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How to use Geometric Software in Courses of Differential Geometry

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KMA FAV ZČU Plzeň

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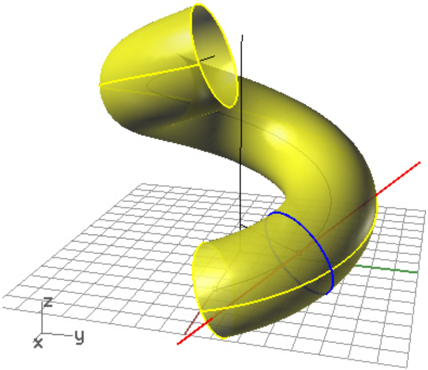
Coimbra 2018

Content



- Course of Differential Geometry.
- How we use software
- Practical lessons of differential geometry

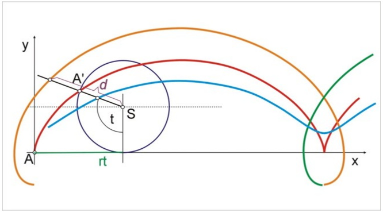
Course of Differential Geometry

- Curve equations, parameterization.
- Curvature, torsion, Frenet equations.
- Surface equations, parameterization.
- The first fundamental form of surfaces: the length of the curve on the surface, projection and developing surfaces, conformal mapping.
- The second fundamental form of surfaces: normal surface curvature, Meusnier proposition, Dupin indikatrix.
- Mean and Gaussian curvature.
- Developable and minimal surfaces.
- The second fundamental form of surfaces: normal surface curvature, Geodetic curvature, geodesic.

How to use software in courses

- Vizualizations of curves
- Vizualizations of surfaces
- Theoretical lectures - ilustration of definitions
- Theoretical lectures - theorems
- Ilustration of exercise or examples
- Ilustration of properties
- Practical lesson with Rhinoceros

How to use software in courses



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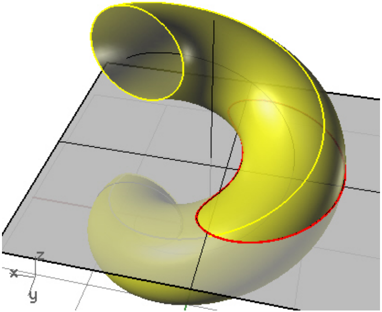

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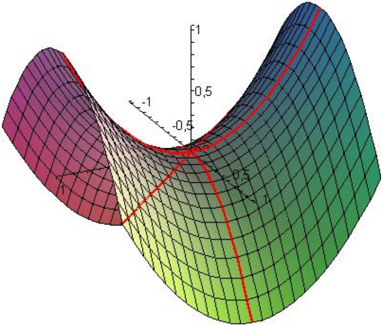

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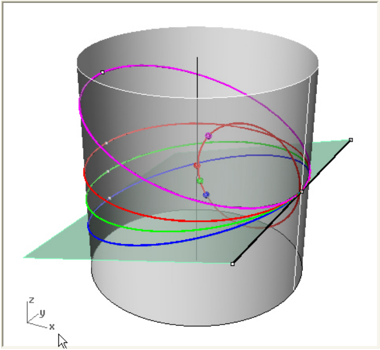

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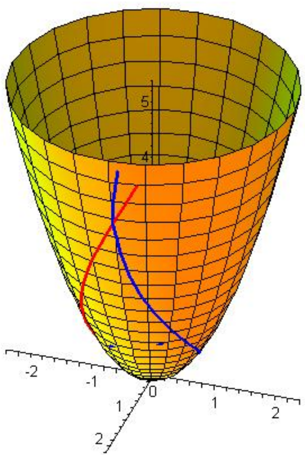

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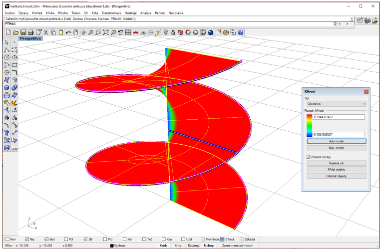

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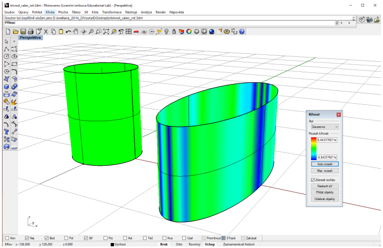

