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The Roland and Astrid Dana-Picard Chair for Mathematics, Education and Judaism

CADGME 2018
UNIVERSIDADE DE COIMBRA

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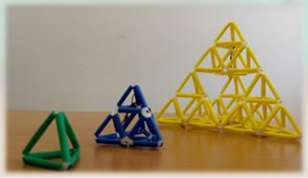
Fractals and tessellations: from K's to cosmology

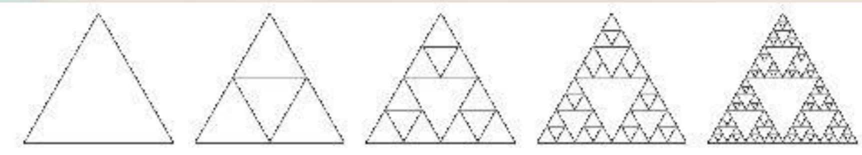
Thierry Dana-Picard and Sara Hershkovitz

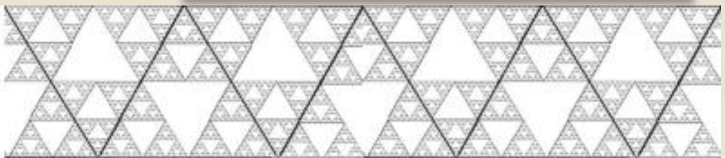
CADGME 6

Coimbra, Portugal

June 26th, 2018







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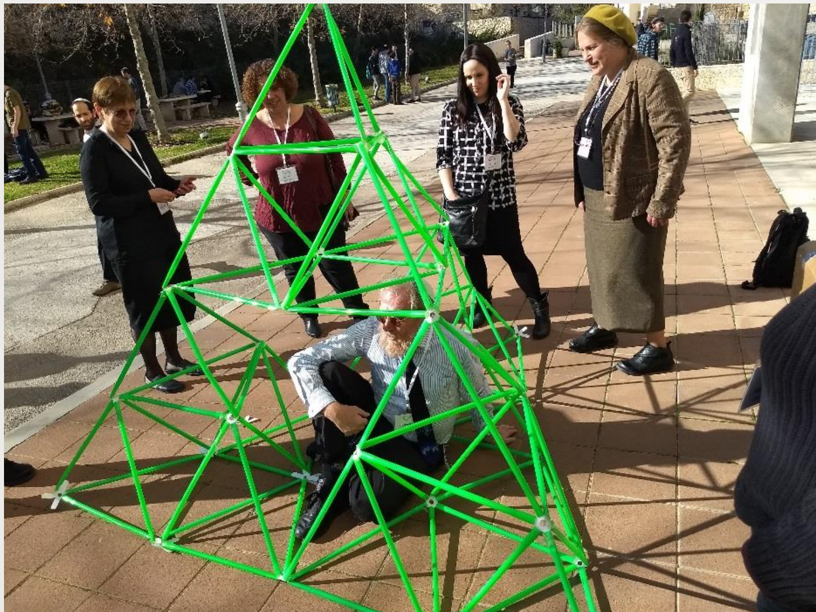
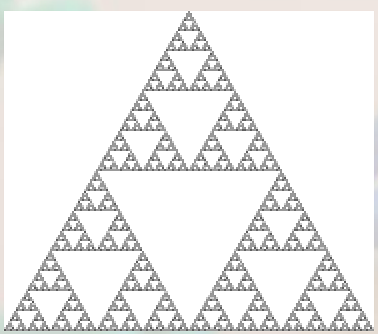
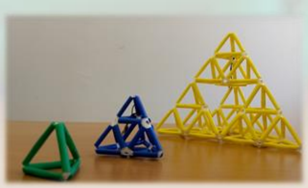
“Doing mathematics” with low tech

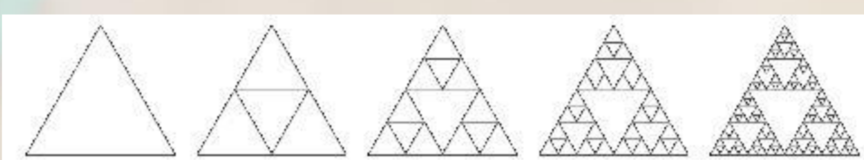


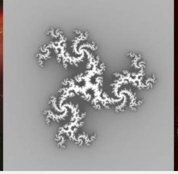
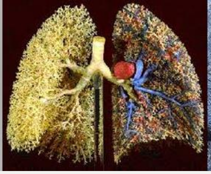
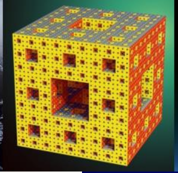
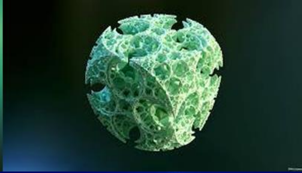
A fractal is a never-ending pattern. Fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop. Driven by recursion, fractals are images of dynamic systems – the pictures of Chaos. … Fractal patterns are extremely familiar, since nature is full of fractals. For instance: trees, rivers, coastlines, mountains, clouds, seashells, hurricanes, etc. Abstract fractals – such as the Mandelbrot Set – can be generated by a computer calculating a simple equation over and over.

Credit: Fractal Foundation

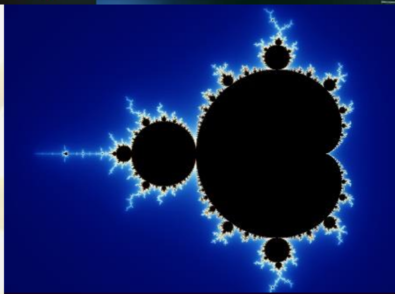


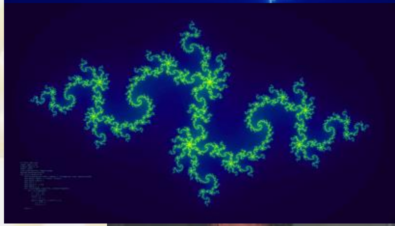




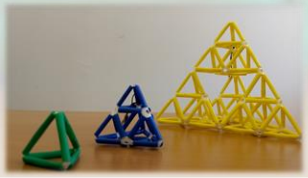
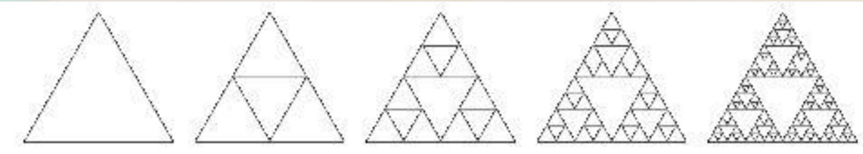
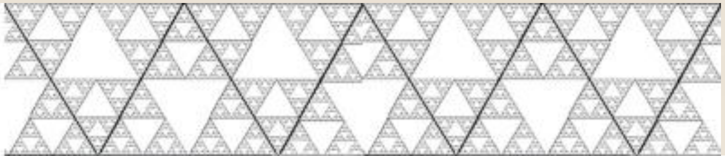


“Popular (more or less…) fractals”

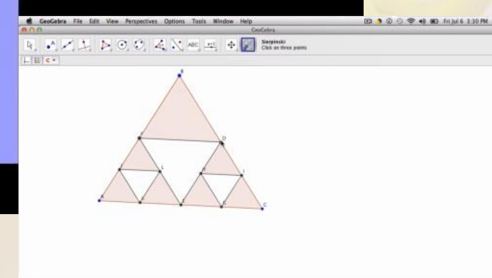


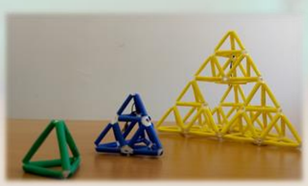


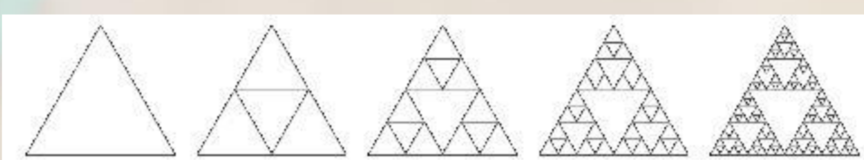
- Mathematics:
  - Mandelbrot and Julia sets
- Botanics:
  - Trees, Cauliflower, Romanesco cabbage, …
- Life sciences:
  - Lungs, blood vessels
- Sciences of Earth:
  - Coastal lines, lightings, ….
- Structure of time in the Jewish calendar



Construction of a Sierpinski triangle with



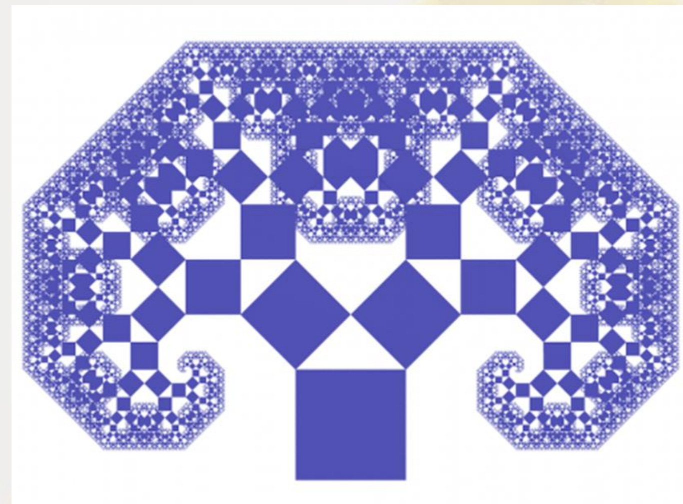


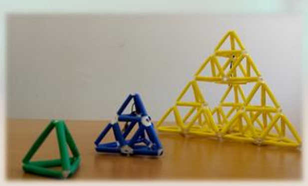


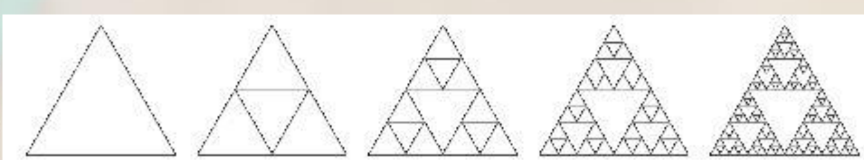
Working in opposite direction

Kids Inspiring Kids for STEAM - Erasmus+ @ European Researchers' Night 2017 Budapest. Kristof Fenyvesi

Pythagorean tree (Etienne Ghys, ENS, Lyon, France)







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Plotting a fractal using a Maple package

Mandelbrot set

> restart; with(Fractals:-EscapeTime);

