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CADGME 7



Organização
das Nações Unidas
para a Educação,
a Ciência e a Cultura



Universidade de
Coimbra – Alta e Sofia
inscrita na Lista do Património
Mundial em 2013

COIMBRA, 26-29 JUNE 2018

CHANGING PLANS: HOW DYNAMIC MERLO ITEMS CAN SUPPORT STUDENTS' UNDERSTANDING IN 3D GEOMETRY

Giulia Bini & Germana Trincherro



DIPARTIMENTO
DI MATEMATICA
GIUSEPPE PEANO
UNIVERSITÀ DI TORINO



**Santorre di
Santarosa**

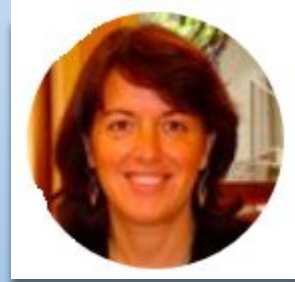
Liceo Linguistico - Liceo delle Scienze Umane
Tecnico Chimica, Materiali e Biotecnologie

THE PROJECT: A JOINT EFFORT

TEACHERS' PROFESSIONAL DEVELOPMENT

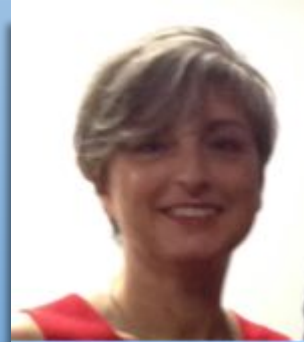
GEOMETRY WITH GEOGEBRA: MEANINGS
AND REPRESENTATIONS (PLSTO 04)

NOV 2017 - MAY 2018



ORNELLA ROBUTTI

GERMANA TRINCERO



GIULIA BINI





THEORETICAL FRAMEWORK:

M.E.R.L.O. PEDAGOGY



MEANING
EQUIVALENCE
REUSABLE
LEARNING
OBJECTS

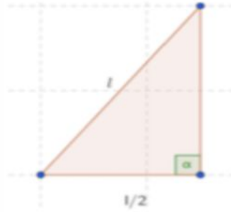
FOSTER STUDENTS' ATTENTION ON **CONCEPTUAL MEANING**, THROUGH INTERACTIVE/FORMATIVE ASSESSMENT BASED ON:

- UNSTRUCTURED TASKS
- FREQUENT AND DETAILED FEEDBACK

M.E.R.L.O. ITEM

COMPOSITION: ONE TASK AND FIVE STATEMENTS

- IN THE BOXES *DIFFERENT REGISTERS* ARE USED: NATURAL LANGUAGE, GRAPHIC, NUMERICAL, ...
- EACH BOX CONTAINS A CORRECT STATEMENT / REPRESENTATION FROM A MATHEMATICAL POINT OF VIEW
- SOME BOXES SHARE THE SAME MEANING (*MEANING EQUIVALENCE*) OTHERS ONLY HAVE A SIMILARITY IN THE REPRESENTATION (*SURFACE SIMILARITY*)
- THE TASK IS: "TICK THE REPRESENTATIONS THAT SHARE THE SAME MEANING (TWO OR MORE); INDICATE THE REASONS THAT GUIDE YOUR CHOICE"

Slope	TS	QS															
1. Mark the statements (two or more) that share the same mathematical meaning; 2. Write the reasons that guided you in the choice	A [] 	B [] The slope of a line is the ratio of the variation of the ordinates and the axis of the increase of two of its points abscissas															
Q2 C [] <table border="1"> <thead> <tr> <th>t</th> <th>s</th> <th>$\frac{\Delta s}{\Delta t}$</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>3</td> <td>---</td> </tr> <tr> <td>2</td> <td>6</td> <td>3</td> </tr> <tr> <td>3</td> <td>9</td> <td>3</td> </tr> <tr> <td>4</td> <td>12</td> <td>3</td> </tr> </tbody> </table>	t	s	$\frac{\Delta s}{\Delta t}$	1	3	---	2	6	3	3	9	3	4	12	3	D [] 	Q4 E [] 
t	s	$\frac{\Delta s}{\Delta t}$															
1	3	---															
2	6	3															
3	9	3															
4	12	3															

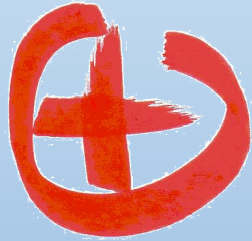


THEORETICAL FRAMEWORK: M.E.R.L.O. PEDAGOGY

MERLO APPROACH TO PARSING AND ANALYSING CONCEPTS IS APPLICABLE TO VARIOUS SUBJECTS FOR RECOGNIZING, REPRESENTING, ORGANIZING, EXPLORING AND MANIPULATING KNOWLEDGE. IT IS PARTICULARLY RECOMMENDED IN MATHEMATICS, WHERE THE ABILITY TO SHIFT FROM ONE TO ANOTHER REPRESENTATION OF THE SAME OBJECT AND THE COORDINATION OF MULTIPLE REPRESENTATIONS IN MORE THAN ONE SEMIOTIC REGISTER ARE FUNDAMENTAL COMPETENCIES, IN ORDER TO ACCESS THE UNDERLYING MEANING AND TO UNDERSTAND MATHEMATICS.

ARZARELLO, F., ROBUTTI O. & CARANTE P. (2015), MERLO: A NEW TOOL AND A NEW CHALLENGE IN MATHEMATICS TEACHING AND LEARNING, PROCEEDINGS OF PME 39,

ADDED VALUE & ADJUSTMENTS



DYNAMIC CONTENT IS ADDED TO
THE MERLO ITEMS ALLOWING
STUDENTS' INTERACTION WITH
THE MATHEMATICAL OBJECTS
THROUGH A CONNECTED DEVICE

THE CLASSICAL MERLO PROTOCOL IS
STRENGTHENED ASKING STUDENTS
TO GIVE ARGUMENTATIONS FOR
BOTH INCLUSIONS AND EXCLUSIONS
AND TO CHOOSE A TITLE

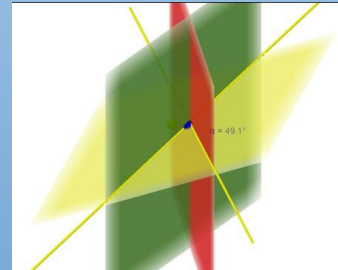
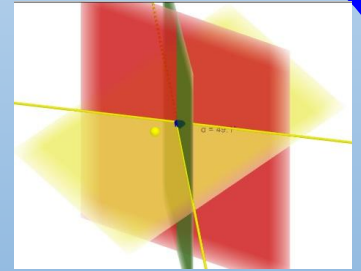
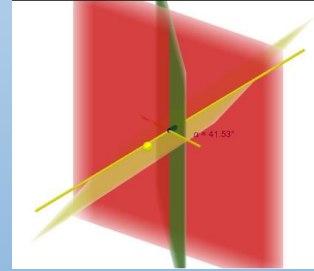
GEOMETRY & DYNAMISM

2 REGISTERS OF SEMIOTIC REPRESENTATIONS ONLY

A

The amplitude of a dihedral is defined as the measurement of the angle obtained by cutting the dihedral with a plane perpendicular to the edge of the dihedral.

LINGUISTIC REGISTER



GRAPHIC REGISTER



THEORETICAL FRAMEWORK:

THE COGNITIVE ROLE OF DRAGGING

DRAGGING SUPPORTS THE PRODUCTION OF CONJECTURES: EXPLORING DRAWINGS BY MOVING THEM, LOOKING AT THE WAYS AFTER WHICH THEIR FORMS CHANGE (OR DO NOT CHANGE), ALLOWS USERS TO DISCOVER THEIR INVARIANT PROPERTIES. THE POSSIBILITY OF DRAGGING OFFERS A FEEDBACK TO THE DISCOVERING PHASE, AND IN THIS WAY IT PROVIDES SUPPORT TO THE ROLE OF PROOFS AS REAL

ARZARELLO, F., OLIVERO, F., PAOLA, D. ET AL., (2002) A COGNITIVE ANALYSIS OF DRAGGING PRACTICES IN CABRI ENVIRONMENTS, ZDM

TECHNOLOGY & PROOF

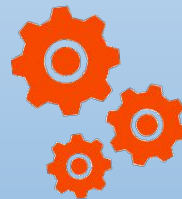
DYNAMIC DRAGGABLE DIAGRAMS STRONGLY AFFECT THE PROVING PROCESS IN THE SENSE OF MEDIATING WHAT KINDS OF CONJECTURES WILL BE MADE [AND] WHAT KINDS OF PROPERTIES WILL BE IDENTIFIED AND THEN ORGANIZED. IN THIS SENSE THE DIAGRAM CONSTRUCTED IN DGE AND THE SCHEMES OF USE ACTIVATED ON IT BY THE STUDENTS CAN BE CONSIDERED AN ACTUAL MEDIATOR [...] PROVIDING CONTINUITY DESPITE THE EPISTEMOLOGICAL DISCONTINUITY BETWEEN A CONJECTURE AND THE CORRESPONDING PROOF

SINCLAIR N., ROBUTTI O. (2012), TECHNOLOGY AND THE ROLE OF PROOF: THE CASE OF DYNAMIC GEOMETRY.