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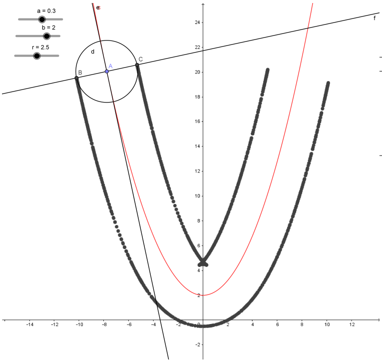
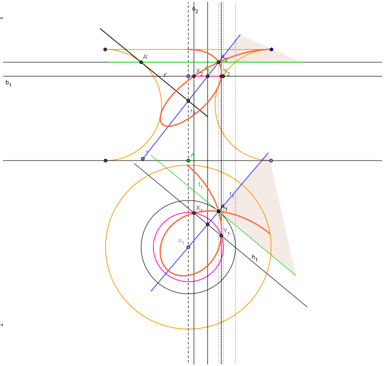
Geometric software

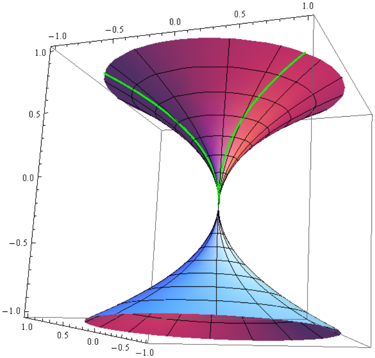
Geogebra

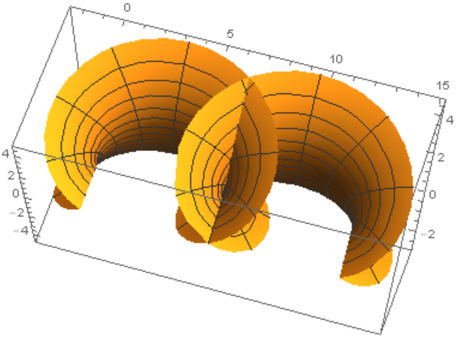




Geometric software

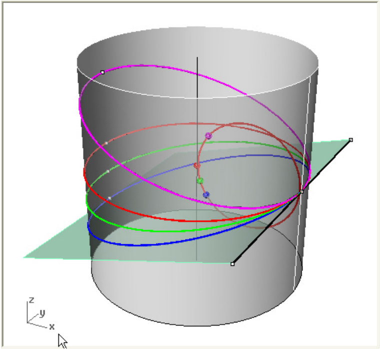
Wolfram Mathematica

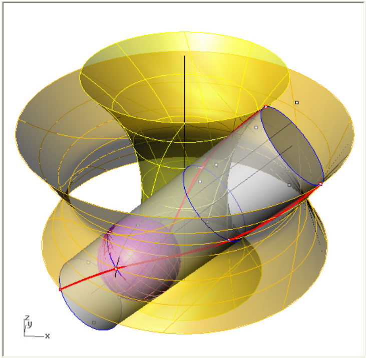




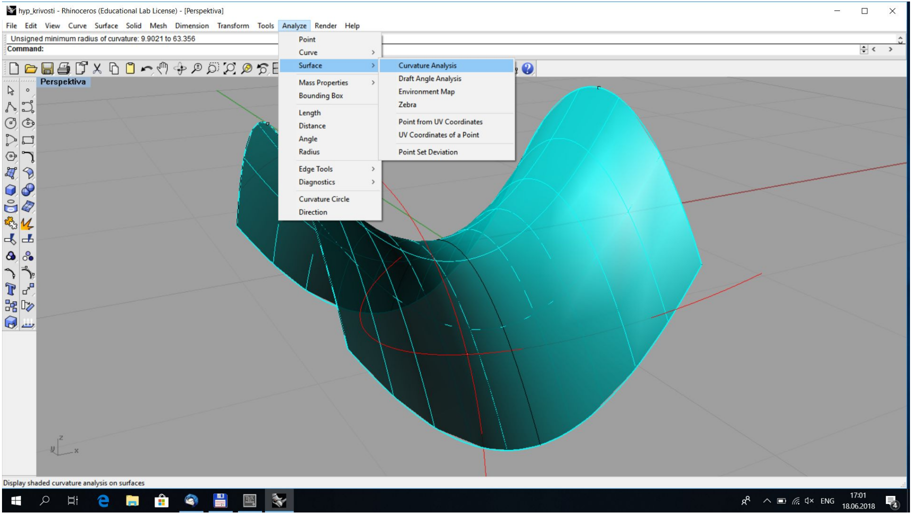
Geometric software

Rhinoceros

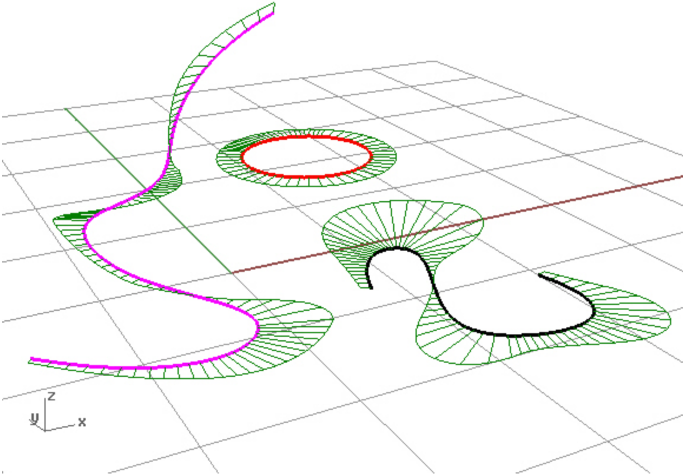




Why Rhinoceros

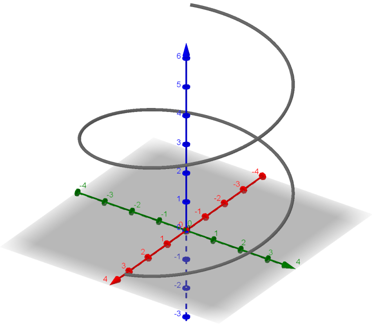


Curvature Graph



Helix

Example 1: Compute the curvature for the helix. Plot the helix and display the curvature graph.