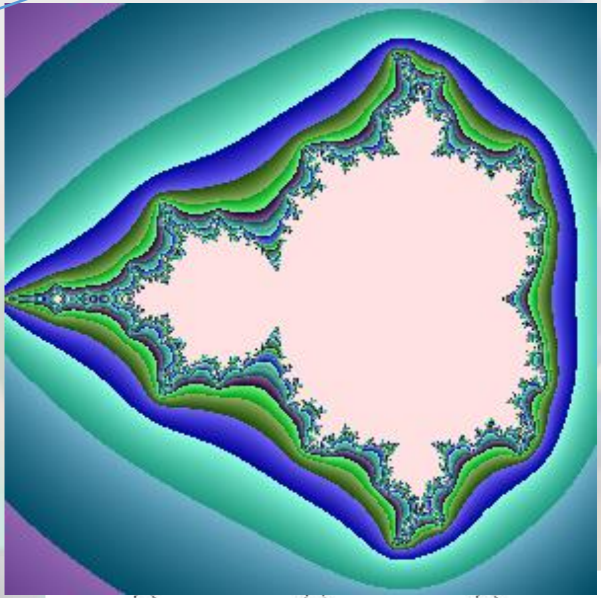
> with(ImageTools);

size

> M := Mandelbrot(300, -2.0-1.35*I, .7+1.35*I);

> Embed(M);

Initial points



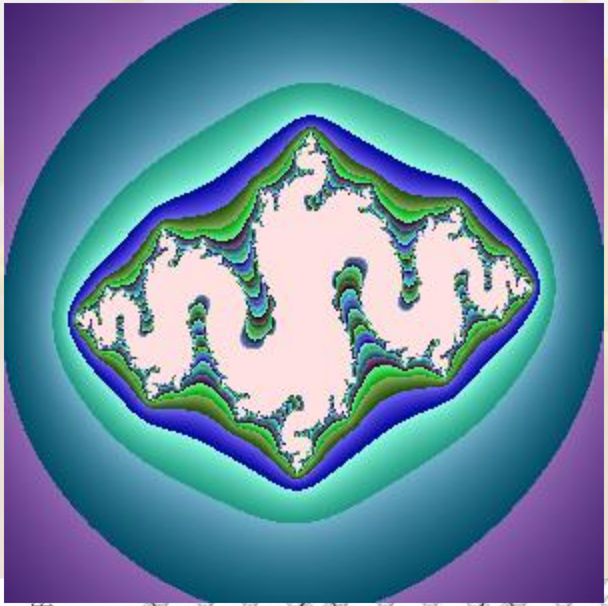
Julia set

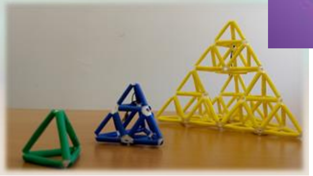
> bl, ur, c := -2.0-1.5*I, 2.0+1.5*I, -.8+.156*I;

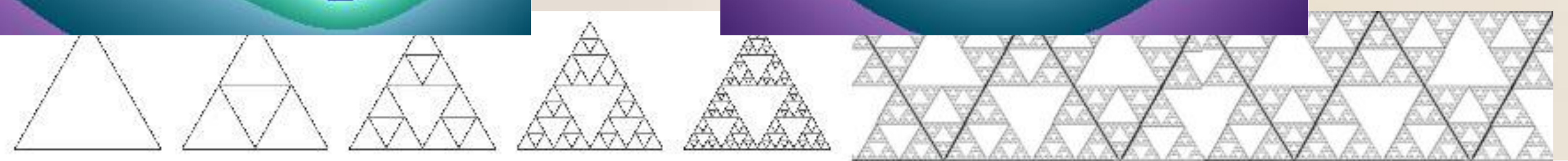
> M := Julia(300, bl, ur, c);

>MAssignArray(%id = 18446744074392415014)

> Embed(M);





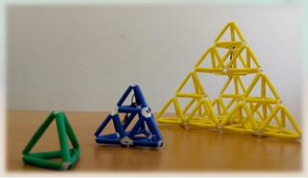


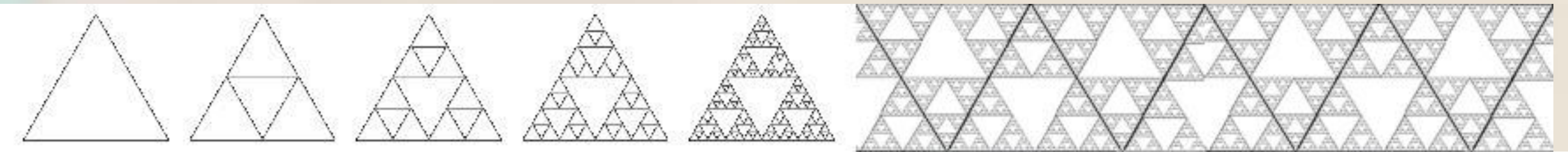
Building a fractal

Angela Gammella-Mathieu & Nicolas Mathieu: **Algorithmique et programmation graphique des fractales de Sierpinski, APMEP, France.**
https://www.apmep.fr/Algorithmique-et-programmation

Exercise:

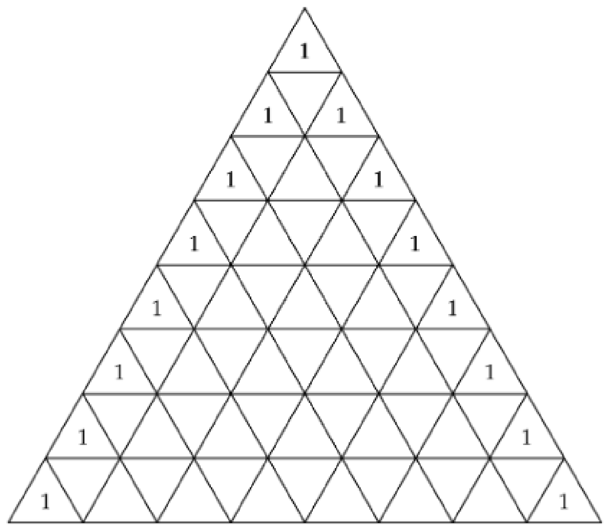
1. **Take an equilateral triangle. We will build a Sierpinski triangle. Denote by u_n the number of white triangles at step number n. Which kind of sequence is (u_n)?**
2. **We build a Sierpinski pyramid. Denote by v_n the number of white triangles on the faces of the original tetrahedron at step n. Which kind of sequences is (v_n)**





Exercise: Fractals meet Pascal

(i) Fill in the missing entries to complete Pascal's triangle (we only use the upside triangles):

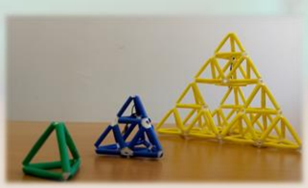


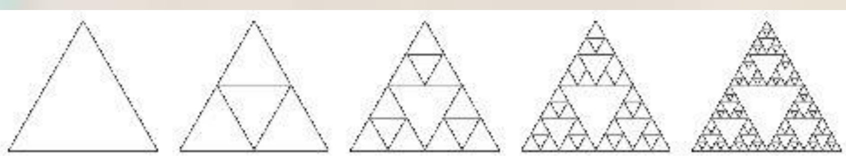
(ii) Give a rule as to which triangles must be colored in order to retrieve Sierpinki's triangle from Pascal's triangle (Sierpinski meets Pascal).

(iii) If n is an integer, write $n = \sum_{i=0}^{k} n_i 2^i$ with $n_i \in \{0, 1\}$ (we call this the binary expansion of n) Similarly, for another integer m, we write $m = \sum_{i=0}^{k} m_i 2^i$. Explain how Lucas formula

$$\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \mod 2$$

explains the "mod 2" pattern of the binomial coefficient $\binom{n}{m}$ in terms of the binary expansions of n and m.





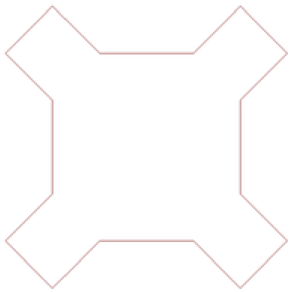
Sierpinski curve: Finite-infinite interplay

Sierpiński curves are a recursively defined sequence of continuous closed plane fractal curves discovered by Wacław Sierpiński, which in the limit $n \to \infty$ completely fill the unit square: thus their limit curve, also called **the Sierpiński curve**, is an example of a space-filling curve.

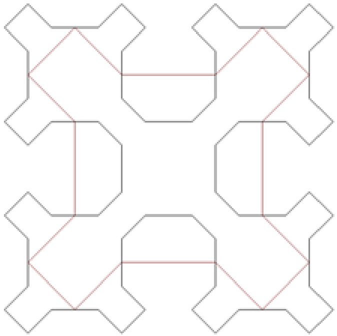
Because the Sierpiński curve is space-filling, its Hausdorff dimension (in the limit $n \to \infty$) is 2.
The Euclidean length of

$$S_n \text{ is } l_n = \frac{2}{3}(1+\sqrt{2})2^n - \frac{1}{3}(2-\sqrt{2})\frac{1}{2^n},$$

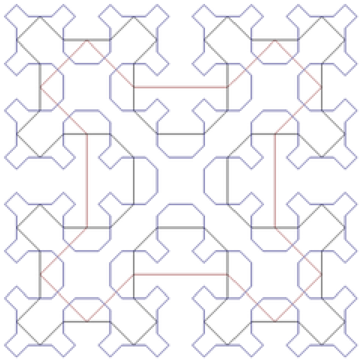
i.e., it grows *exponentially* with n beyond any limit, whereas the limit for $n \to \infty$ of the area enclosed by S_n is $5/12$ that of the square (in Euclidean metric).



