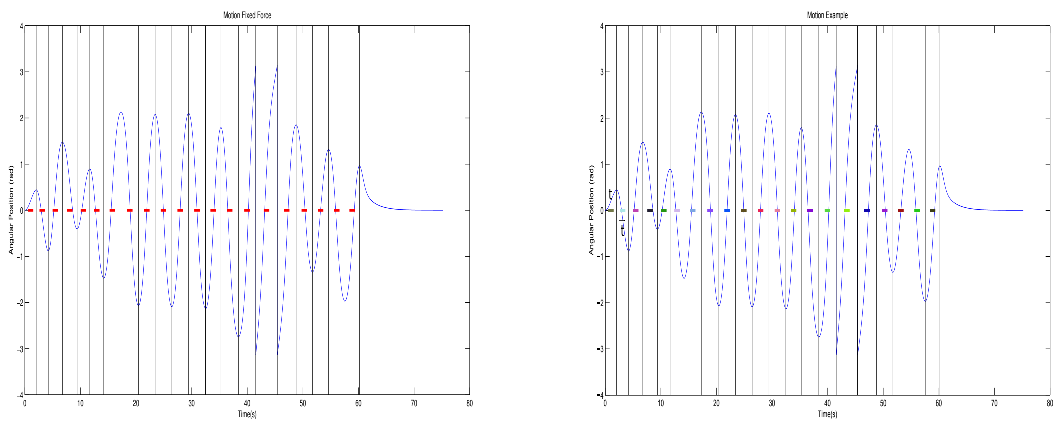
Hence the problem can be represented as minimisation of time for which force was applied.
With the second approach we assume that power still is constant but has some levels, so in each interval of time the force is constant but it varies from one interval to another (see Figure 3.7b), so the problem can be represented as minimisation of time multiplied by force. Hence, the algorithm for the second approach only differs from the previous one in the last step:

1. Divide the desired path into sub-intervals.

2. Deduce the sign of force from the path.

3. Estimate the time of applying the force *and* its value in every sub-interval, so that desired path is approximated (see Figure 3.6).

We then compared the different versions to see which of the approaches should be chosen.



(a) Fixed forces (b) Variable forces

Figure 3.7: Approaches

Optimisation

To optimise the ride we constructed two objective functions: one for each of prescribed approaches (Formulas (3.2) and (3.3) correspondingly). Here t stands for time of applying the force, φ is angular position, F is the force and β is a parameter of optimisation which allows to choose whether the solution must be closer to the desired path or to consume less energy.

$$\min_{t} \sum_{i}^{n} t(i) + \beta \cdot \|\varphi_{new} - \varphi_{desired}\|_2 \tag{3.2}$$

$$\min_{t,F} \sum_{i}^{n} t(i)F(i) + \beta \cdot \|\varphi_{new} - \varphi_{desired}\|_2 \tag{3.3}$$

$$t(i) \geq 0, F(i) \geq 0 \quad \forall i \in I \tag{3.4}$$

$$\beta > 0 \tag{3.5}$$

Where I is the index set of the chosen intervals.

3.4 Tools and methods

We used for all of our calculations the MATLAB software, since it is a powerful and standard tool for numerical simulations. Specifically for solving the ordinary differential equation (3.1) the *ode45*-function of MATLAB has been applied. This function has been chosen as (3.1) is non-stiff. Moreover, we used the standard time-steps for the *ode45* in MATLAB, which are adaptive to minimise error. This means they most likely are finer-grained compared to the original time-steps. Therefore, a linear interpolation has been applied.

3.5 Results

As referred in Section 3.3 we have optimised the path according to two different strategies. The first one with respect to the time intervals while keeping the force applied in each interval constant, the second with respect to the time intervals and also the force values in each interval.

In Figure 3.8 one can see the optimisation with respect to the time intervals and in Figure 3.10 the optimisation with respect to both the time intervals and force values.