RESEARCH QUESTIONS



IS THE DYNAMIC ELEMENT ADDED TO MERLO ITEMS USING THE GEOGEBRA SW EFFECTIVE IN HELPING STUDENTS UNDERSTAND GEOMETRIC CONCEPTS?



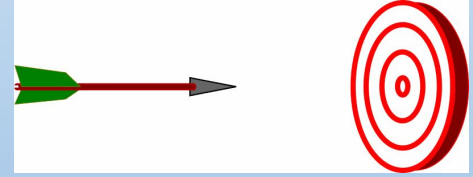
ARE DIGITAL TOOLS ADDING EFFICACY TO MERLO ITEMS IN THE DIRECTION OF ENHANCING STUDENTS' CONCEPTUAL THINKING & ARGUMENTATION?



THE CONTENT CHOICE: THE AMPLITUDE OF

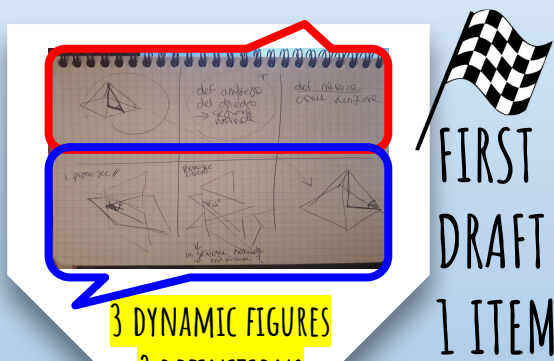
Pragmatic

A DIHEDRAL ANGLE



A TRICKY POINT IN 3D
EUCLIDEAN GEOMETRY
WHOSE KNOWLEDGE IS
FOCAL WITHIN THE ITALIAN
NATIONAL CURRICULUM

" ..THE STUDY OF GEOMETRY WILL CONTINUE WITH THE EXTENSION TO SPACE OF SOME OF THE THEMES OF PLANE GEOMETRY, ALSO TO DEVELOP GEOMETRIC INTUITION. IN PARTICULAR, THE RECIPROCAL POSITIONS OF LINES AND PLANES IN SPACE, THE PARALLELISM AND THE PERPENDICULARITY, AS WELL AS THE PROPERTIES OF THE MAIN GEOMETRIC SOLIDS (IN PARTICULAR OF THE POLYHEDRA AND ROTATION SOLIDS) WILL BE STUDIED ... "



FIRST
DRAFT:
1 ITEM

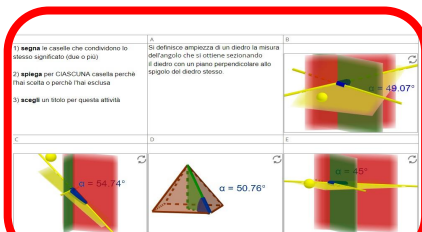
3 DYNAMIC FIGURES
2 DEFINITIONS

THE TASK DESIGN

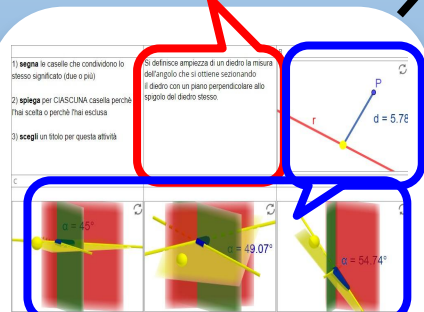


FINAL VERSION: 2 ITEMS

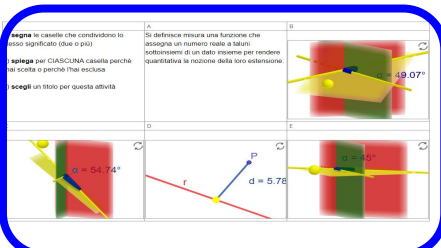
- 1 DEFINITION EACH
- 4 DYNAMIC FIGURES



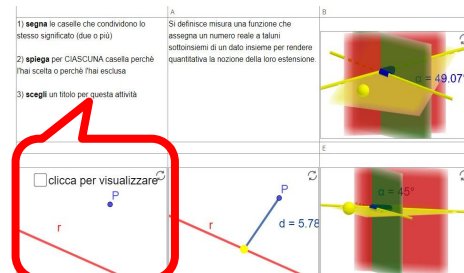
DIHEDRAL ANGLE



1 DEFINITION
4 DYNAMIC FIGURES

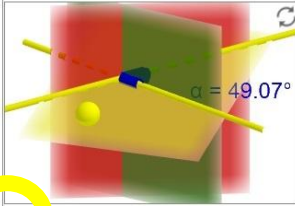
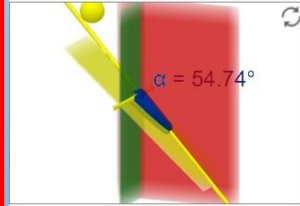
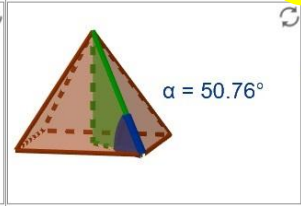
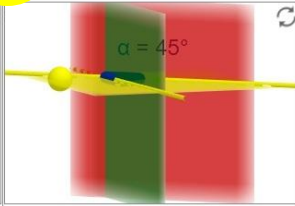


MEASURE

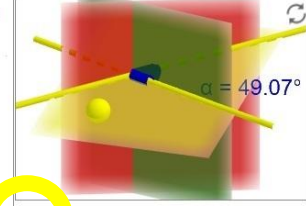
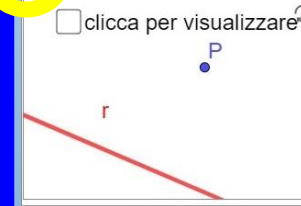
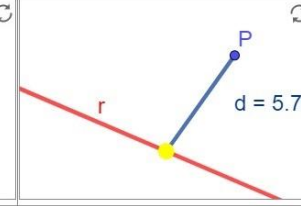
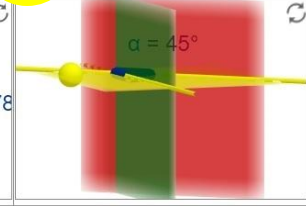


LINKING 2D & 3D MEASURES

FINAL VERSION: EXPECTED CHOICES

1) mark the boxes that share the same meaning (two or more) 2) explain for EACH box why you chose it or why you excluded it 3) choose a title for this activity	A Amplitude of a dihedral is defined as the measurement of the angle obtained by cutting the dihedral with a plane perpendicular to the edge of the dihedral.	B 
C 	D 	E 

ITEM 1 - DIHEDRAL ANGLE A + E

1) mark the boxes that share the same meaning (two or more) 2) explain for EACH box why you chose it or why you excluded it 3) choose a title for this activity	A Measure is defined as a function that assigns a real number to certain subsets of a given set to make quantitative the notion of their extension	B 
C ☐ clicca per visualizzare 	D 	E 