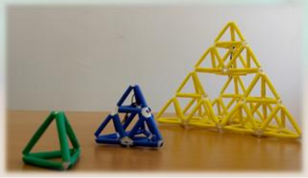
Sierpiński curve of first order

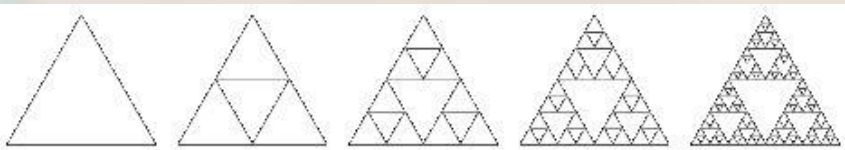


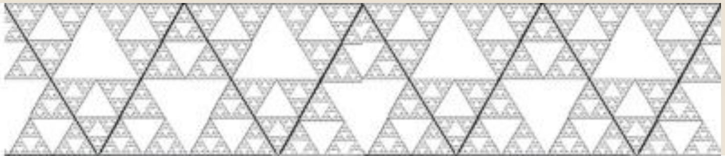
Sierpiński curves of orders 1 and 2



Sierpiński curves of orders 1 to 3







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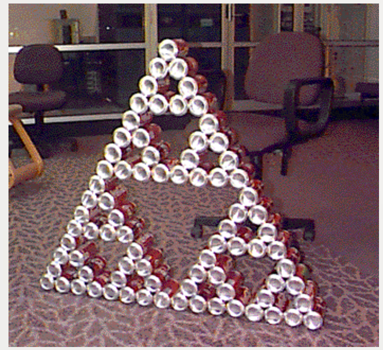
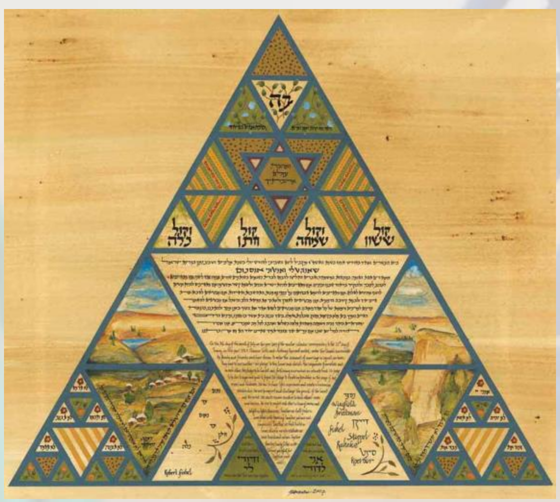
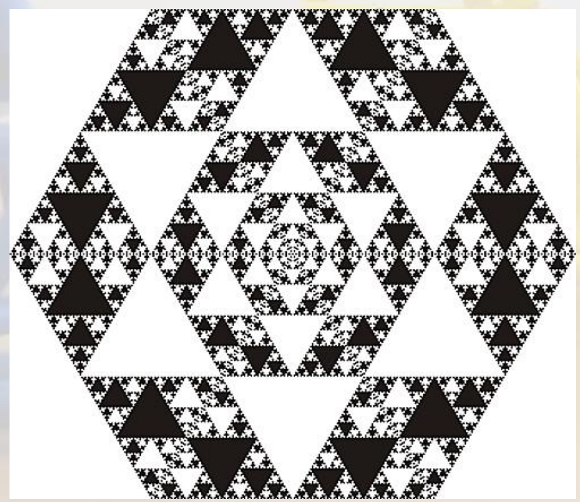
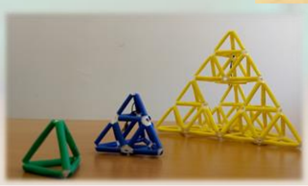
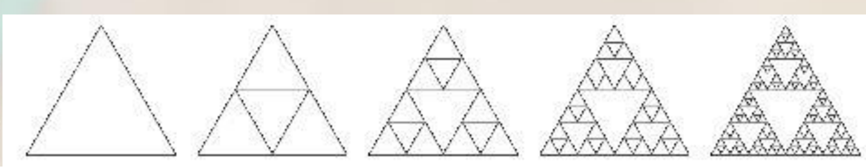
Art pieces based on Sierpinski triangle and pyramid






Jewish contract of marriage (Norman Slepkov, Jerusalem)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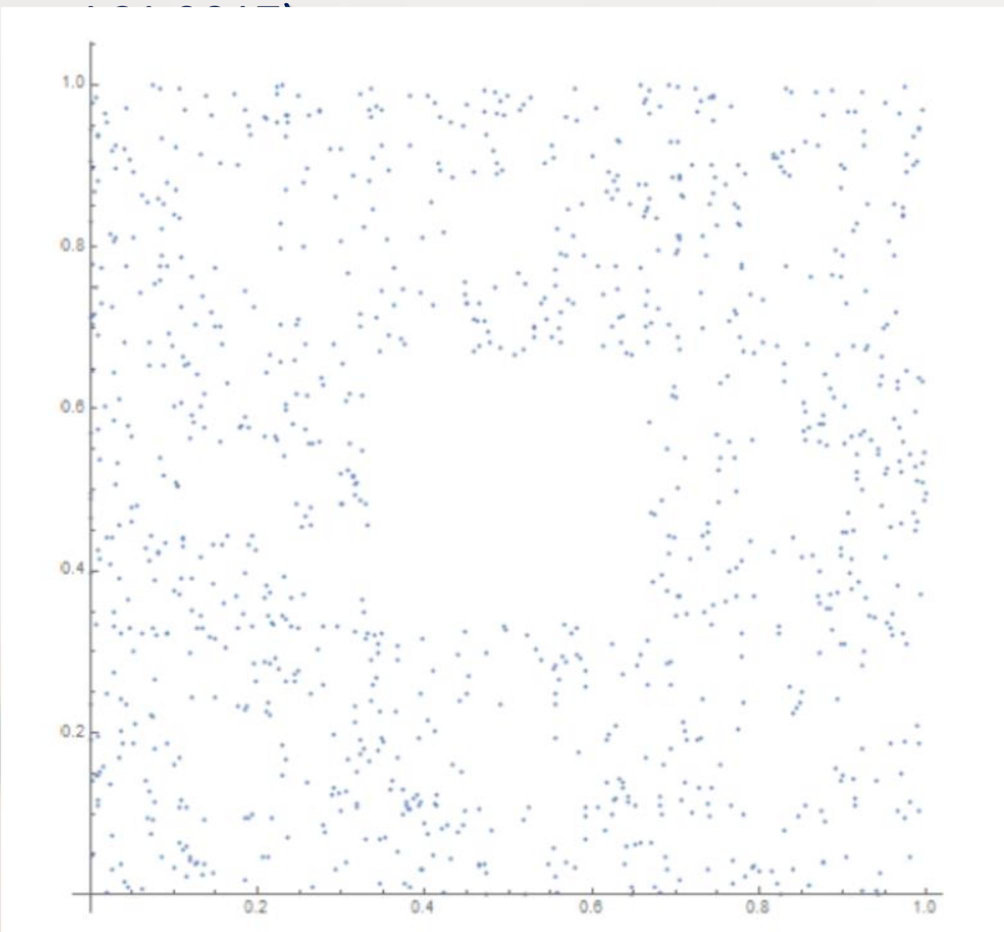
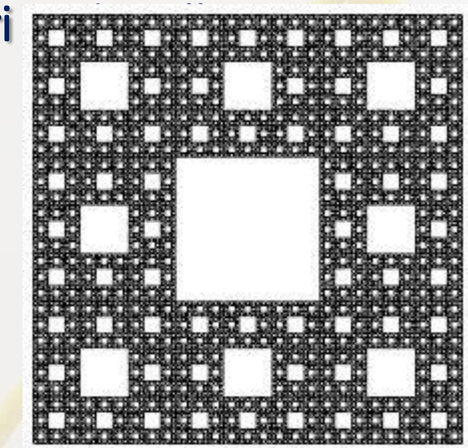
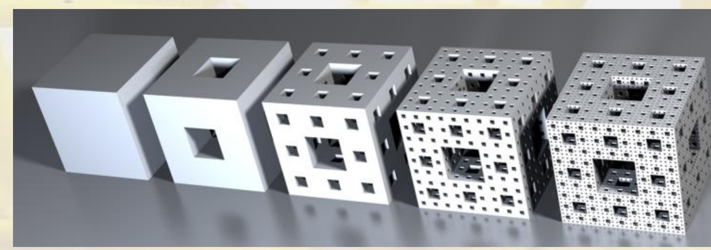
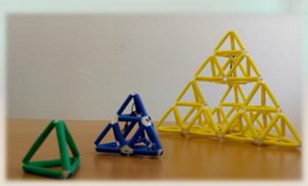
Cosmologic simulation (Benjamin, Mylläri

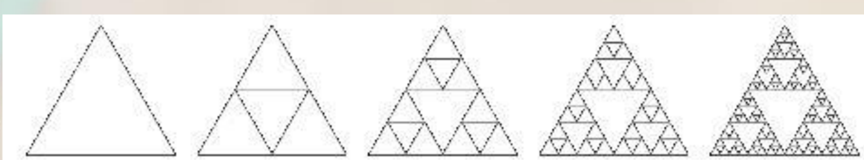




Sierpinski carpet

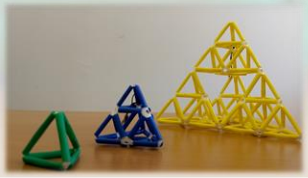


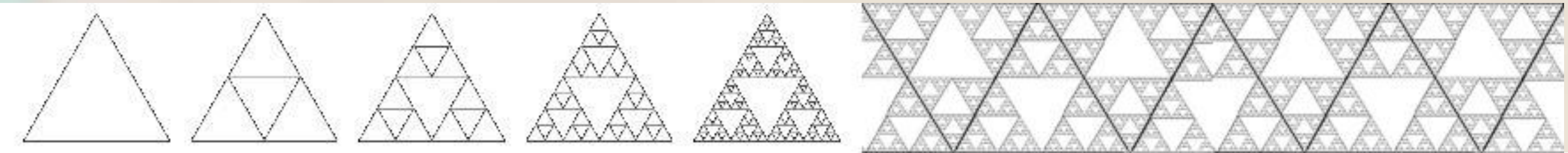




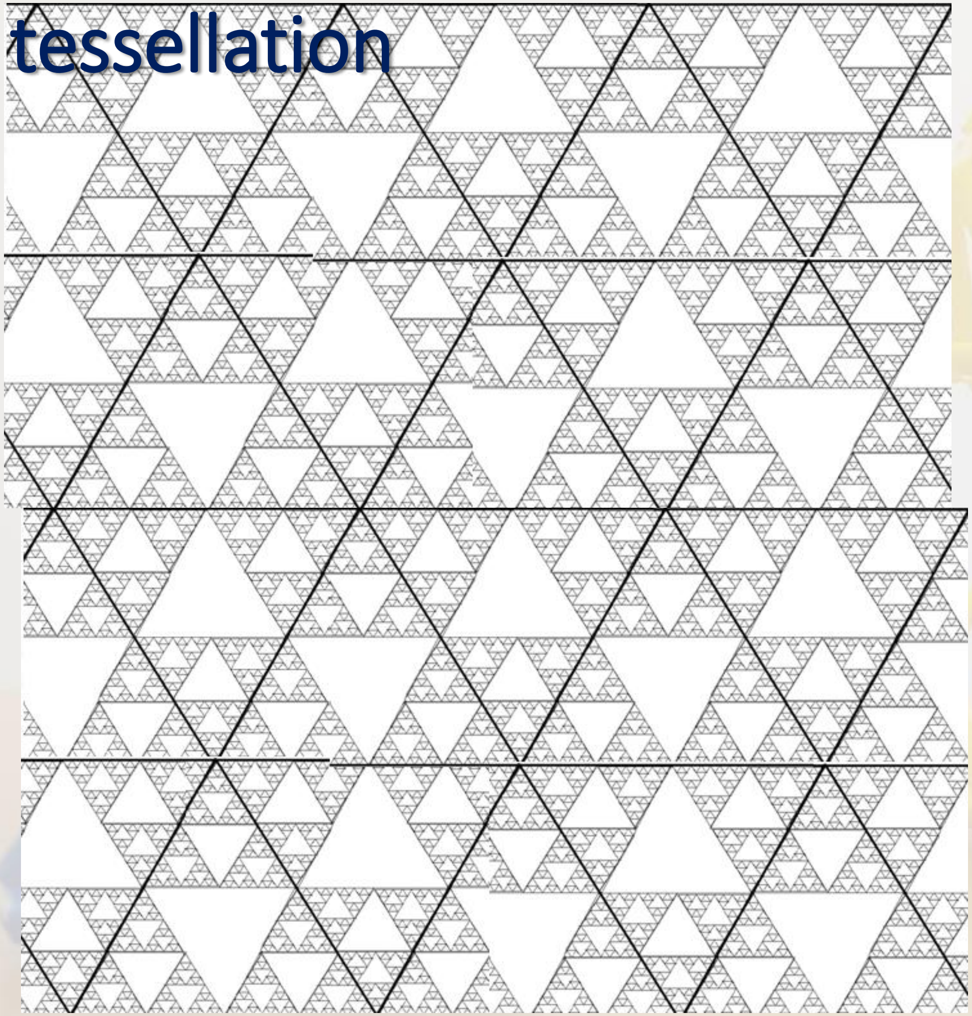
Tessellations

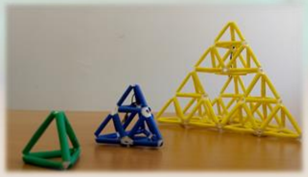
A **tessellation** of a flat surface is the tiling of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. In mathematics, tessellations can be generalized to higher dimensions and a variety of geometries.