$\boldsymbol{P}(s) = (r\cos\frac{s}{\sqrt{r^2+c^2}}, r\sin\frac{s}{\sqrt{r^2+c^2}}, \frac{cs}{\sqrt{r^2+c^2}}),\ s \in (-\infty, \infty)$

$\dot{\boldsymbol{P}}(s) = (-r\frac{1}{\sqrt{r^2+c^2}}\sin\frac{s}{\sqrt{r^2+c^2}}, r\frac{1}{\sqrt{r^2+c^2}}\cos\frac{s}{\sqrt{r^2+c^2}}, \frac{c}{\sqrt{r^2+c^2}})$

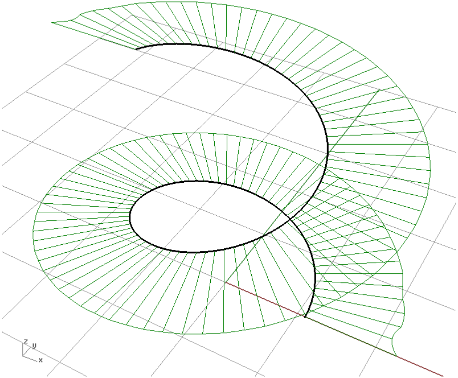
$\ddot{\boldsymbol{P}}(s) = (-r\frac{1}{r^2+c^2}\cos\frac{s}{\sqrt{r^2+c^2}}, -r\frac{1}{r^2+c^2}\sin\frac{s}{\sqrt{r^2+c^2}}, 0)$

${}^1k = \parallel \ddot{\boldsymbol{P}} \parallel = \sqrt{(r\frac{1}{r^2+c^2})^2} = \frac{r}{r^2+c^2}$

Helix

Example 1:

Compute the curvature for the helix. Plot the helix and display the curvature graph.