ITEM 2 - MEASURE A + C + E

FINAL VERSION: EXPECTED ARGUMENTATIONS

1) mark the boxes that share the same

The amplitude of a dihedral is defined as the

A - TARGET STATEMENT

B - SECTION PLANE INCLINATION IS NOT FIXED

C - SECTION PLANE INCLINATION IS FIXED BUT NOT NORMAL

D - THE ANGLE SHOWN IS THE ONE BETWEEN THE EDGE AND THE BASE PLANE AND NOT A DIHEDRAL

E - SECTION PLANE IS COHERENT WITH THE TARGET STATEMENT

ITEM 1 - DIHEDRAL ANGLE $A + E$

A - TARGET STATEMENT

B - THE INDICATED VALUE IS NOT A MEASURE OF THE AMPLITUDE OF THE DIHEDRAL BECAUSE IT VARIES

C - THE INDICATED VALUE IS THE WELL-DEFINED UNIVOCAL MEASURE OF THE DISTANCE BETWEEN A POINT AND A LINE

D - THE INDICATED VALUE IS NOT A MEASURE OF THE DISTANCE BETWEEN A POINT AND A LINE BECAUSE IT VARIES

E - THE INDICATED VALUE IS THE WELL-DEFINED UNIVOCAL MEASURE OF THE AMPLITUDE OF THE DIHEDRAL

ITEM 2 - MEASURE $A + C + E$

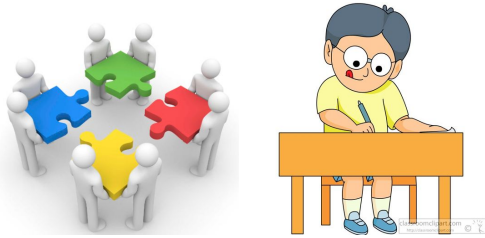
TEACHING EXPERIMENTS SITUATED IN A REAL EDUCATIONAL CONTEXT

Grounded



WE OBSERVED 4 TEACHING EXPERIMENTS
INVOLVING DYNAMIC MERLO ITEMS
CARRIED OUT AT THE SECONDARY SCHOOL
LEVEL (12TH GRADE) IN 3 HIGH
SCHOOLS IN TURIN AND MILAN

M.E.R.L.O. SETTING AND TEACHING PRACTICE



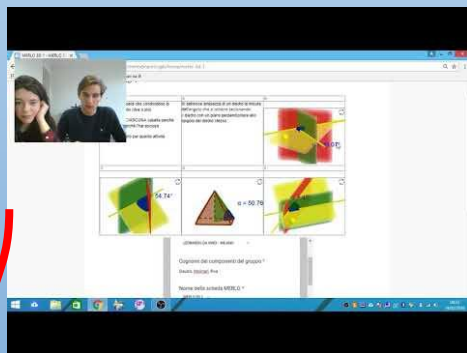
A TYPICAL LESSON BASED ON THE USE OF MERLO ITEMS ENGAGES STUDENTS IN:

- A) SMALL GROUP DISCUSSIONS
- B) RESPONSES
- C) CLASS DISCUSSION MEDIATED BY THE TEACHER

*ARZARELLO, F., KENETT, R. S., ROBUTTI, O., SHAFRIR, U.,
PRODROMOU, T., & CARANTE, P. (2015), TEACHING AND
ASSESSING WITH NEW METHODOLOGICAL TOOLS (MERLO): A NEW
PEDAGOGY?*

COMBINING QUALITATIVE RESEARCH METHODS

Integrative



1 - OBSERVING PROCESSES: AUDIO
& VIDEO + SCREEN ACTIVITY
RECORDING WITH



MERLO DINAMICO - SCHEDA STUDENTE

*Campo obbligatorio

Nome della scuola *

Scegli

Cognomi dei componenti del gruppo *

La tua risposta

Nome della scheda MERLO *

Scegli

Quali caselle secondo te condividono lo stesso significato matematico? (due o più) *

☐ A

☐ B

☐ C

☐ D

☐ E

Per ogni casella spiega perché l'hai scelta o perché l'hai esclusa

Casella A *

La tua risposta

2 - ANALYZING DATA:
ANSWER FORMS

Nome e Cognome _____ Classe _____

Scienze _____ Data _____

LA MISURA DELL'AMPIEZZA DI UN ANGOLO DIEDRO - MERLO ASSESSMENT

1. Dai la definizione di ampiezza di un diedro.

2. Quali condizioni geometriche devono essere verificate affinché gli angoli MERLO e MERLO in figura rappresentino rispettivamente la ampiezza dei diedri aventi come faccia:

- la base della piramide e la faccia laterale BC'D'?
- la base della piramide e la faccia laterale AD'D'?

E' possibile che tali condizioni siano verificate contemporaneamente?

3. Facendo riferimento alla definizione di ampiezza di angolo diedro, spiega perché in una piramide regolare di vertice V l'angolo che una faccia laterale forma con il piano della base corrisponde all'angolo tra l'ipotenusa della piramide e l'ipotenusa del triangolo di base.

4. Supponi di osservare una piramide regolare quadrangolare di vertice V con piani contenenti l'altezza della piramide che forma angoli in rosso? In quale caso l'angolo di vertice V del poligono sezione avrà ampiezza superiore? Puntua?

3 - ASSESSING UNDERSTANDING:
WRITTEN TEST (1 MONTH AFTER)

5. Ci sono diedri figure in fianco? Quali? Come li misureresti?

6. "L'ampiezza planare tra due piani su una superficie sferica è la lunghezza dell'arco di circonferenza appartenente alla superficie sferica avente per estremi i punti dove i piani toccano la sfera". Spiega perché la definizione di ampiezza data non conterrà il concetto di misura in che modo la correggerai?

7. Puntua il quesito rispetto a come giustifichi la tua risposta (senza calcoli).

8. In un parallelogramma rettangolo misurano 6, 8 e 10 cm. Si calcoli in gradi e in' l'ampiezza dell'angolo che la diagonale manda da un vertice forma con l'altezza del parallelogramma.

4 - WRAPPING IT ALL
UP: FEEDBACK FORMS

MERLO FEEDBACK

*Campo obbligatorio

Nome della scuola *

Scegli

ANGOLO DIEDRO

1. Spiega in quali casi la comprensione di un angolo diedro è più complessa e perché.

2. Spiega per quali attività di apprendimento la tua comprensione di un angolo diedro è più complessa e perché.

3. Spiega in quali casi la comprensione di un angolo diedro è più complessa e perché.

Ritieni che l'attività MERLO svolta a febbraio ti abbia aiutato a capire il concetto di ampiezza di un angolo diedro? *

per niente 1 2 3 4 5 moltissimo

Ricordi quale/i aspetto/i dell'attività ti ha/hanno maggiormente aiutato? (puoi scegliere più caselle) *

☐ la definizione

FIRST & SECOND TEACHING EXPERIMENTS: LICEO SCIENTIFICO LEONARDO DA VINCI, MILAN & LICEO DELLE SCIENZE UMANE SANTORRE DI SANTAROSA, TURIN

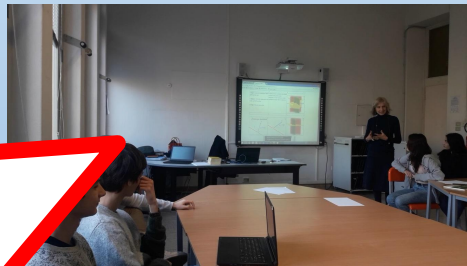
TEACHER & RESEARCHER PRESENT
15 STUDENTS SPLIT INTO GROUPS OF 3
PEOPLE EACH

60 MIN GROUP WORK

- 2 GROUPS WORK ON ITEM 1
- 3 GROUPS WORK ON ITEM 2

30 MIN GUIDED CLASS DISCUSSION

DATA COLLECTED THROUGH VIDEO
REGISTRATION AND WRITTEN
QUESTIONNAIRE



BINI & TRINCHERO - CADGME 2018

TEACHER/RESEARCHER PRESENT
15 STUDENTS SPLIT INTO GROUPS OF 3
PEOPLE EACH

60 MIN GROUP WORK

- 2 GROUPS WORK ON ITEM 1
- 3 GROUPS WORK ON ITEM 2

30 MIN GUIDED CLASS DISCUSSION

DATA COLLECTED THROUGH VIDEO
REGISTRATION AND WRITTEN
QUESTIONNAIRE



DATA: OBSERVING SIGNS AND PROTOCOLS



THE UNPREDICTED GIFTS OF MERLO PEDAGOGY

EXAMPLE 1:
AN UNEXPECTED
(WRONG?!) CHOICE
WITH RIGHT
ARGUMENTATIONS

EXAMPLE 2:
AN EXPECTED
(RIGHT?!) CHOICE
WITH WRONG
ARGUMENTATIONS

EXAMPLE 3:
SMALLER EXPECTATIONS
ALLOW LARGER
PERCEPTIONS

OBSERVING SIGNS AND PROTOCOLS: **EXAMPLE 1**



S1 "IN FACT WE CAN CONSIDER THE BASE (OF THE PYRAMID) AS ONE FACE OF THE DIHEDRAL AND THE OTHER PLANE THAT YOU HAVE TO IMAGINE IS THIS" (POINTS TO THE FIGURE)

S2 MAKES A GESTURE MIMICKING THE TWO PLANES DESCRIBED BY S1

S3 "I DO NOT KNOW IF WE CAN INCLUDE IT AND THEN WRITE THAT THE PLANE IS NOT THERE"

S2 "BECAUSE IT SEEMS TO ME THAT THERE IS EVERYTHING"

S3 "THERE IS EVERYTHING WHAT?"