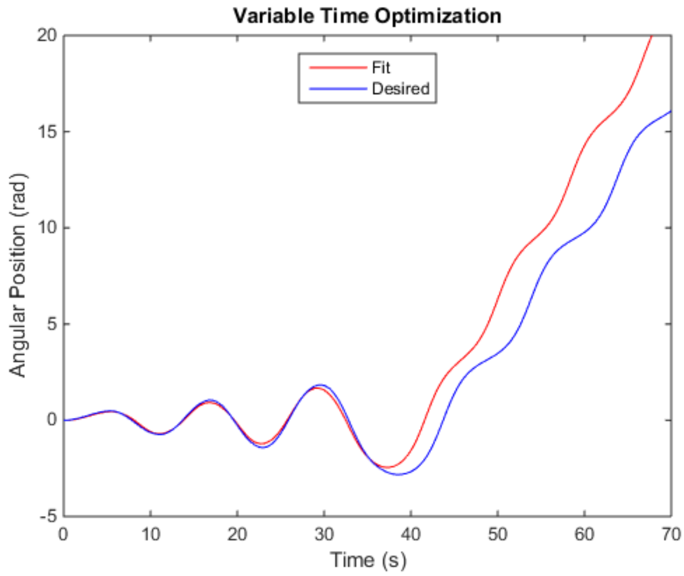


Figure 3.8: Fit of desired path by the model with constant force and variable time

One can notice that in Figure 3.8 the gondola takes a similar motion to the original path, except the motion is achieved in a much shorter time period. This means that the angular velocity is higher in the optimised path. The acceleration felt on-board the gondola is presented on Figure 3.9.

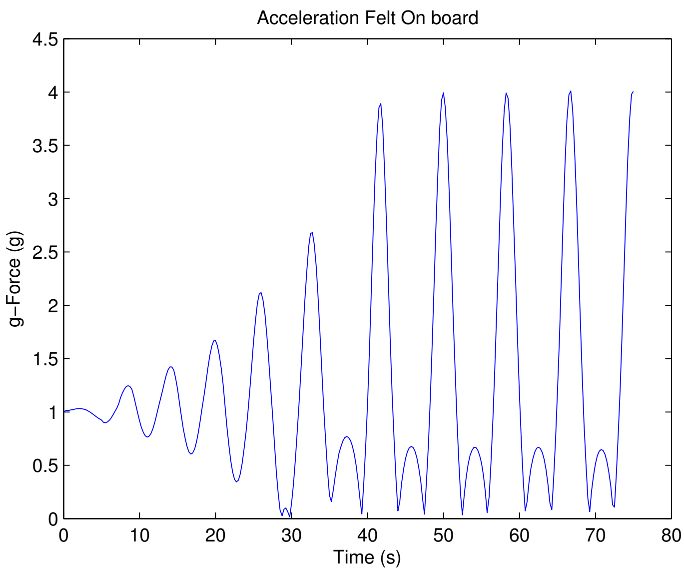


Figure 3.9: g-Forces of the approximating path with constant force and variable time

The acceleration values are between 0 and $4g$ (g is gravity). Knowing that g-forces up to $5g$ can be handled without issues [3], we can rest assure it is quite safe.

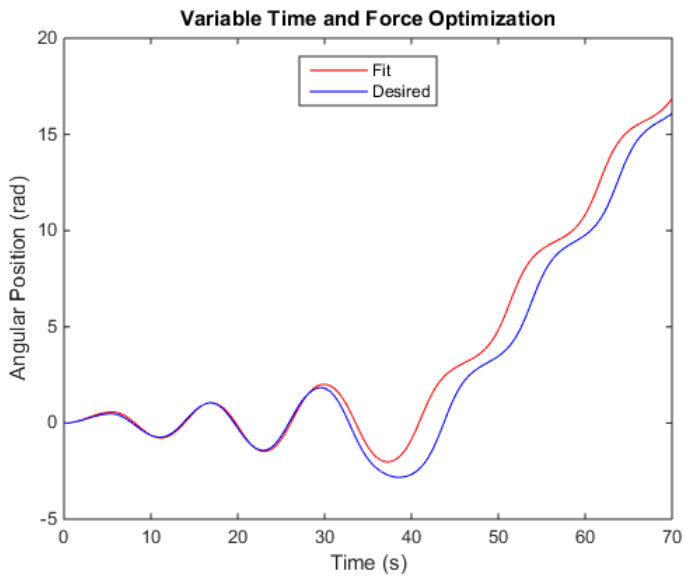


Figure 3.10: Fit of desired path by the model with variable force and variable time

From Figure 3.10 we can see that the optimised path is much similar to the original path, following the same curves, but shifted in time. The two paths show a bigger difference around the 40 seconds mark, prior to the consequent loops. We can foresee an energy saving in this stage since the gondola does not reach out too far until it goes back.

The acceleration felt on-board the gondola is plotted on Figure 3.11.