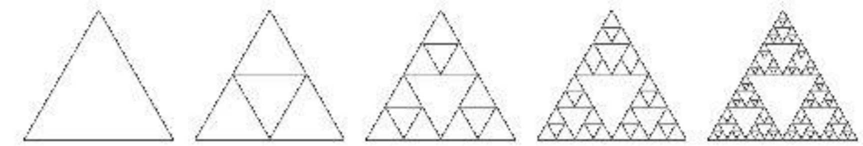


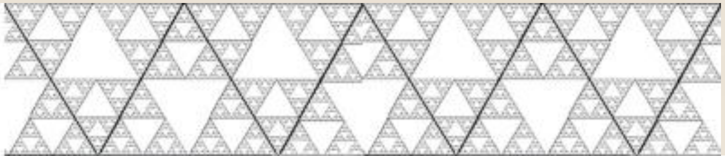


Sierpinski tessellation







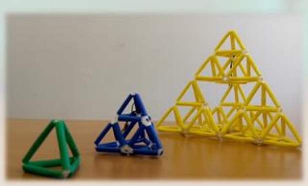


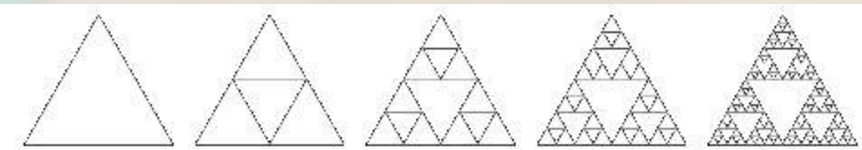
Hens

Plymouth rock

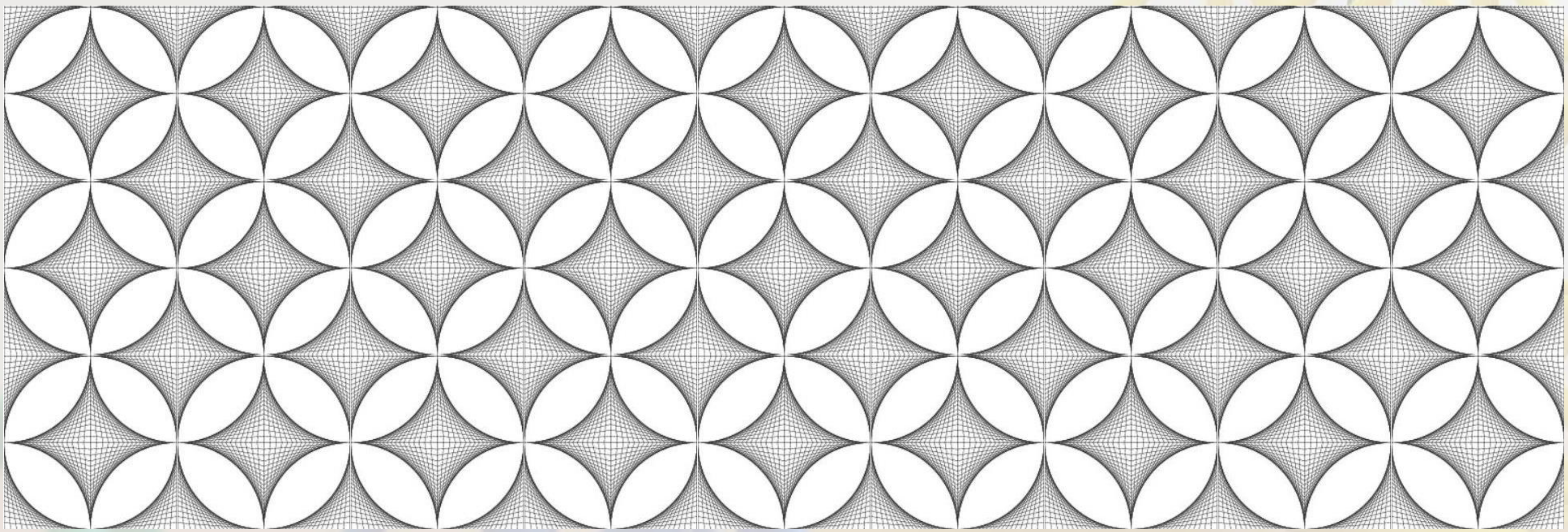
Speckled Sussex Chicken

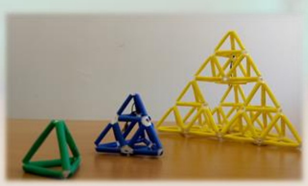
עוף בראקל

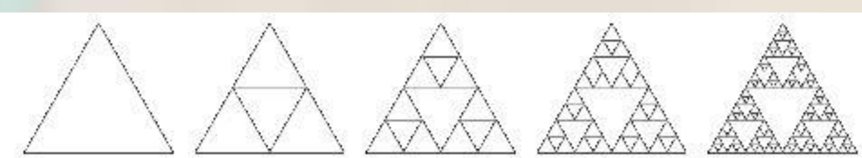




Tessellations using envelopes







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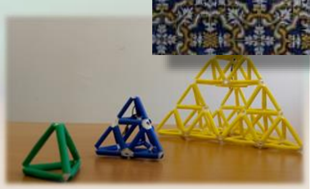
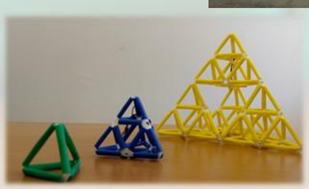
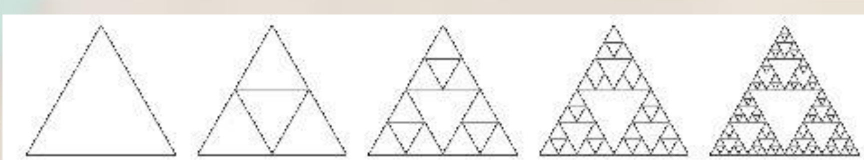
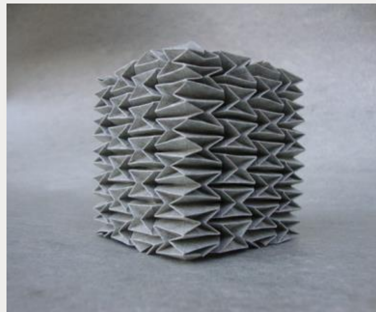


A living tessellation (Please applause!!!!)





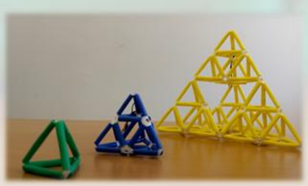
3D extensions

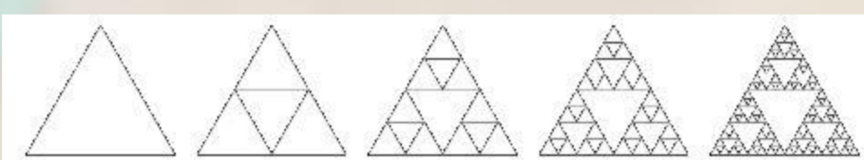


Andrea Russo, Italy
(Paper works)