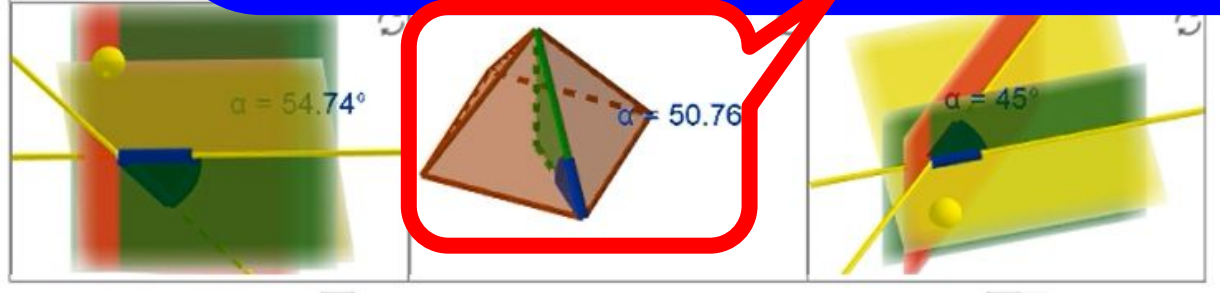
S2 "THERE IS EVERYTHING WE NEED"

S3 "NO BECAUSE YOU SEE HERE THERE ARE TWO PLANES AND HERE IS A PLANE AND AN EDGE"

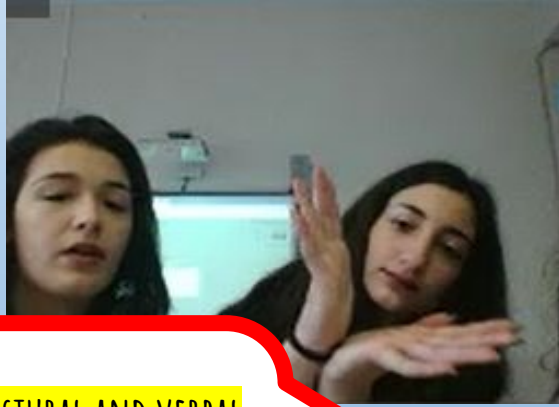
~~ABCDE~~

ITEM 1

DIHEDRAL ANGLE

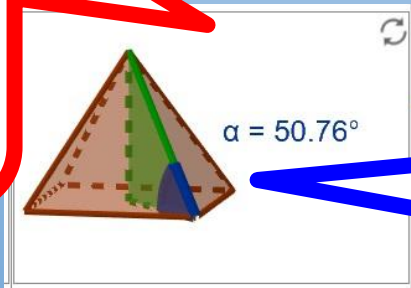


OBSERVING SIGNS AND PROTOCOLS: EXAMPLE 1



A GESTURAL AND VERBAL
THOUGHT EXPERIMENT

TAKES PLACE
(SINCLAIR ET AL., 2013)



ITEM 1

DIHEDRAL ANGLE

PRODUCTIVE REASONING [...] INCLUDES A PERMANENT
INTERPLAY BETWEEN CONCEPTUAL AND IMAGINATIVE
DYNAMICS (FISCHBEIN, 1993)

OBSERVING SIGNS AND PROTOCOLS: **EXAMPLE 2**

THE THIRD GROUP MAKES THE EXPECTED CHOICE, BUT ARGUMENTATIONS REVEAL
THEY MISMATCHED DOMAIN AND RANGE OF THE MEASURE FUNCTION:

BOX A: WE START FROM THE DEFINITION TO GET TO THE GRAPHICAL REPRESENTATION.

BOX B: DOES NOT CORRESPOND TO DEFINITION A BECAUSE A SINGLE SUBSET CORRESPONDS TO A REAL NUMBER.

BOX C: MANY SUBSETS CORRESPOND TO A REAL NUMBER D (DISTANCE)

BOX D: THERE ARE TOO MANY VARIABLES TO DEFINE A FUNCTION

BOX E: CALLING X THE HEIGHT OF THE YELLOW PLANE, WHEN X VARIES THERE IS ONLY ONE REAL NUMBER WHICH IS $\alpha = 45^\circ$

ITEM 2
MEASURE

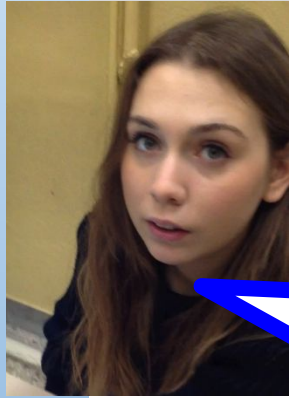
A B C D E



SUGGESTED TITLE:

FUNCTIONS APPLIED TO TWO-DIMENSIONAL AND
THREE-DIMENSIONAL FIGURES

OBSERVING SIGNS AND PROTOCOLS: **EXAMPLE 3**



that share the same
(ore)
CH box why you chose it or
or this activity

A

A measure is defined as a function that assigns a quantity to a positive real number obtained by comparing the quantity with a unit of measurement

B

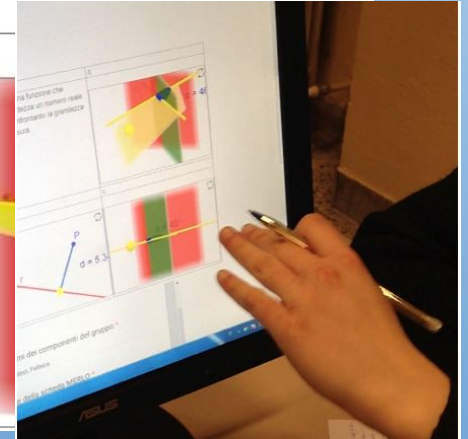
~~ABCDE~~

BY MOVING THE BALL, WE NOTICE THAT THE ALPHA ANGLE VARIES: *IF THE PLANE* ON WHICH THE BALL RESTS *IS PERPENDICULAR* TO THE OTHER TWO PLANES, **THE ANGLE WILL BE THE BIGGEST**; ON THE CONTRARY, IF THE *PLANE* WITH THE BALL IS IN A *MORE INCLINED* POSITION, THE *AMPLITUDE OF THE ANGLE WILL DECREASE* MORE AND MORE

☐ clicca per visualiz



ITEM 2
MEASURE



DATA: OBSERVING SIGNS AND PROTOCOLS

THE COGNITIVE VALUE OF MERLO PEDAGOGY

EXAMPLE 4: BRINGING OUT STUDENTS'
MISCONCEPTIONS ABOUT
ARGUMENTATION



OBSERVING SIGNS AND PROTOCOLS: **EXAMPLE 4**



IN THEIR QUESTIONNAIRE,
STUDENTS **MIX UP DESCRIPTION**
WITH ARGUMENTATION: ACCORDING
TO THIS, SURFACE SIMILARITY LEADS
TO THE FIRST WRONG CHOICE

ITEM 2
MEASURE

Nome della scuola *

LEONARDO DA VINCI - MILANO

Cognomi dei componenti del gruppo

castiglioni, ferlazzo, guida

Nome della scheda MERLO *

MERLO 3D 2

Quali caselle secondo te condividono lo stesso significato matematico? (due o più) *

☐ A

☒ B

☐ C

☐ D

☒ E

1) **segna** le caselle che condividono lo stesso significato (due o più)

2) **spiega** per CIASCUNA casella perchè l'hai scelta o perchè l'hai esclusa

3) **scegli** un titolo per questa attività

Si definisce misura una funzione che assegna un numero reale a taluni sottoinsiemi di un dato insieme per quantificare la nozione della loro estensione.