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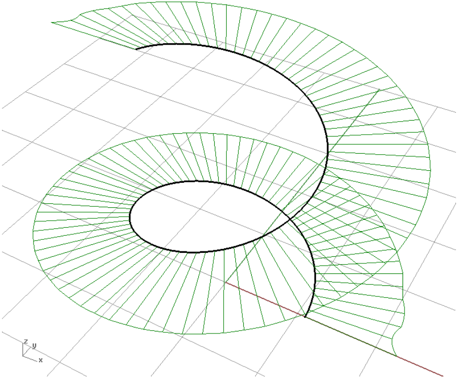
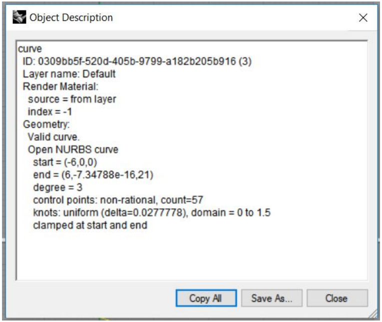
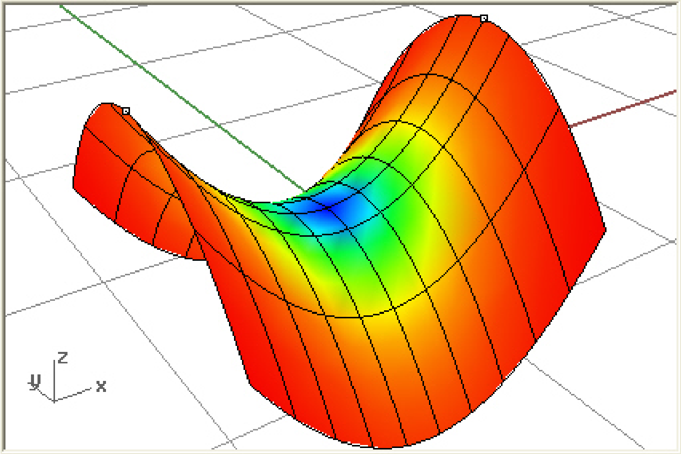




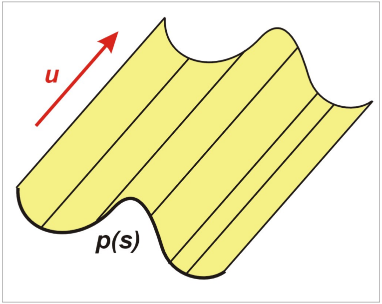
Curvature Analysis

false-color mapping



Developable surfaces

Ruled surfaces which are characterized by property that they can be mapped isometrically into the plane.

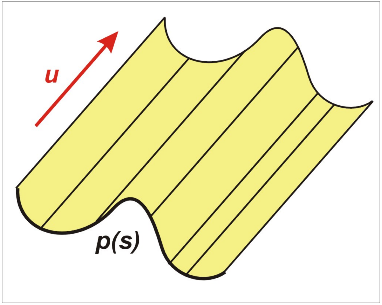


Three basic types of developable surfaces:

- cylinders
- cones
- tangent surfaces of space curves

Gaussian curvature

Example 2: Find the Gauss curvature for the cylinder. Find a patch for some cylinders compare your calculation and visualisation of Gauss curvature in Rhinoceros.

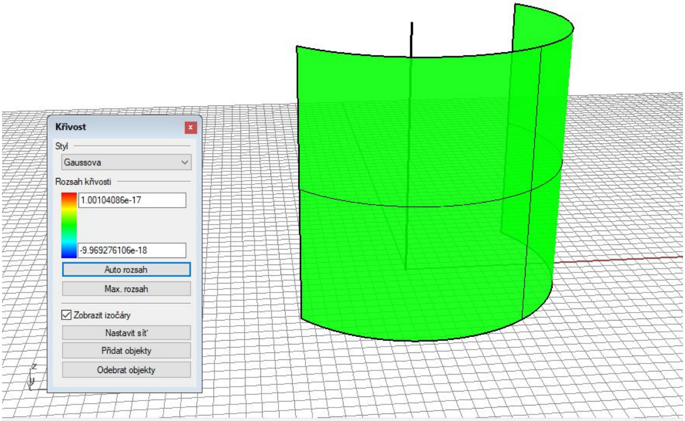


$$\boxed{\boldsymbol{X}(s,t) = \boldsymbol{P}(s) + t\boldsymbol{u},\ s,t \in \mathbb{R}}$$

$$g_{11} = \boldsymbol{P}' \cdot \boldsymbol{P}'$$
$$g_{12} = \boldsymbol{P}' \cdot \boldsymbol{u}$$
$$g_{22} = \boldsymbol{u} \cdot \boldsymbol{u}$$
$$h_{11} = \boldsymbol{P}'' \cdot (\boldsymbol{P}' \times \boldsymbol{u}) \cdot \frac{1}{||\boldsymbol{P}' \times \boldsymbol{u}||}$$
$$h_{12} = 0$$
$$h_{22} = 0$$