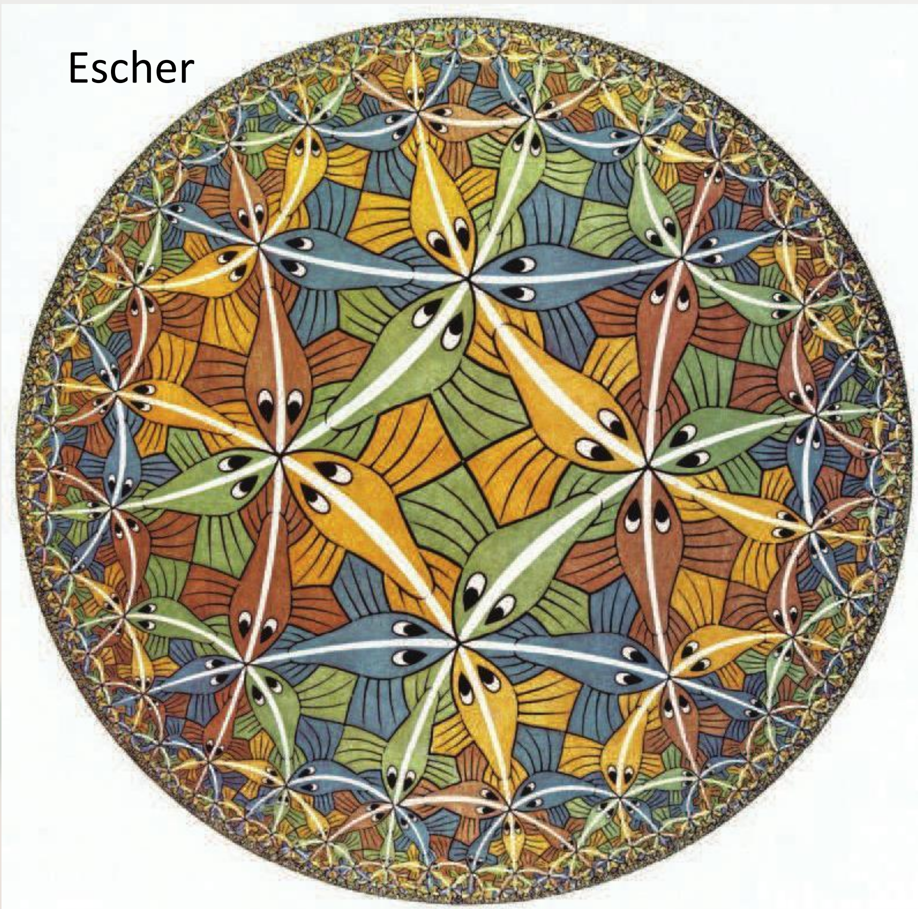
Resch pattern

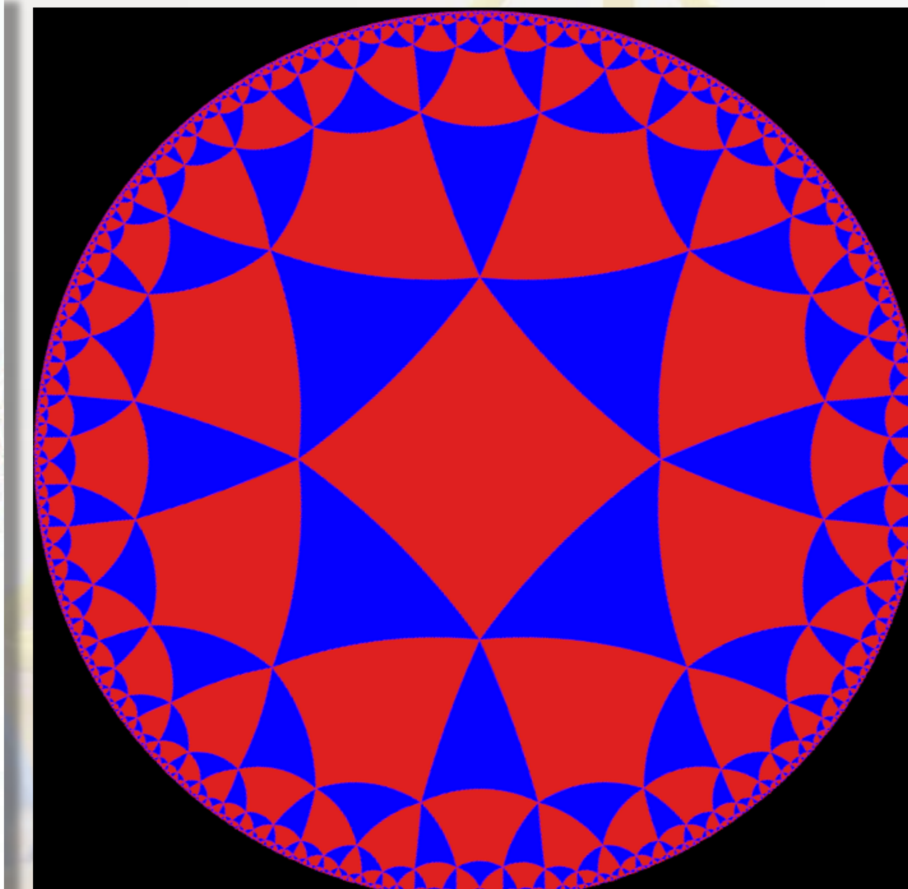
Origami tessellation –
generalization of a Resch
pattern
Every point on the surface has
zero Gaussian curvature.

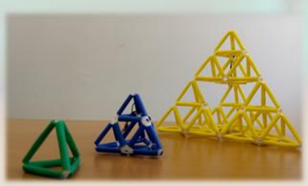


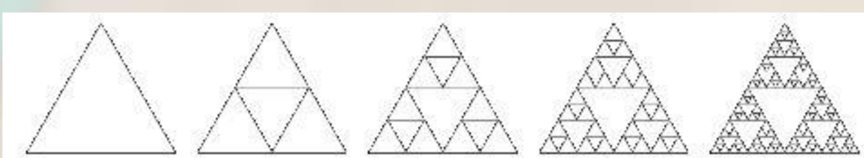


Tessellations on a hyperbolic plane


Escher


Tomruen at en.wikipedia





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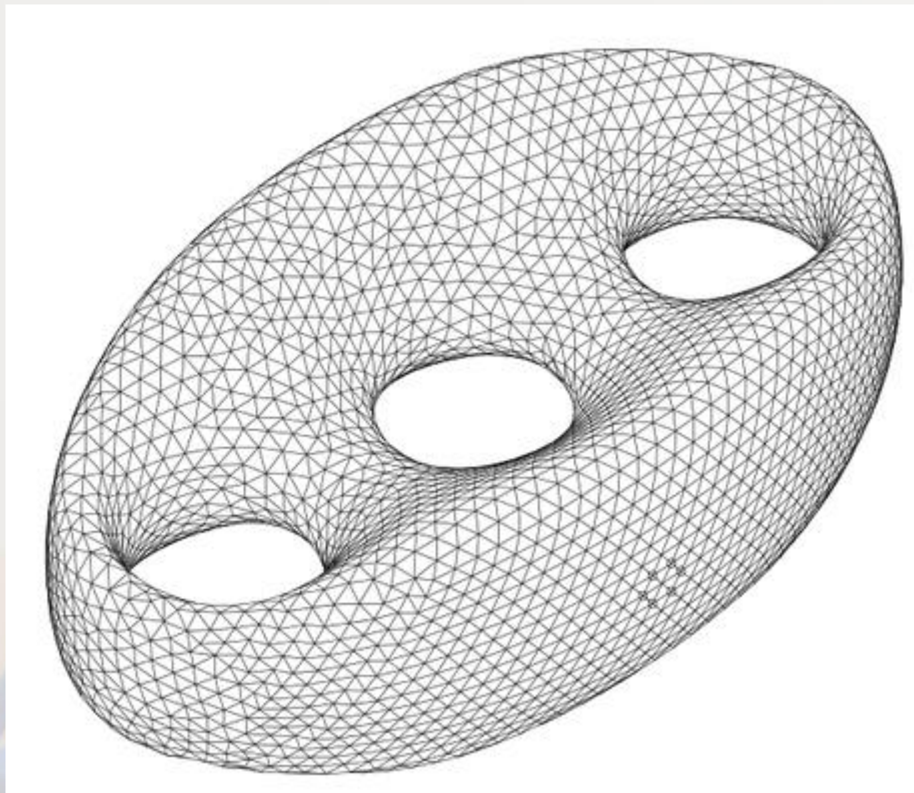
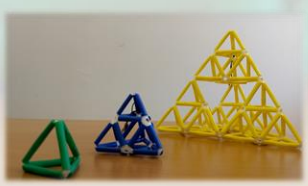
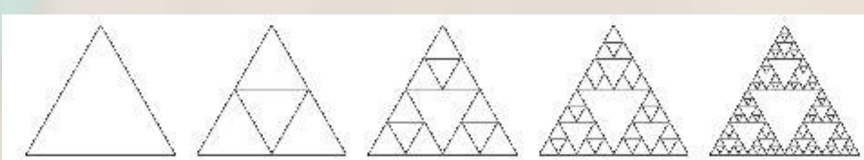
Triangulation, not a tessellation



Triangulation of an implicit surface of genus 3



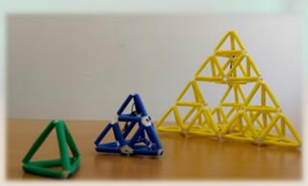


Isoptic curves

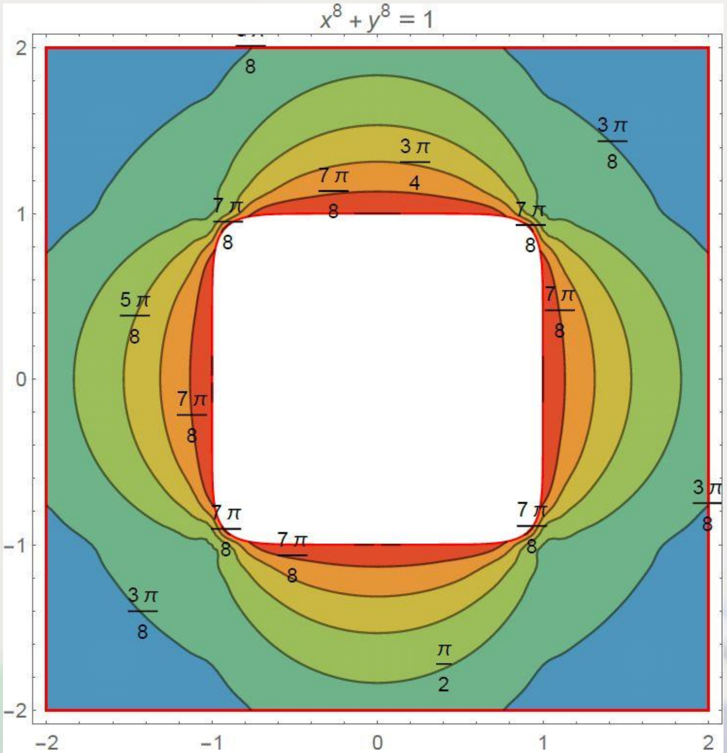
Let be given a plane curve C and an angle θ.

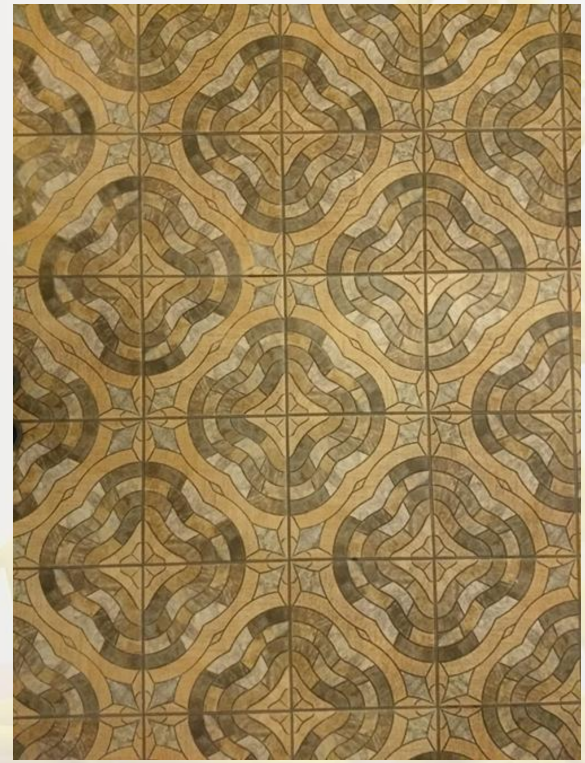
If it exists, the geometric locus of points through which passes a pair of tangents to C making an angle equal to θ is called an *isoptic curve* of C.

The name comes from the fact that from points on this geometric locus the curve C is seen under an angle equal to θ.