clicca per visualizzare

$\alpha = 49.07^\circ$

$\alpha = 45^\circ$

$d = 5.78$

ABCDE

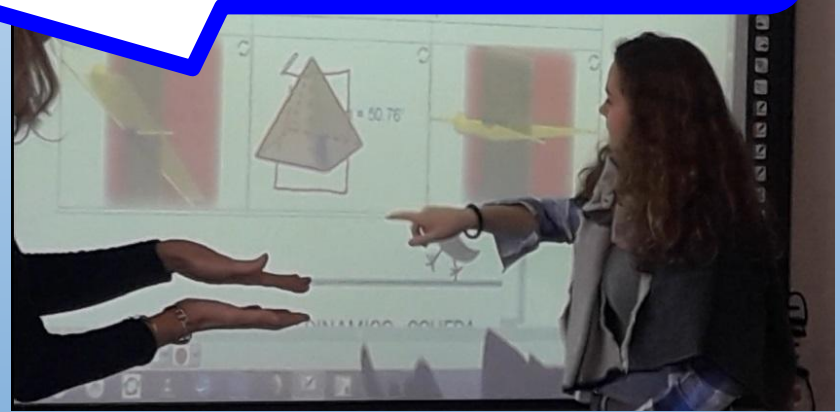
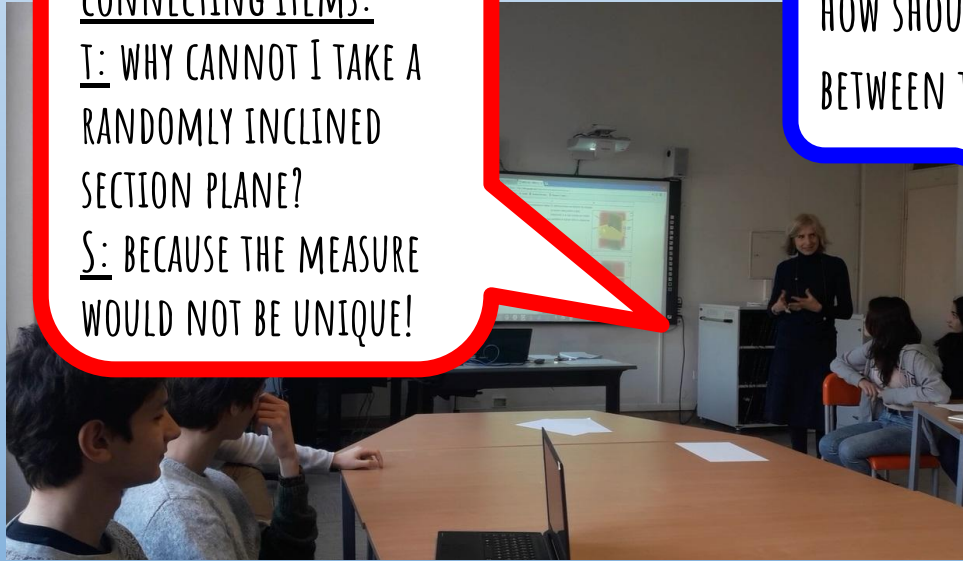
OBSERVING SIGNS AND PROTOCOLS: **EXAMPLE 5**

CONNECTING ITEMS:

T: WHY CANNOT I TAKE A RANDOMLY INCLINED SECTION PLANE?

S: BECAUSE THE MEASURE WOULD NOT BE UNIQUE!

HOW SHOULD WE ADD A PLANE TO BOX D TO MEASURE THE ANGLE BETWEEN THE BASE OF THE PYRAMID AND THE SIDE FACE?



**REFLECTIVE
DISCUSSION**

MERLO PEDAGOGY APPROACH AS AN "OPPORTUNITY TO DISCUSS"

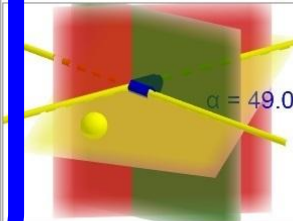
THE FIFTH & SIXTH DRAFT: ADJUSTING THE DEFINITION OF MEASURE

Flexible

AFTER THE 1ST
EXPERIMENT:

A measure is defined as a function that assigns a quantity to a positive real number obtained by comparing the quantity with a unit of measurement

INVOLVING
UNITS

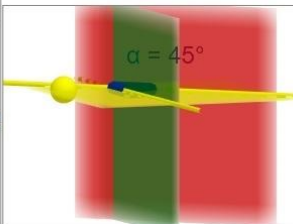
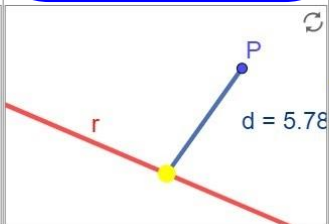


mark the boxes that share the same meaning (two or more)

explain for EACH box why you chose it or why you excluded it

choose a title for this activity

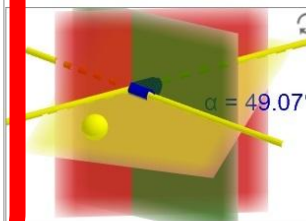
☐ clicca per visualizzare



AFTER THE 2ND
EXPERIMENT:

In mathematics, a measure is defined as a function that assigns to a quantity a positive real number

ELIMINATING
UNITS

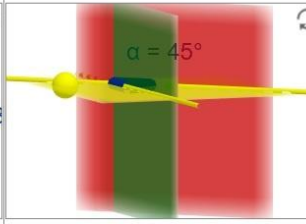
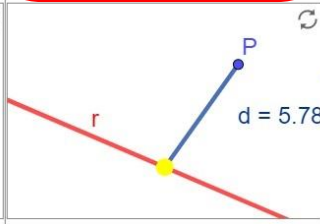
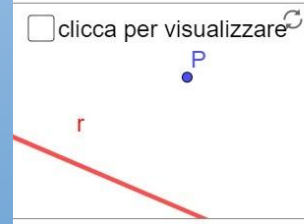


mark the boxes that share the same meaning (two or more)

explain for EACH box why you chose it or why you excluded it

choose a title for this activity

☐ clicca per visualizzare



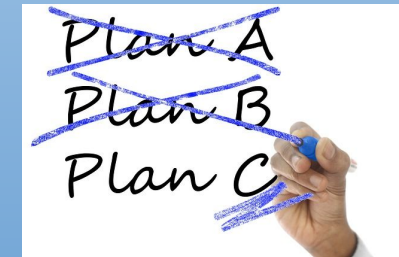
Interactive



ASSESSMENT QUESTION ON DIHEDRON	ASSESSMENT QUESTION ON MEASURE
YES	YES
YES	YES
YES	NO
ABSENT DURING ASSESSMENT	
YES	NO
YES	YES
YES	YES
YES	NO
YES	NO
YES	YES
YES	YES
YES	NO
YES	NO
YES	NO
YES	YES
YES	NO
YES	NO
YES	NO
YES	YES
YES	NO

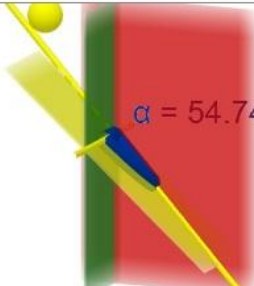
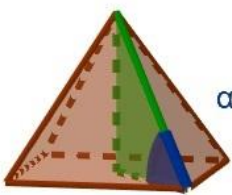
CHANGING PLANS

THE ASSESSMENT TEST ADMINISTERED ONE MONTH AFTER THE TEACHING EXPERIMENTS SHOWED THAT THE TOTALITY OF THE STUDENTS IN MILAN AND THE MAJORITY IN TURIN MASTERED THE CONCEPT OF DIHEDRAL ANGLE, WHILE THE IDEA OF MEASURE AS A FUNCTION WAS PROBLEMATIC: THIS LEAD TO A RADICAL SHIFT IN THE TASK DESIGN AND EVENTUALLY IN A *CHANGE* OF THE SECOND ITEM TS



BACK TO SQUARE 1: THE DIHEDRAL ANGLE ALONE

Iterative

1) mark the boxes that share the same meaning (two or more) 2) explain for EACH box why you chose it or why you excluded it 3) choose a title for this activity	A The amplitude of a dihedral is defined as the measurement of the angle obtained by cutting the dihedral with a plane perpendicular to the edge of the dihedral.	B DEVOLUTION: ELICITING THE CONCEPT OF MEASURE AS A FUNCTION FROM STUDENTS THEMSELVES Why is it necessary to choose a particular inclination of the section plane to define the amplitude of a dihedral angle? Testo risposta lunga
C  $\alpha = 54.74^\circ$	D  $\alpha = 50.7^\circ$	Why is the particular inclination chosen precisely the one perpendicular to the edge of the dihedral? Testo risposta lunga

THIRD TEACHING EXPERIMENT: ISTITUTO TECNICO CHIMICO SANTORRE DI SANTAROSA, TURIN



TEACHER PRESENT

18 STUDENTS SPLIT INTO GROUPS OF 3
PEOPLE EACH

60 MIN GROUP WORK

- 6 GROUPS WORK ON ITEM 1

30 MIN GUIDED CLASS DISCUSSION

DATA COLLECTED THROUGH WRITTEN
QUESTIONNAIRE

OBSERVING SIGNS AND PROTOCOLS: EXAMPLE 6

WHY IS THE INCLINATION
CHOSEN TO DEFINE THE
AMPLITUDE OF THE DIHEDRAL
ANGLE THE ONE PERPENDICULAR
TO THE EDGE OF THE DIHEDRAL?