$$\boxed{K = \frac{h_{11}h_{22} - (h_{12})^2}{g_{11}g_{22} - (g_{12})^2} = 0}$$

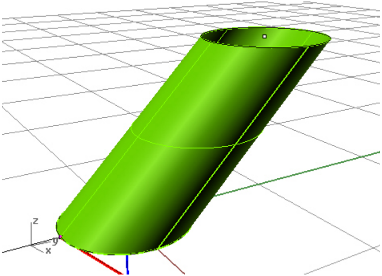
Gauss curvature



Exercise

Find the the Gauss and mean curvature for the oblique circular cylinder (the circle in the plane xy, centre $S[0,0,0]$, radius $r=2$, direction of straight lines $\boldsymbol{a}=(0,1,1)$) in the point $U[0,-2,0]$.

$X(u,v)=(2\cos v, u+2\sin v, u)$ point U for $v=\frac{3\pi}{2}$, $u=0$



$$\frac{dX}{du}=(0,1,1)$$
$$\frac{dX}{dv}=(-2\sin v, 2\cos v, 0)$$
$$\frac{d^2X}{du^2}=(0,0,0)$$
$$\frac{d^2X}{dv^2}=(-2\cos v,-2\sin v,0)$$
$$\frac{d^2X}{dudv}=(0,0,0)$$
$$\boldsymbol{n}=(-2\cos v,-2\sin v,2\sin v)\cdot\frac{1}{2\sqrt{1+\sin^2 v}}$$

$$g_{11}=2$$
$$g_{12}=2\cos v$$
$$g_{22}=4$$

$$h_{11}=0$$
$$h_{12}=0$$
$$h_{22}=\frac{2}{\sqrt{1+\sin^2 v}}$$

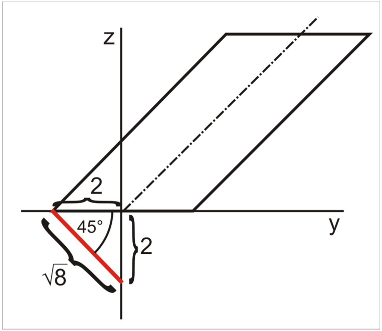
$$H=\frac{1}{2}\cdot\frac{g_{11}\cdot h_{22}-2g_{12}\cdot h_{12}+g_{22}\cdot h_{11}}{g_{11}g_{22}-(g_{12})^2}=$$
$$=\frac{1}{2\sqrt{1+\sin^2 v}(2-\cos^2 v)},$$ for $v=\frac{3\pi}{2}$ $H=\frac{\sqrt{2}}{8}$

$$K=\frac{h_{11}h_{22}-(h_{12})^2}{g_{11}g_{22}-(g_{12})^2}=0$$

Exercise

Find the the Gauss and mean curvature for the oblique circular cylinder (the circle in the plane xy, centre $S[0,0,0]$, radius $r=2$, direction of straight lines $\boldsymbol{a}=(0,1,1)$) in the point $U[0,-2,0]$.

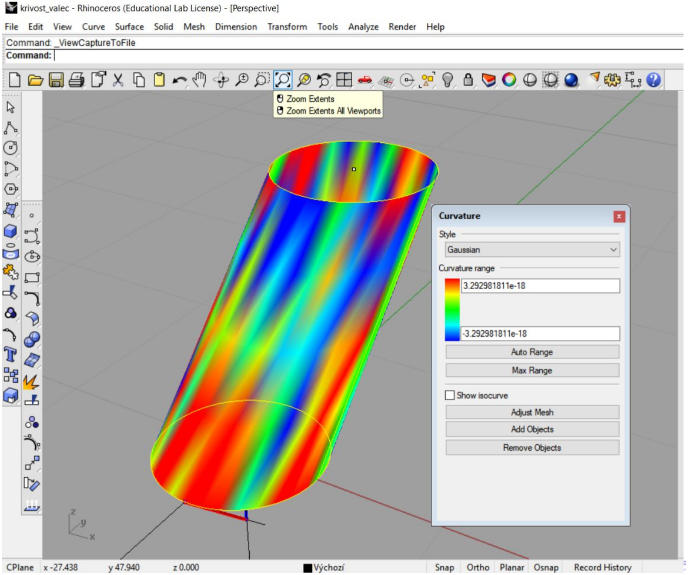
Ussing Meusnier theorem:



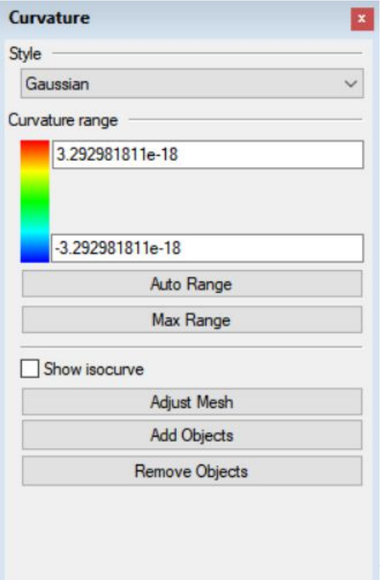
$$H=\frac{1}{2}({}^{n}k_{min}+{}^{n}k_{max})==\frac{1}{2}(0+\frac{1}{\sqrt{8}})=\frac{\sqrt{2}}{8}$$

$$K={}^{n}k_{min}\cdot{}^{n}k_{max}=0\cdot\frac{1}{\sqrt{8}}=0$$

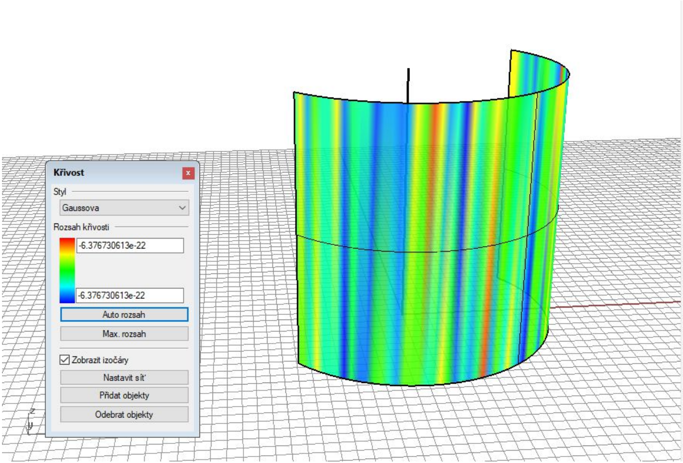
Gauss curvature



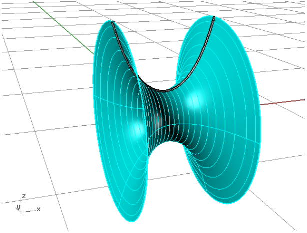
Gauss curvature



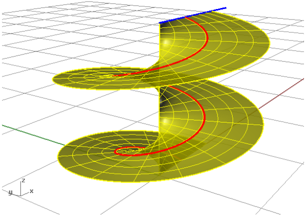
Gauss curvature



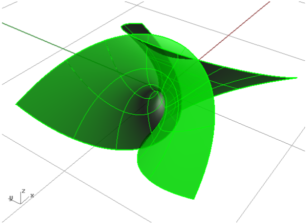
Minimal surfaces



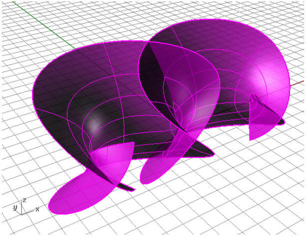
Catenoid



Helicoid

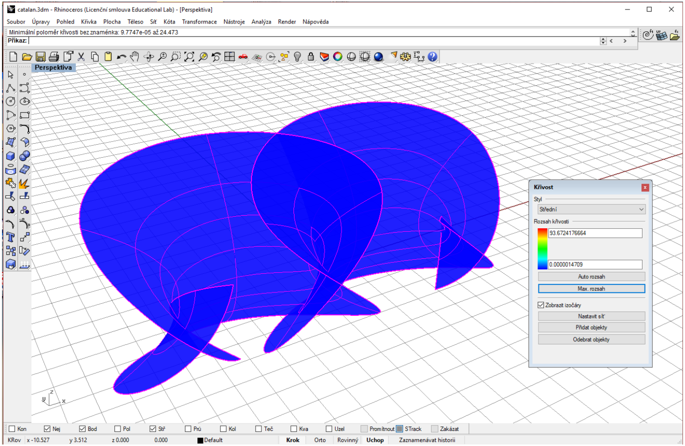