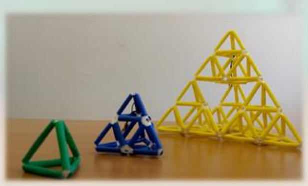
Example: isoptics of a Fermat curve

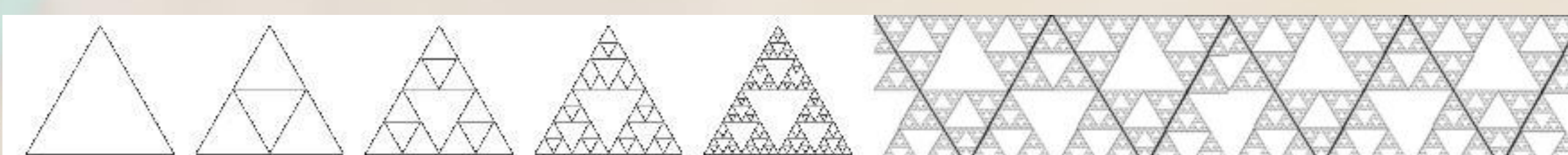




Floor at in the lobby of an old synagogue in Budapest

Ref: Th. D-P and A. Naiman (2017): Isoptics of Fermat Curves, ACA 2017





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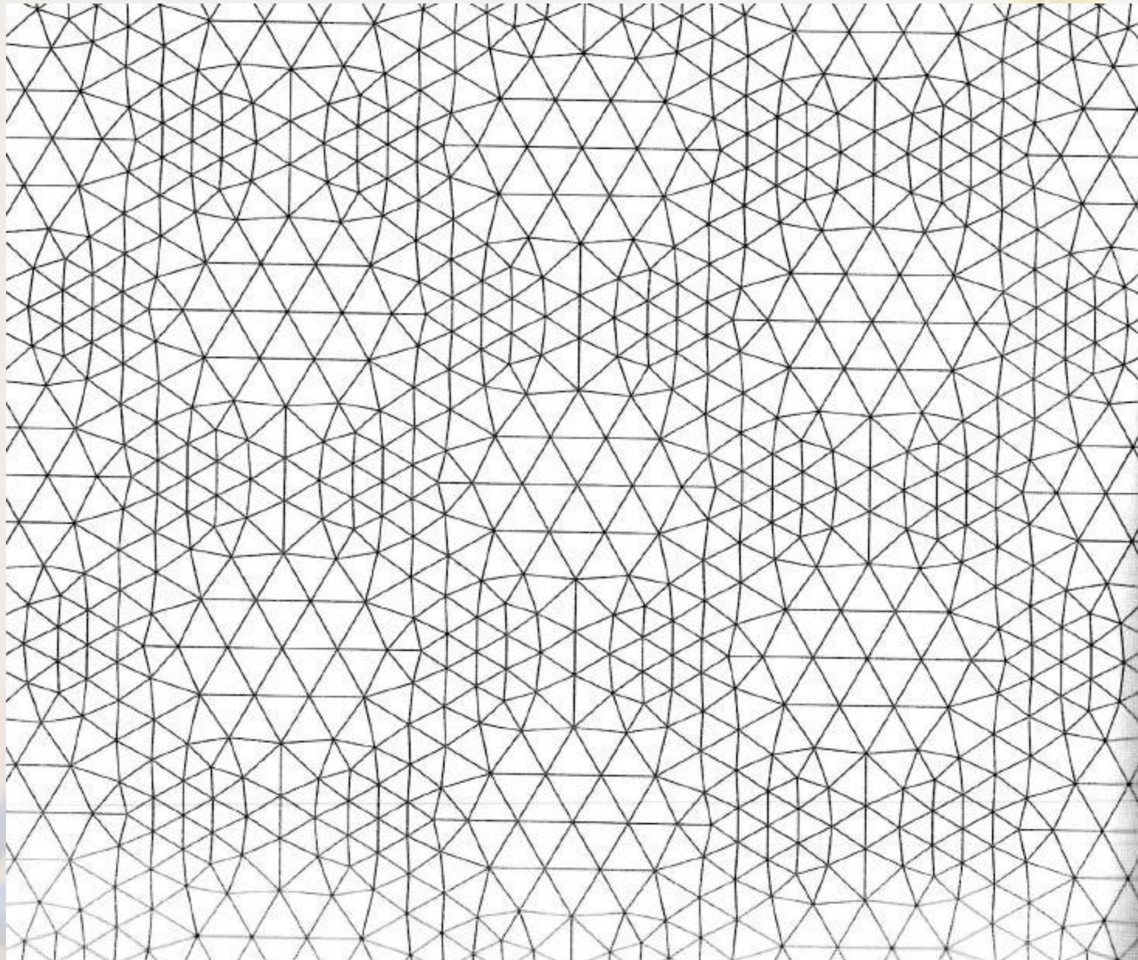
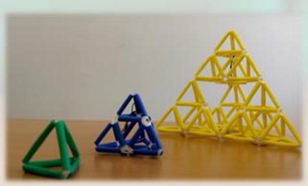
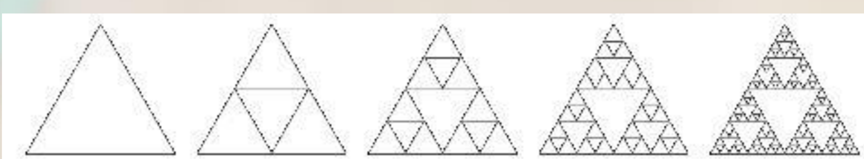
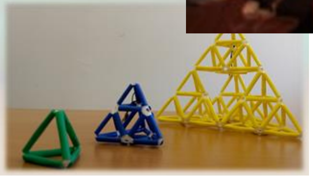
Low tech







Low tech building of a tessellation – doing m



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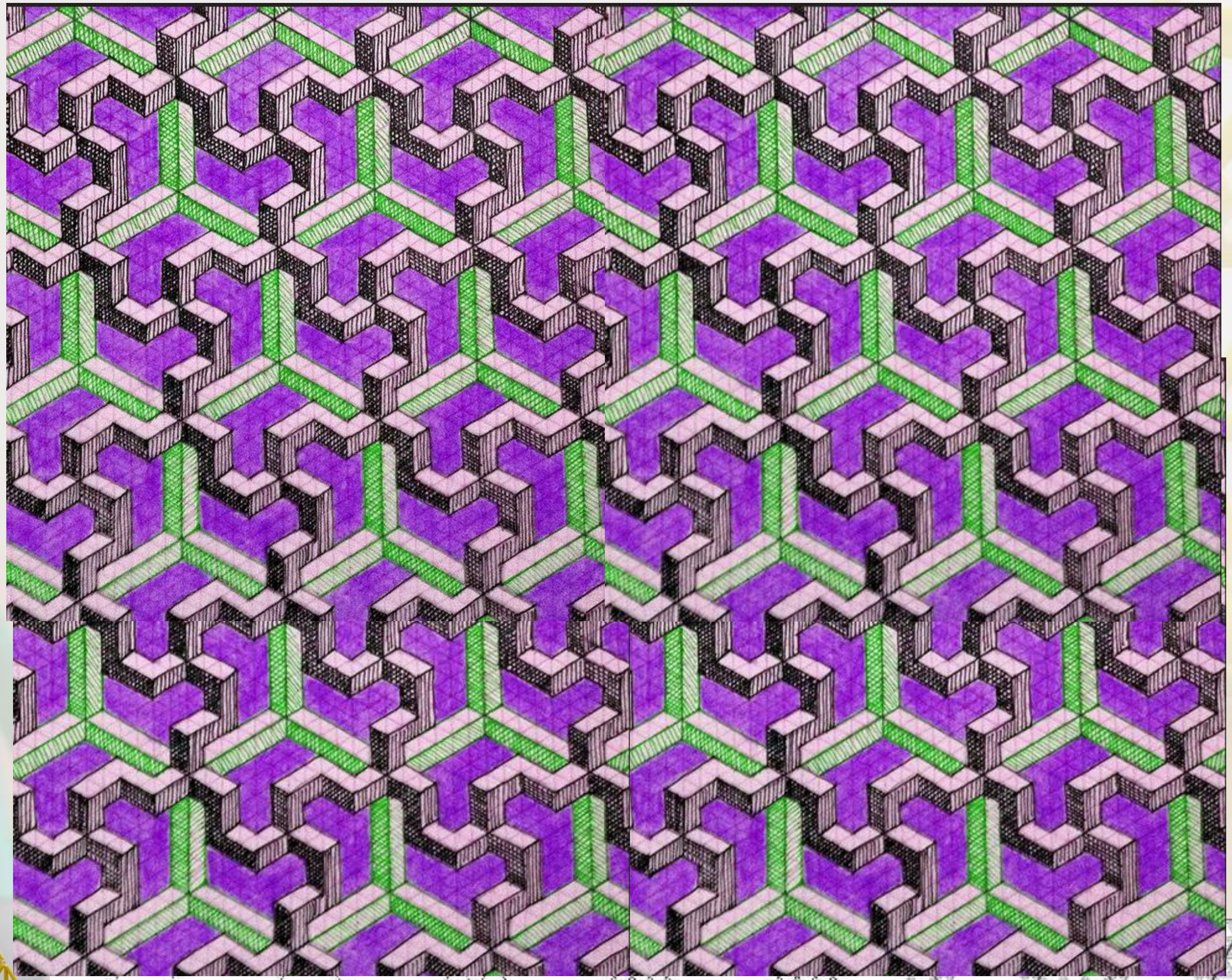
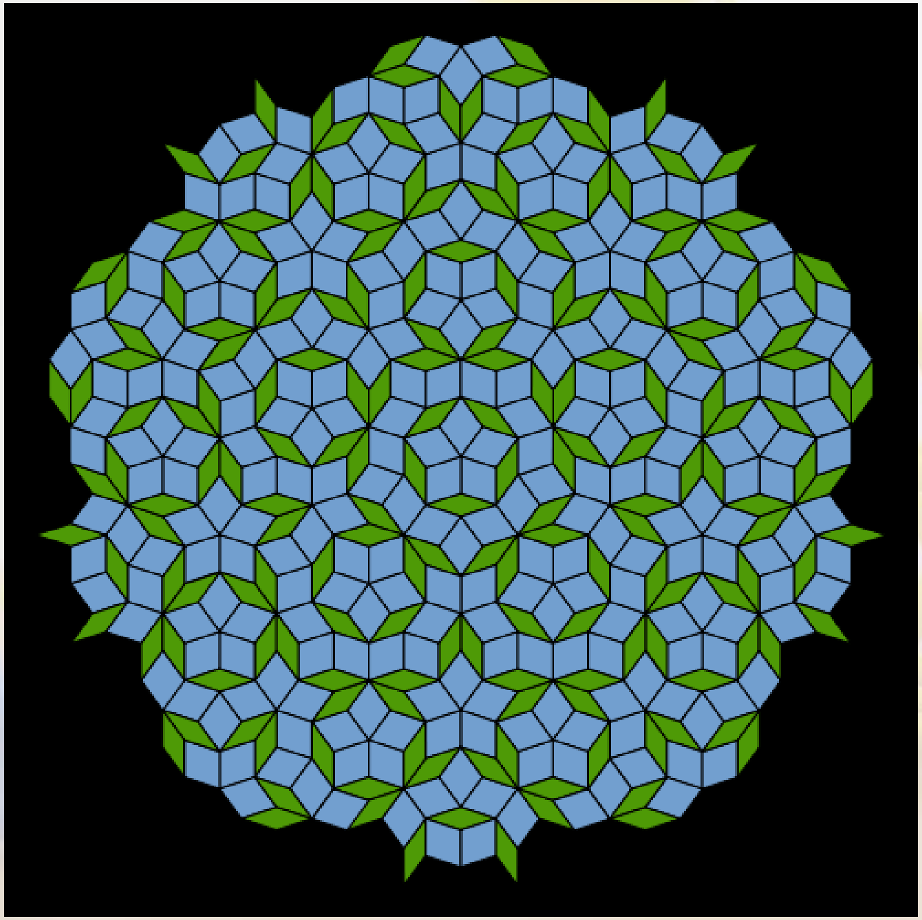
A tessellation built on a triangular grid