THE ROLE OF DYNAMICITY & THE
EMERGENCE OF NEW MATHEMATICAL IDEAS

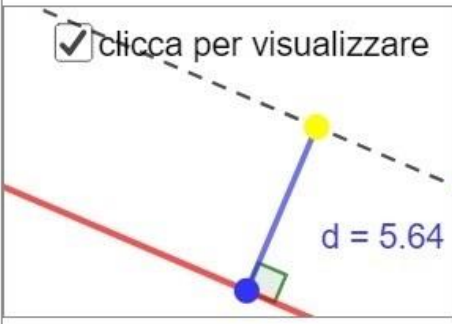
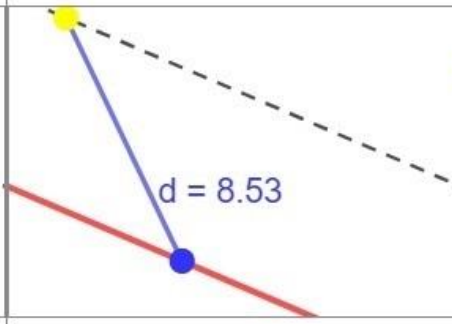
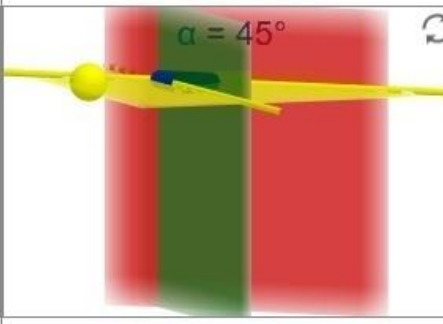
ITEM 1
DIHEDRAL
ANGLE

"CONSIDERING THAT BY VARYING THE INCLINATION THE MEASURE WOULD VARY, A STANDARD MEASURE WAS PROBABLY CHOSEN, IN ORDER TO MAKE ALL MEASUREMENTS OF THE DIHEDRAL WIDTHS COMPARABLE".

BECAUSE IN THIS
WAY YOU HAVE THE
MAXIMUM WIDTH

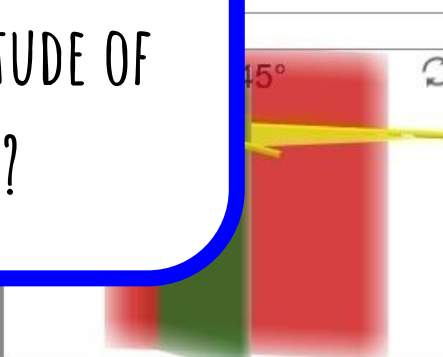
FROM MEASURE TO INVARIANTS

Flexible

1) mark the boxes that share the same meaning (two or more) 2) explain for EACH box why you chose it or why you excluded it 3) choose a title for this activity	A <u>an invariant is a quantity that remains unaltered during a transformation</u>	B 
C ☒ clicca per visualizzare 	D 	E 

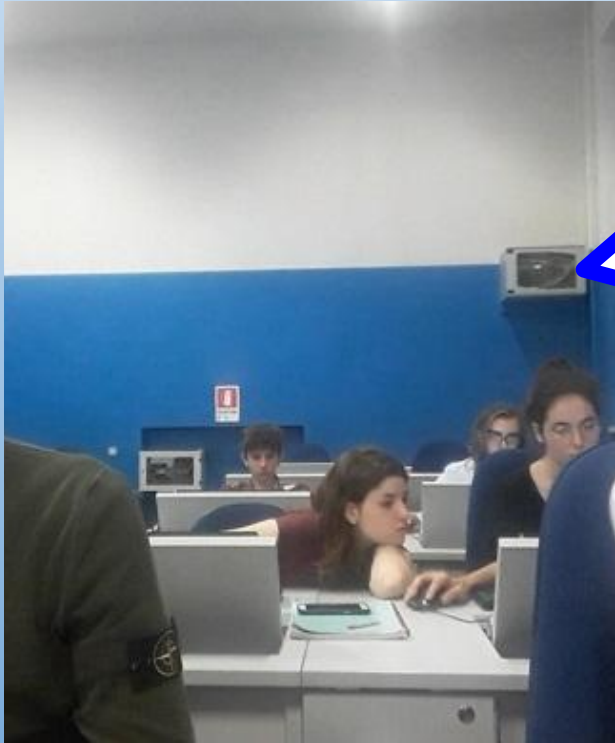
IDENTIFYING THE REAL SHARED MEANING

WRAPPING IT ALL UP

	A	B
1) mark the boxes that share the same meaning (two or more)	An invariant is a quantity that remains unaltered during a transformation	
2) explain for EACH box why it or why you excluded it		
3) choose a title for this		
C		
☒ clicca per v		

WHICH CONNECTION DO YOU SEE BETWEEN THIS ACTIVITY AND THE DEFINITION OF THE AMPLITUDE OF THE DIHEDRAL ANGLE?

FOURTH TEACHING EXPERIMENT: LICEO SCIENTIFICO GALILEO FERRARIS, TURIN



20 STUDENTS SPLIT INTO GROUPS OF 2/3 PEOPLE EACH

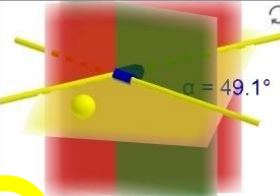
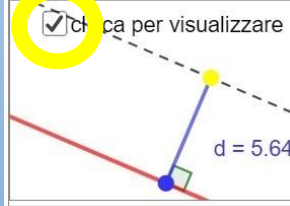
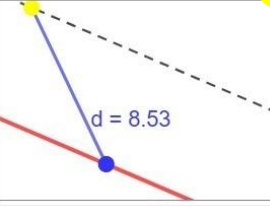
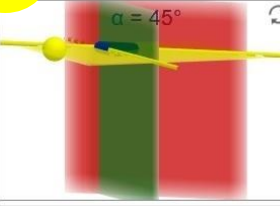
60 MIN GROUP WORK

- 7 GROUPS WORK ON ITEM 2

30 MIN GUIDED CLASS DISCUSSION

DATA COLLECTED THROUGH VIDEO REGISTRATION AND
WRITTEN QUESTIONNAIRE

ITEM 2 FINAL VERSION: EXPECTED & MADE CHOICES

1) mark the boxes that share the same meaning (two or more) 2) explain for EACH box why you chose it or why you excluded it 3) choose a title for this activity	A Invariant is a quantity that remains unaltered during a transformation	B 
C ☒ scelta per visualizzare 	D 	E 

ITEM 2 - INVARIANT A + C + E

ALL GROUPS HAVE SELECTED THE
EXPECTED BOXES, JUSTIFYING THEIR
CHOICES WITH APPROPRIATE
ARGUMENTS

~~A~~~~B~~~~C~~~~D~~~~E~~

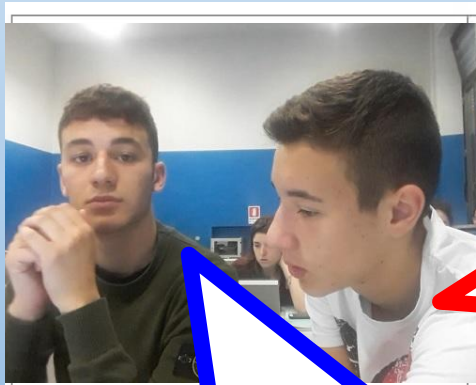
DATA: OBSERVING SIGNS AND PROTOCOLS

DYNAMIC MERLO PEDAGOGY APPROACH AS A SETTING FOR
TRANSFORMATIONAL REASONING (M.A.SIMON)



EXAMPLE 7: THE "ABILITY TO
CONSIDER A DYNAMIC PROCESS BY
WHICH A CONTINUUM OF STATES
ARE GENERATED"

OBSERVING SIGNS AND PROTOCOLS: EXAMPLE 7



"KNOWLEDGE AS THE RESULT OF
RUNNING THE SYSTEM"

ITEM 2
INVARIANTS

An invariant is a quantity
unaltered during a trans

GROUP 1: RELEVANT STATIC ASPECTS IN SELECTED BOXES:

AMONG THE **STATIC** ASPECTS WE HAVE RECOGNIZED AS
RELEVANT: IN BOX C **THE DISTANCE** BETWEEN THE TWO POINTS;
IN BOX E **THE ANGLE** BETWEEN THE PLANES

GROUP 2: RELEVANT DYNAMIC ASPECTS IN SELECTED BOXES:

THE CHOSEN BOXES **REMAIN UNCHANGED DURING THE
TRANSFORMATION**. THE WIDTH OF THE DIHEDRAL ANGLE AND
THE DISTANCE BETWEEN TWO PARALLEL LINES NEVER
CHANGES.