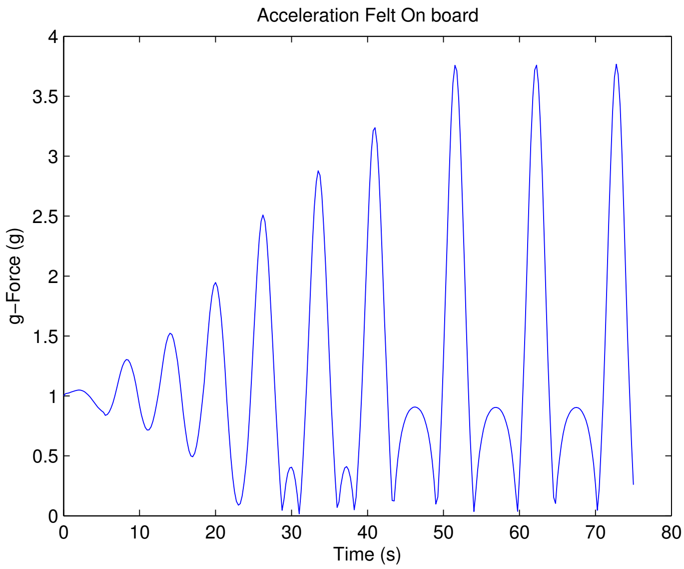


Figure 3.11: g-Forces of the approximating path with variable force and variable time

The acceleration range is practically the same as in the first scheme, meaning that it is also safe.

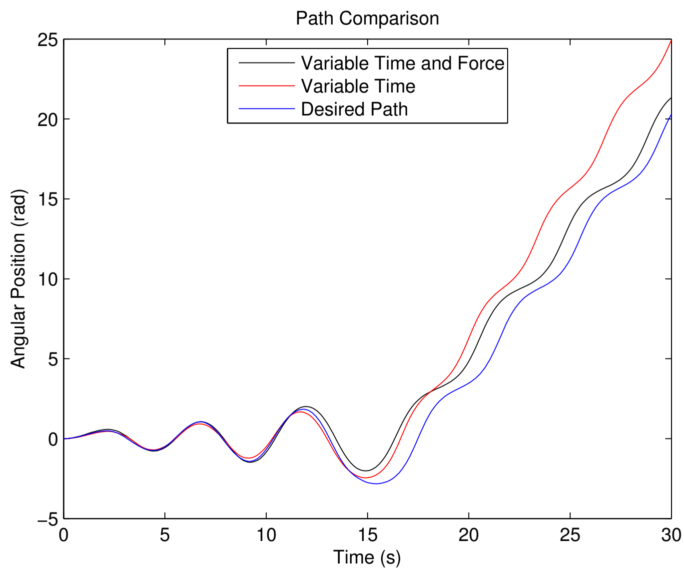


Figure 3.12: Comparing the optimised paths

The achieved solutions through our 2 schemes have been plotted for comparison in Figure 3.12. One can notice that both the first and the second schemes lead to similar result regarding the shape of the curve, i.e. motion of the gondola would be almost the same, except shifted in time. Both the schemes yield a slightly faster motion in comparison with the original path. Also, the first scheme (variable time intervals with fixed force) yields a faster motion than the second one (variable force within variable time intervals).

At this point, we can argue that we will reach better results (i.e. consume less energy) with the results obtained through the second scheme, since there are more variables we can optimised with respect to.

In Table 3.2 we can verify the energy consumption of original path and the paths yielded by the optimisation schemes.

	Energy	Reduction
Desired path	6.4912e+05	-
Variable time, constant force	2.4569e+05	62.15%
Variable force and time	2.3176e+05	64.3%

Table 3.2: Optimisation results

As expected, the energy reduction obtained with the second scheme was slightly better, with an extra 2% of savings.
The results vary a lot upon the current state of the energy function. In this sample case, we achieved really high energy savings because the initial force function was not adequate at all for the ride.
Finally, a quick note about the optimisation. We can obtain higher energy savings in case we decide to prioritise the savings in detriment of the ride being similar to the original ride. We can do this by varying the parameter β in the objective functions stated in Section 3.3. A high value of β will increase the similarity between the optimised and original paths. The opposite also holds true. In conclusion, one first has to know what to prioritise for more suitable results.

3.6 Further work

Further studies can be conducted in miscellaneous directions. Let us consider those we think most promising:

- Using the physical pendulum as a basic model instead of mathematical one.

- Regarding the shape of the gondola to consider the resistance of the air.

- Creating continuous functions of the force to apply the laws of functional analysis.
- Researching the field of solvers for differential equations as well as for optimisation.

3.7 Conclusion

- Optimisation can give good energy reductions if original path is bad enough.
- Following the desired path more closely will give higher energy consumption.
- We cannot be sure if there does not exist a better solution with the same (or closer) distance to the desired path.
- The contradiction between good approximation of the path and slow energy consumption can be adjusted with β.

Bibliography

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