Non periodic tiling – Penrose tiling

Made of two kinds of quadrilaterals



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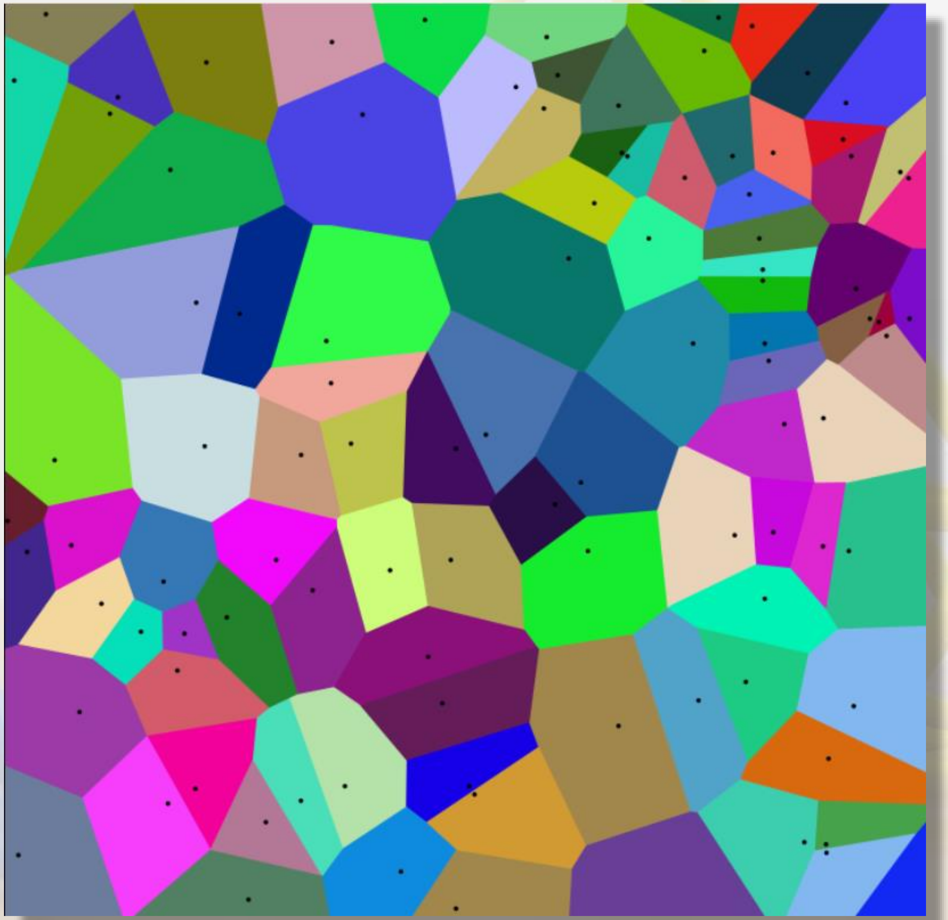
Voronoi (or Dirichlet) tessellation

A *Voronoi diagram* is a partitioning of a plane into regions based on distance to points in a specific subset of the plane. That set of points (called seeds, sites, or generators) is specified beforehand, and for each seed there is a corresponding region consisting of all points closer to that seed than to any other. These regions are called *Voronoi cells*.
(Wikipedia)



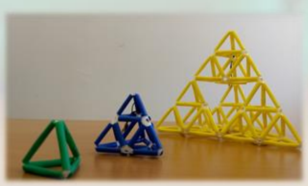
Here all the cells are convex polygonial cells

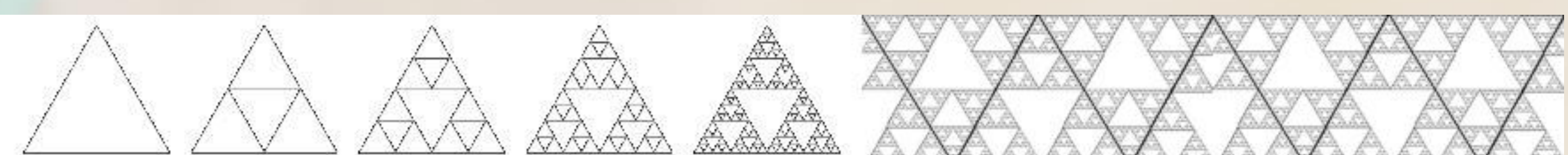
Can a Voronoi diagram solve a biblical problem?

Deuteronomy 21:

1. If a slain person be found in the land which the Lord, your God is giving you to possess, lying in the field, [and] it is not known who slew him,
2. then your elders and judges shall go forth, and **they shall measure to the cities** around the corpse.

Ref: Deuteronomy Chapter 21 - E. Merzbach, Higayon

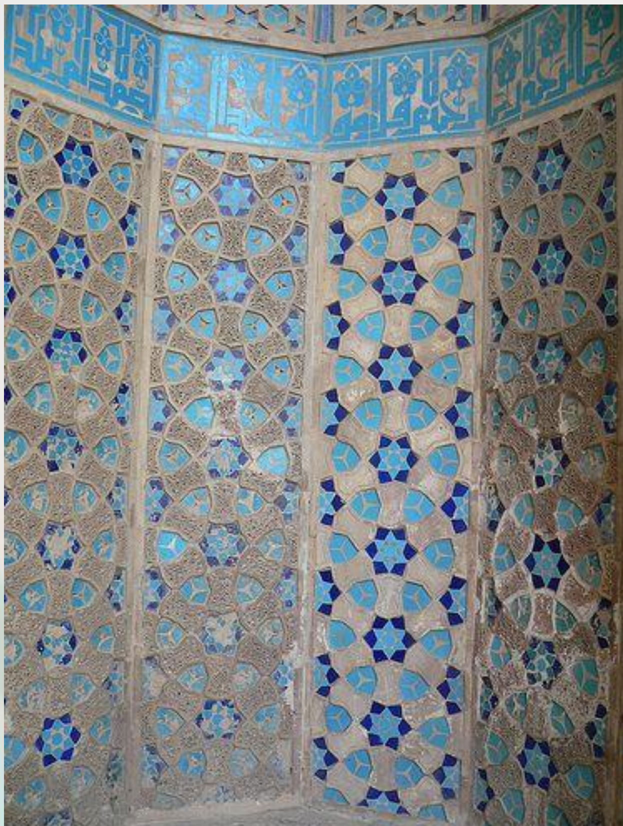




Tessellations on a surface topologically equivalent to a plane

No pattern change

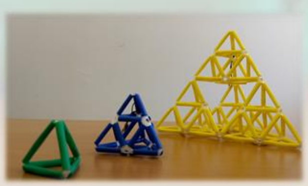
Ilkhanid Emamzadeh-ye-Abd al-Samad, Natanz, Iran

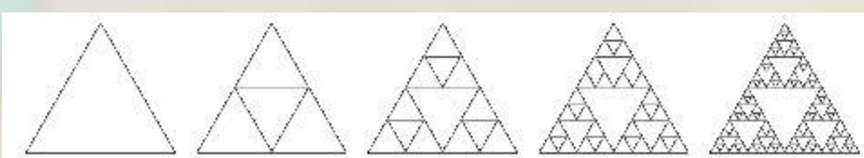


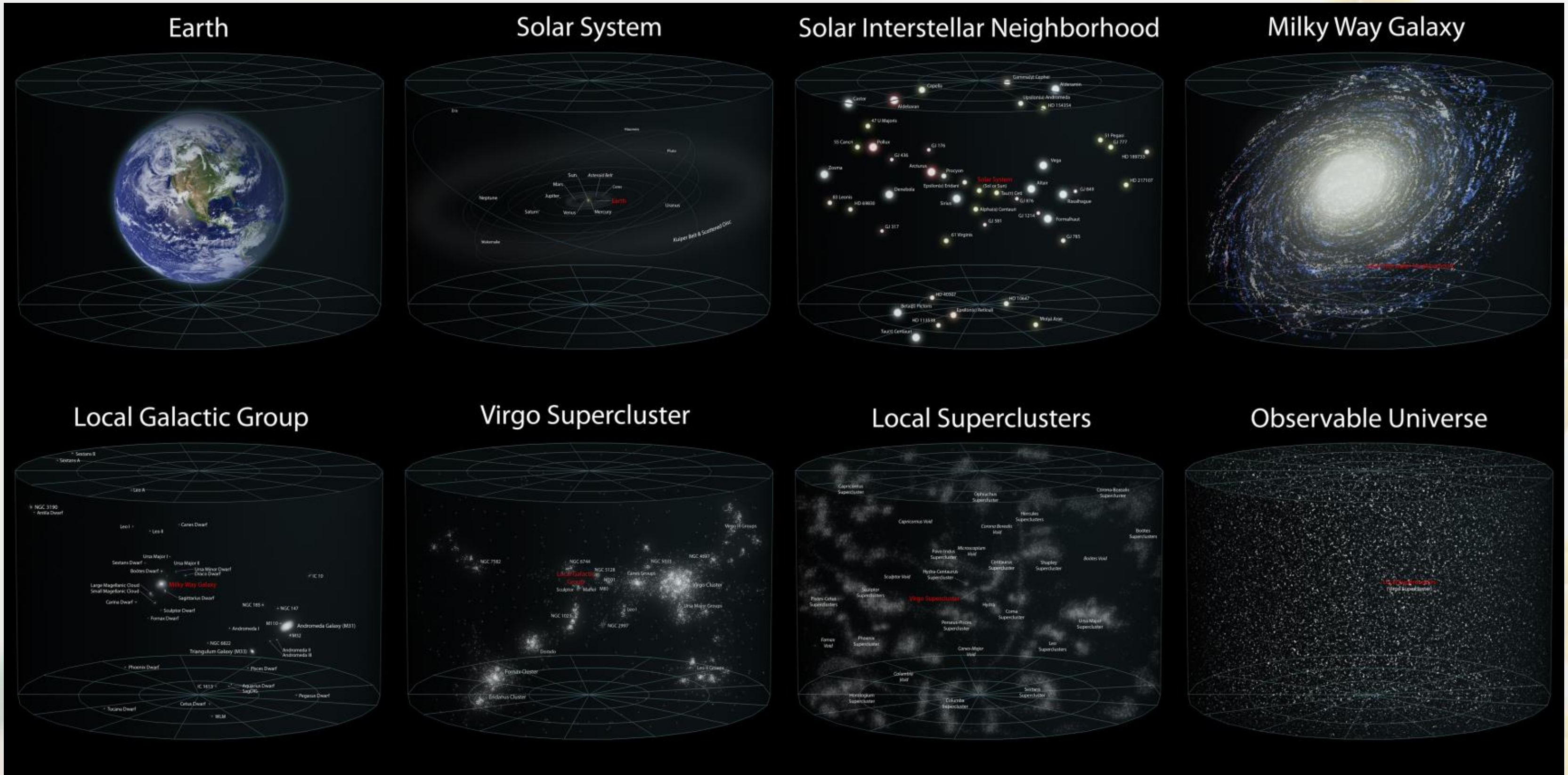
With pattern change

Jame (Friday) Mosque, Yazd, Iran

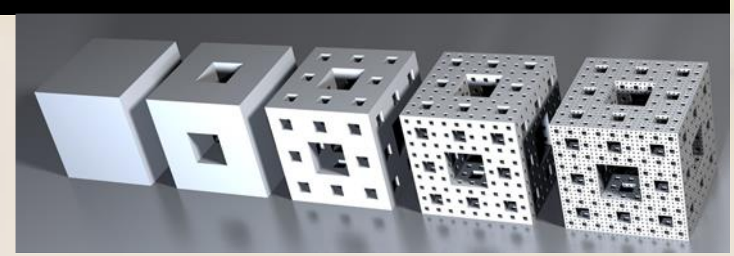
The rosette changes bottom-up from 10-fold to 5-fold