GROUP 3: **ONCE THE IMAGES WERE CHOSEN**, WE NOTICED THE
AFFINITY OF THE DEFINITION WITH THESE [IMAGES]

RESULTS & CONCLUSIONS

THE TEACHERS' PERSPECTIVE

THE STUDENTS' PERSPECTIVE

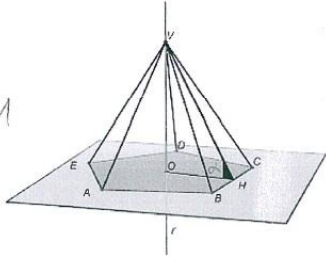
THE MATHEMATICIANS'
PERSPECTIVE

THE RESEARCHERS'
PERSPECTIVE

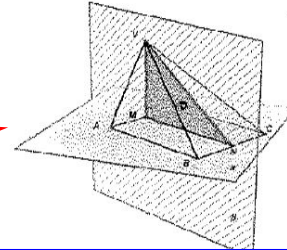
DID IT WORK? THE TEACHERS' PERSPECTIVE

IDENTIFYING THE DIHEDRAL ANGLE

3. Facendo riferimento alla definizione di ampiezza di angolo diedro, spiega perché in una piramide regolare di vertice V l'angolo che una faccia laterale forma con il piano della base corrisponde all'angolo tra l'apotema della piramide e l'apotema del poligono di base.



VH apotema $\Rightarrow BH \equiv HC$
 $\Rightarrow \angle VHB \equiv \frac{\pi}{2} \equiv \angle VHC \Rightarrow VH \perp BC$ ✓
 OH apotema base $\Rightarrow OH \perp BC$ per
 caratteristiche dell'angolo di base ✓
 per due rette passa un solo piano
 $VH \perp BC$
 $OH \perp BC$
 $\Rightarrow \angle VHO$



2. Quali condizioni geometriche devono essere verificate affinché gli angoli MNV e NMV in figura esprimano rispettivamente le ampiezze dei diedri aventi come facce

- la base della piramide e la faccia laterale BCV ?
che il piano di...
...sia perpendicolare alla spigola BC ...
- la base della piramide e la faccia laterale ADV ?
che il piano di...
...sia perpendicolare alla spigola AD ...

DEFINING THE MEASURE



6. "Si definisce distanza dell'arco di circonferenza estremi i punti formulata è sbagliata. E' una parte di circonferenza che si può misurare." ✓

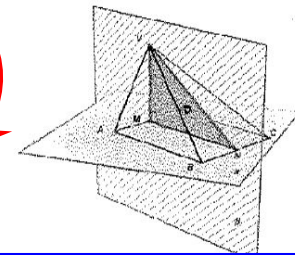
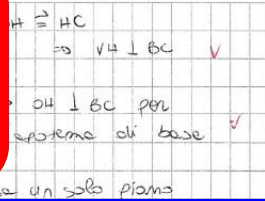
"THE DISTANCE BETWEEN TWO POINTS ON A SPHERICAL SURFACE IS DEFINED AS THE LENGTH OF THE ARC OF CIRCUMFERENCE BELONGING TO THE SPHERICAL SURFACE HAVING THE TWO POINTS AS EXTREMITIES"
EXPLAIN WHY THIS DEFINITION OF DISTANCE ON THE SPHERE IS WRONG AND SUGGEST HOW YOU WOULD CORRECT IT

DID IT WORK? THE TEACHERS' PERSPECTIVE

IDENTIFYING THE DIHEDRAL ANGLE

ALTERNATIVE CONCEPTUAL THINKERS SCORE HIGHER THAN GOOD CONCEPTUAL THINKERS

U. SHAFRIR



2. Quali condizioni geometriche devono essere verificate affinché gli angoli MNV e NMV in figura esprimano rispettivamente le ampiezze dei diedri aventi come facce
- la base della piramide e la faccia laterale BCV?
 - la base della piramide e la faccia laterale ADV?

DEFINING THE MEASURE



6. "Si definisce distanza dell'arco di circonferenza estremi i punti A e B" formulata è sbagliata perché si trovano sulla stessa circonferenza che si può scegliere la più piccola o la più grande.

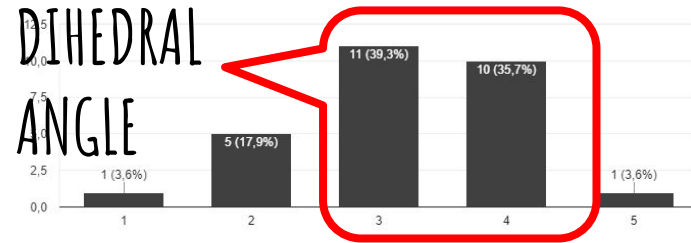
"THE DISTANCE BETWEEN TWO POINTS ON A SPHERICAL SURFACE IS DEFINED AS THE LENGTH OF THE ARC OF CIRCUMFERENCE BELONGING TO THE SPHERICAL SURFACE HAVING THE TWO POINTS AS EXTREMITIES" EXPLAIN WHY THIS DEFINITION OF DISTANCE ON THE SPHERE IS WRONG AND SUGGEST HOW YOU WOULD CORRECT IT:

IT IS WRONG BECAUSE GIVEN TWO POINTS WE HAVE INFINITE CIRCLES LYING ON THE SPHERE; WE MUST THEREFORE SPECIFY THAT THE CIRCUMFERENCE THAT WE CONSIDERED IS THAT OF MAXIMUM RADIUS

DID IT WORK? THE STUDENTS' PERSPECTIVE

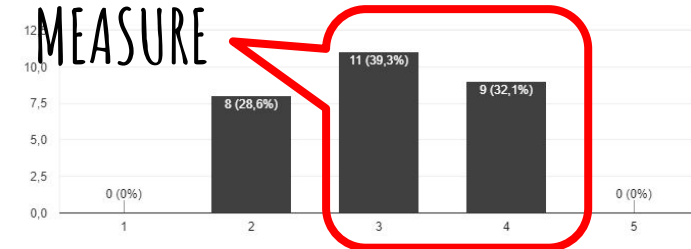
Ritieni che l'attività MERLO ti abbia aiutato a capire il concetto di ampiezza di un angolo diedro?

28 risposte



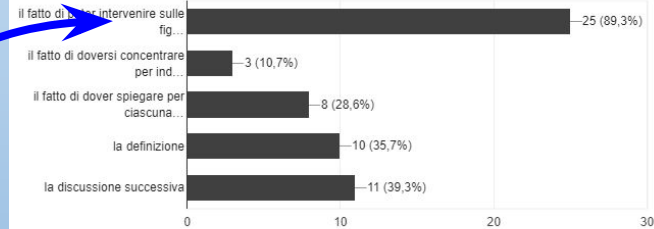
Ritieni che l'attività MERLO ti abbia aiutato a capire il concetto di misura come invariante?