Eneper surface

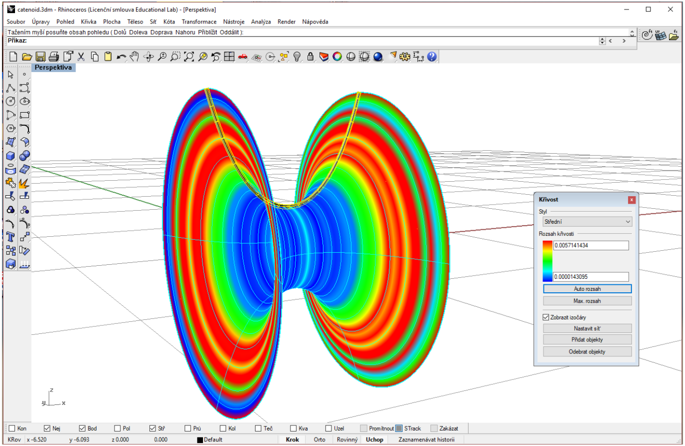


Catalan surface

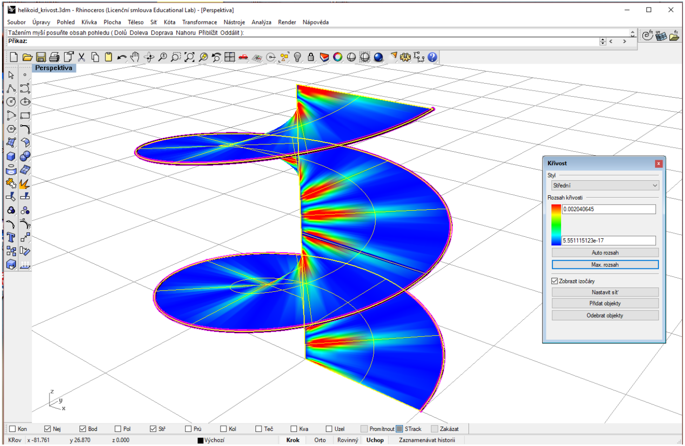
Mean curvature



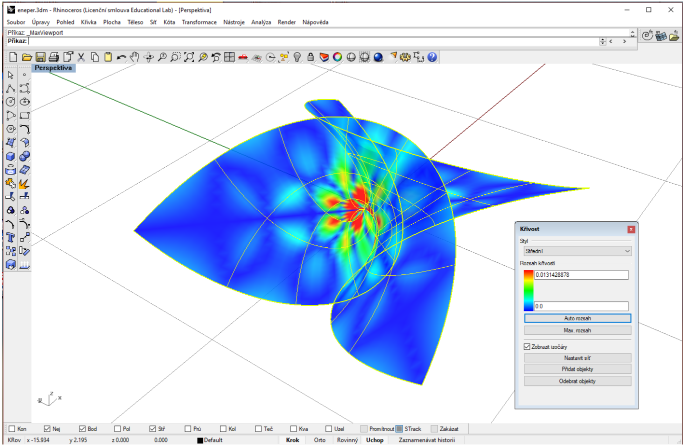
Mean curvature



Mean curvature

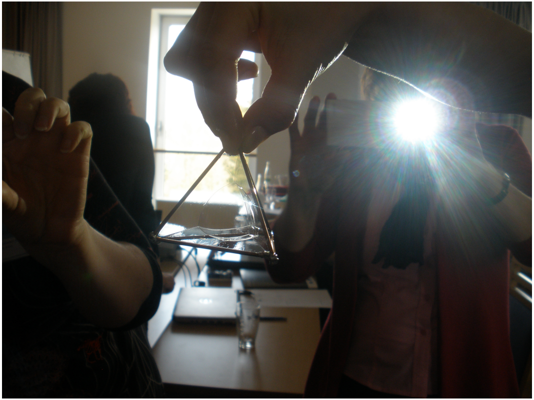


Mean curvature



Conclusion

- Terms, definition, theorems, properties
- Visualization
- Discussion problems of visualization of properties
- Connection between different parts of geometry
- Motivation for course Geometric and Computer modelling



Thank you for your attention