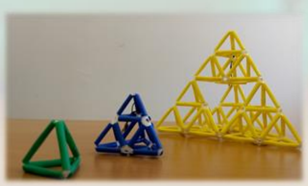


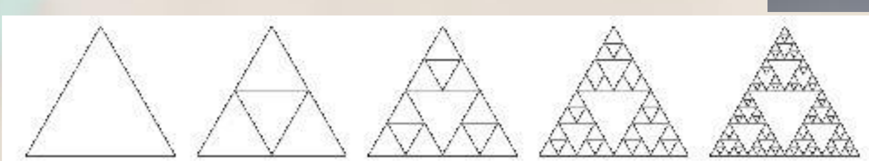




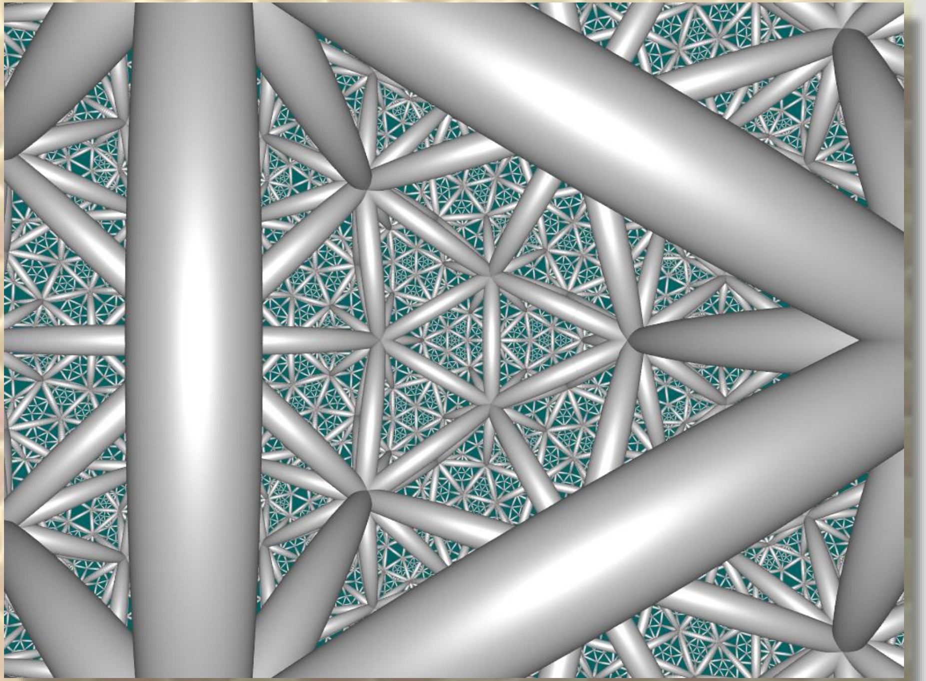
Menger sponge







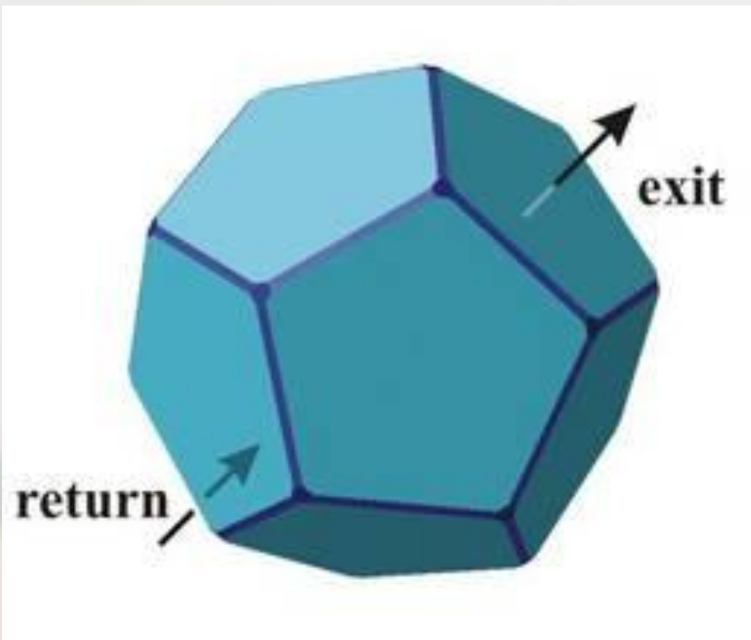
Bees and hives

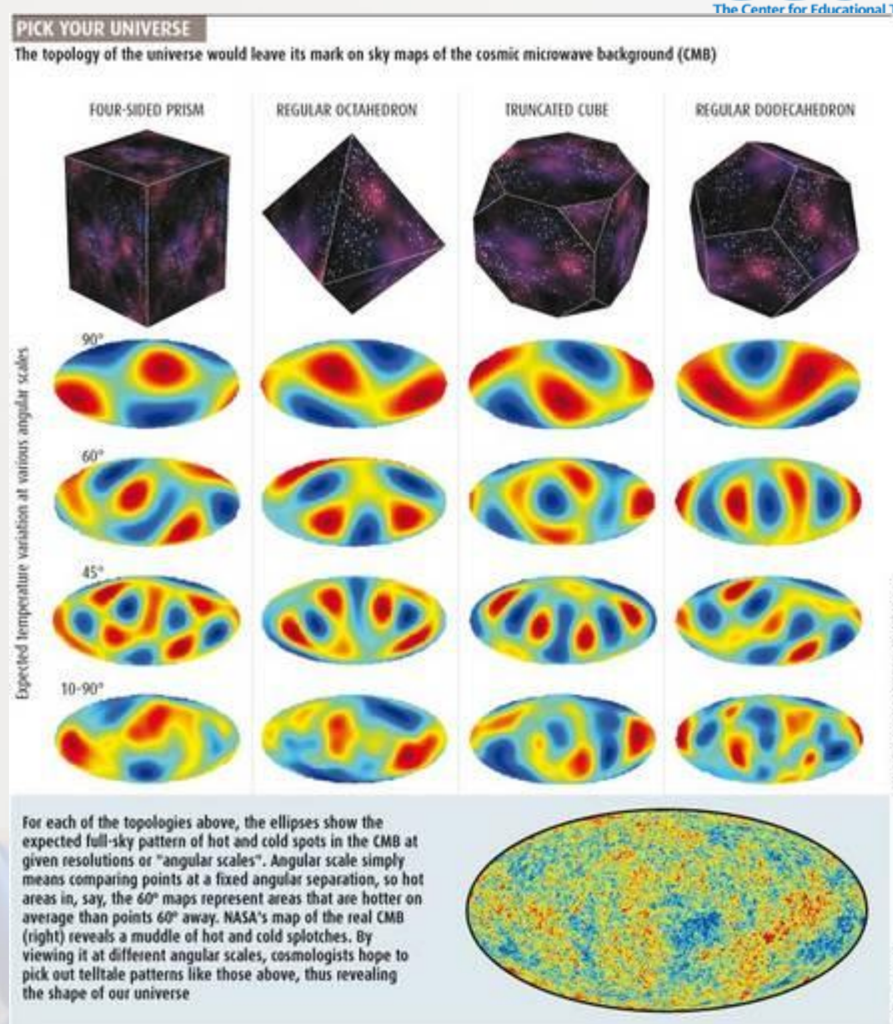


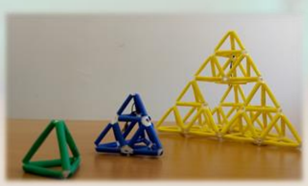
Icosehedral honey comb
in hyperbolic space

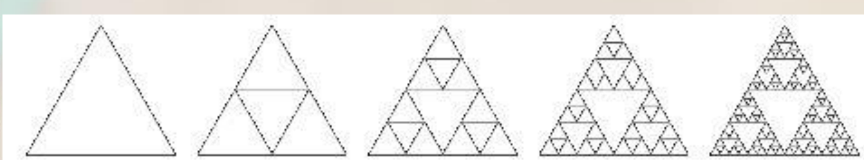
Is the universe finite?

The Poincaré dodecahedron

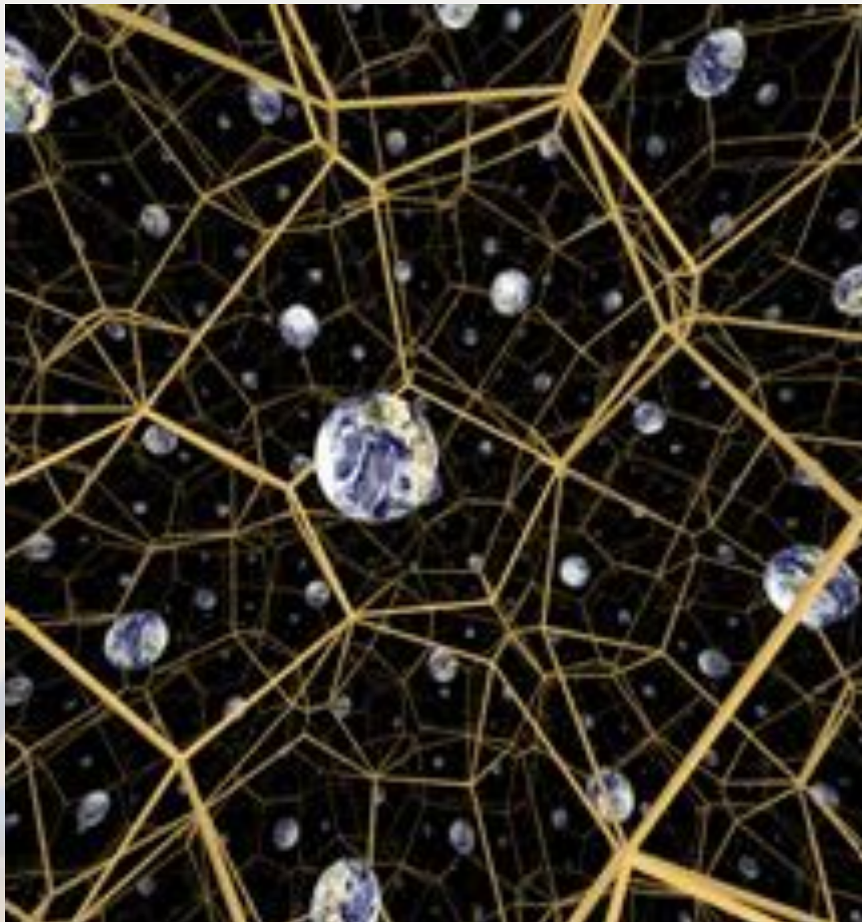








A cosmic hall of fame (J.P. Luminet)



Thank you for your attention

and good appetite!