28 risposte



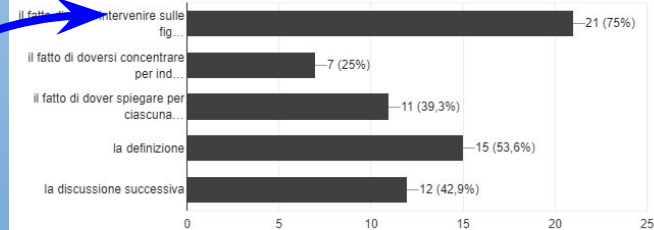
Ricordi quale/i aspetto/i dell'attività ti ha/hanno maggiormente aiutato? (puoi scegliere più caselle)

28 risposte



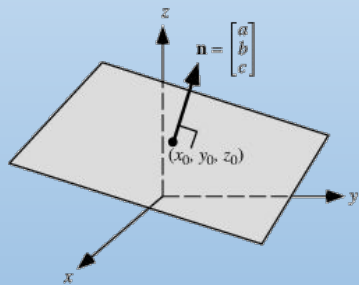
Ricordi quale/i aspetto/i dell'attività ti ha/hanno maggiormente aiutato? (puoi scegliere più caselle)

28 risposte



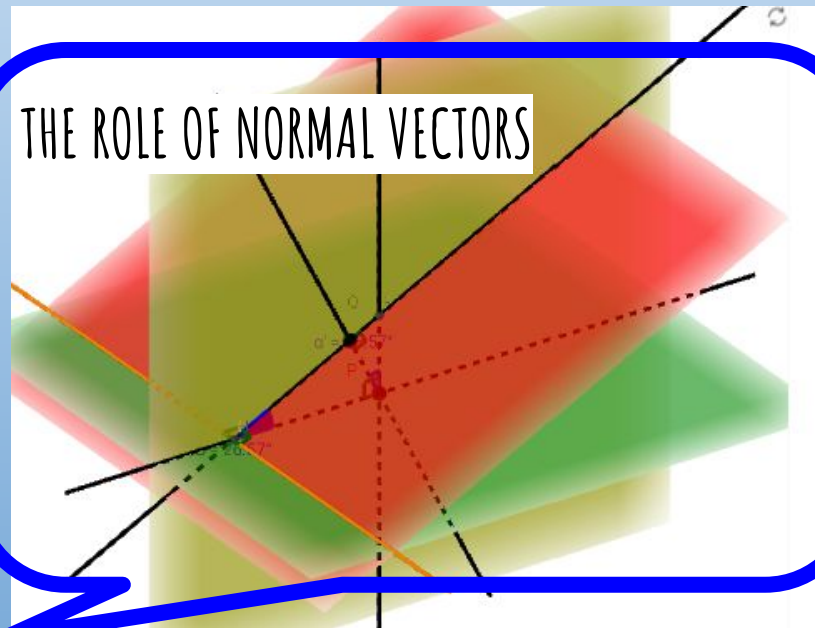
DID IT WORK? THE MATHEMATICIANS' PERSPECTIVE

RETHINKING THE DEFINITION



1. DOES THE ANGLE DEFINED THROUGH THE NORMAL SECTION CORRESPOND TO AN EXTREME VALUE OF THE AMPLITUDE?
2. CAN WE PROVE THAT THE PLANE CONTAINING THE TWO NORMAL VECTORS IS PERPENDICULAR TO THE DIHEDRAL EDGE?

THE ROLE OF NORMAL VECTORS





DID IT WORK? THE RESEARCHERS' PERSPECTIVE

RQ1: IS THE DYNAMIC ELEMENT ADDED TO MERLO ITEMS USING THE GEOGEBRA SW EFFECTIVE IN HELPING STUDENTS UNDERSTAND GEOMETRIC CONCEPTS?

- ★ EXAMPLE 3 BIGGEST ANGLE
- ★ EXAMPLE 5 CLASS DISCUSSION
- ★ EXAMPLE 6 MEASURE

EXAMPLES 3, 5 AND 6 SHOW THAT THE PERCEPTION OF THE NECESSITY UNIQUENESS FOR A WELL DEFINED MEASURE AND THE GUESSING OF THE MAXIMUM VALUE FOR THE CHOSEN ONE WERE EFFECTIVELY DRIVEN BY THE DYNAMICITY OF THE ITEMS

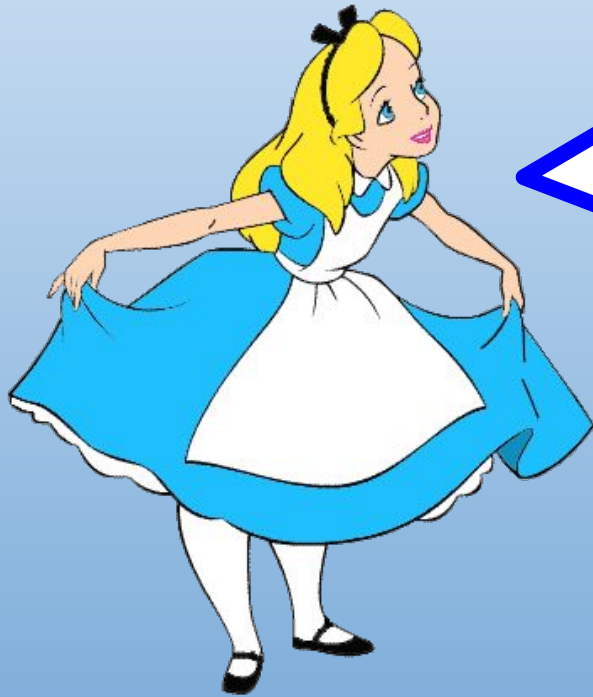
DID IT WORK? THE RESEARCHERS' PERSPECTIVE



RQ2: ARE DIGITAL TOOLS ADDING EFFICACY TO MERLO ITEMS IN THE DIRECTION OF ENHANCING STUDENTS' CONCEPTUAL THINKING & ARGUMENTATION?

- ★ EXAMPLE 1 THOUGHT EXPERIMENT
- ★ EXAMPLE 2 RIGHT ANSWER, WRONG ARGUMENTATION
- ★ EXAMPLE 4 DESCRIPTION VS ARGUMENTATION
- ★ EXAMPLE 7 STATIC VS DYNAMIC INVARIANTS

EXAMPLES 1, 2, 4 AND 7 SHOW THAT ALLOWING STUDENTS TO INTERACT WITH MATHEMATICAL OBJECTS BY THE MEANS OF DIGITAL TOOLS CAN PROMOTE VALID ARGUMENTATIONS, HELP IN FOCALIZING INCORRECT ONES AND FOSTER TRANSFORMATIONAL REASONING



THANK YOU
OBRIGADA
MERCİ
DANKE
GRAZIE MILLE!

REFERENCES



- ARZARELLO, F., OLIVERO, F., PAOLA, D. ET AL., (2002) A COGNITIVE ANALYSIS OF DRAGGING PRACTICES IN CABRI ENVIRONMENTS, ZENTRALBLATT FÜR DIDAKTIK DER MATHEMATIK 34: 66.
- ARZARELLO, F., ROBUTTI O. & CARANTE P. (2015), MERLO: A NEW TOOL AND A NEW CHALLENGE IN MATHEMATICS TEACHING AND LEARNING, PROCEEDINGS OF PME 39, 2 57-65
- ARZARELLO, F., KENETT, R. S., ROBUTTI, O., SHAFRIR, U., PRODRIMOU, T., & CARANTE, P. (2015), TEACHING AND ASSESSING WITH NEW METHODOLOGICAL TOOLS (MERLO): A NEW PEDAGOGY? PROCEEDINGS OF THE IMA INTERNATIONAL CONFERENCE ON BARRIERS AND ENABLERS TO LEARNING MATHS: ENHANCING LEARNING AND TEACHING FOR ALL LEARNERS, M.A. HERSH AND M.KOTECHA EDITORS, 10-12TH JUNE, GLASGOW, UK.
- ETKIND, M., & SHAFRIR, U. (2013).TEACHING AND LEARNING IN THE DIGITAL AGE WITH PEDAGOGY FOR CONCEPTUAL THINKING AND PEER COOPERATION. IN:PROC. 7H INTERNATIONAL TECHNOLOGY, EDUCATION AND DEVELOPMENT CONFERENCE (INTED) (PP. 5347-5352). VALENCIA, SPAIN.
- FISCHBEIN, E. THE THEORY OF FIGURAL CONCEPTS, EDUC STUD MATH 24: 139, SPRINGER INTERNATIONAL
- SIMON, M. (1996). BEYOND INDUCTIVE AND DEDUCTIVE REASONING: THE SEARCH FOR A SENSE OF KNOWING. EDUCATIONAL STUDIES IN MATHEMATICS, 30(2), 197-209.
- SINCLAIR N., ROBUTTI O. (2012), TECHNOLOGY AND THE ROLE OF PROOF: THE CASE OF DYNAMIC GEOMETRY. IN: CLEMENTS M., BISHOP A., KEITEL C, KILPATRICK J., LEUNG F. (EDS) THIRD INTERNATIONAL HANDBOOK OF MATHEMATICS EDUCATION. SPRINGER INTERNATIONAL HANDBOOKS OF EDUCATION, VOL 27. SPRINGER, NEW YORK, NY