Augmented Reality, Maths Walks and GeoGebra

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Augmented Reality (AR) is a fast-growing technology (c.f [13]), being involved, in recent years, in diverse applications for Science Education ([11], [12]) and, in particular, for Mathematical Education. In some of them AR aims to provide a better understanding of the spatial vision of mathematical objects, while in other instances it helps enhancing the interaction between reality and mathematical operations involving objects placed in a real context, i.e. solving mathematical equations written in a blackboard, graphing functions described by some parametrization, etc. (cf. [2], [14]).

On the other hand, there is an increasing trend to establish routes (of all kinds: through the countryside or inside towns and villages, focusing on landscape and nature or on human marks, etc.) proposing and carefully documenting diverse mathematical activities that could be attempted by the traveller along these tours.

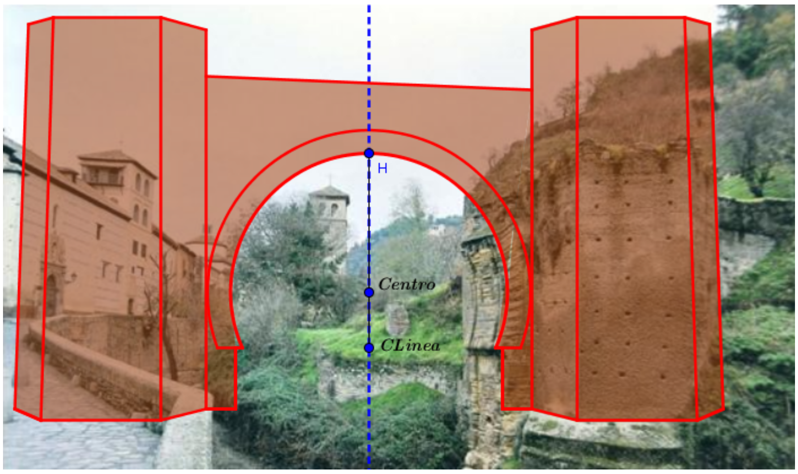

Figure 1. Reconstruction, through a GeoGebra layer, of the ruins of a medieval bridge in Granada (see [7]).

A paradigmatic example is the development of promenades along the historical quarters of European towns, highlighting the mathematical aspects of the

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artistic monuments and buildings, as described in [7] or [10]. This is done in a way that emphasizes both scientific and artistic divulgation, by presenting mathematical explanations of concepts used by artists and architects, considering mathematical walks as a suitable framework to gain added insight – through mathematics-- into the artistic interpretation (symbolic, decorative or functional) of monuments, see [8].

We would like to emphasize that, in these references, the dynamic geometry software GeoGebra[4] has played a fundamental role. In fact, it has been used for adding mathematical layers to monuments' photographic images, thus highlighting, quite easily, their geometric or mathematical features. GeoGebra allows us to draw, over each picture, some math curves that follow a particular shape on the represented object; to display symmetries, alignments, parallelisms, etc. in the given picture; or to measure lengths, heights, angles, determine ratios…, and many other mathematical aspects of the different parts of the image. It is in this way that more than 400 applets of GeoGebra illustrate the book [7], some of them also shown in [9].

If the connection of GeoGebra with math walks is quite well established, some recent trends consider the connection of math promenades and augmented reality. Two on-going, and very different, examples could be, first, the just about started, European Erasmus+ project "Mobile Math Trails for Euorpe, MoMaTrE[5]", involving a mobile phone app that intends to be an essential help for the localization of key points and objects along a trail, and for the subsequent call to some documents containing a proposal of mathematical activities to be carried out by the student, but it must be remarked that augmented reality is practically limited to this *geolocalization* feature.

A second example could be the on-going project from the University of Aveiro, "EduPark[6]", that explicitly mentions augmented reality in its description, and includes some activities related to mathematics.

Finally, in this *trio* (augmented reality, GeoGebra, math walks), we should mention the recent launching, on September 2017, of a GeoGebra Augmented Reality (AR) app[7], helping to visualize math figures generated by GeoGebra 3D, allocating them on a surface chosen by the user (e.g., onto the image of his/her office desk, as displayed on a smarthphone screen), and making possible to walk around or inside these 3D objects, while taking screenshots from different points of view.

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[4] http://geogebra.org
[5] http://www.momatre.eu
[6] http://edupark.web.ua.pt
[7] https://itunes.apple.com/us/app/geogebra-augmented-reality/id1276964610?mt=8

In our CADGME communication we propose to describe some aspects describing our on-going work towards the integration of the three above mentioned elements in the *trio* (as outlined in [6]).

Thus, we plan to explore the applications of computational techniques to aid with and to enhance the dissemination of the characteristics of our cultural heritage elements. Previous experiences [9] have shown the utility of image enrichment with vector information. Hence, in the context of mathematical walks, we plan to produce, through GeoGebra, a vectorial .svg file (by exporting the .ggb file). This file will be used in order to add a mathematical layer with *markerless* AR technology, that should be automatically displayed over the image of a given artistic object, boosting our sentiments by the simultaneous appreciation of the real object in front of us, together with its associated mathematical aspects.

However, state of the art computational techniques in the fields of Computer Vision and Computer Graphics, together with ubiquitous mobile devices and powerful desktop computing platforms provide new functionalities, which can be used to leverage dissemination of the mathematical properties of buildings and artworks.

In fact, *marker* and *markerless* [1] augmented reality allows mobile devices users to overlay virtual models on top of video streams, including the life video stream from the device camera. Such virtual models are aligned in real-time with the real objects visible on the video frames. This allows the usage of a video stream recorded on-site to visually relate real objects images with their derived geometrical structures.

Three-dimensional models acquisition is usually a cumbersome process, which requires expensive equipment (laser or structured light scanners, for instance). However, several authors have designed new computational photography and photogrammetry techniques (called *multiview stereo* [4]), which allow acquisition of 3D models from real objects, by using just an unstructured dense set of photographs of those objects.

The availability of open source implementations [5] allows producing virtual models of cultural heritage elements, without the need of complex hardware. This leads to advanced capabilities such as obtaining new views, not present in the original, obtained on-site, photograph set; to make direct distance measurements; or to add new geometric forms to the 3D model. We think this functionality is particularly well fitted to explore and disseminate mathematical properties of our cultural heritage items.

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