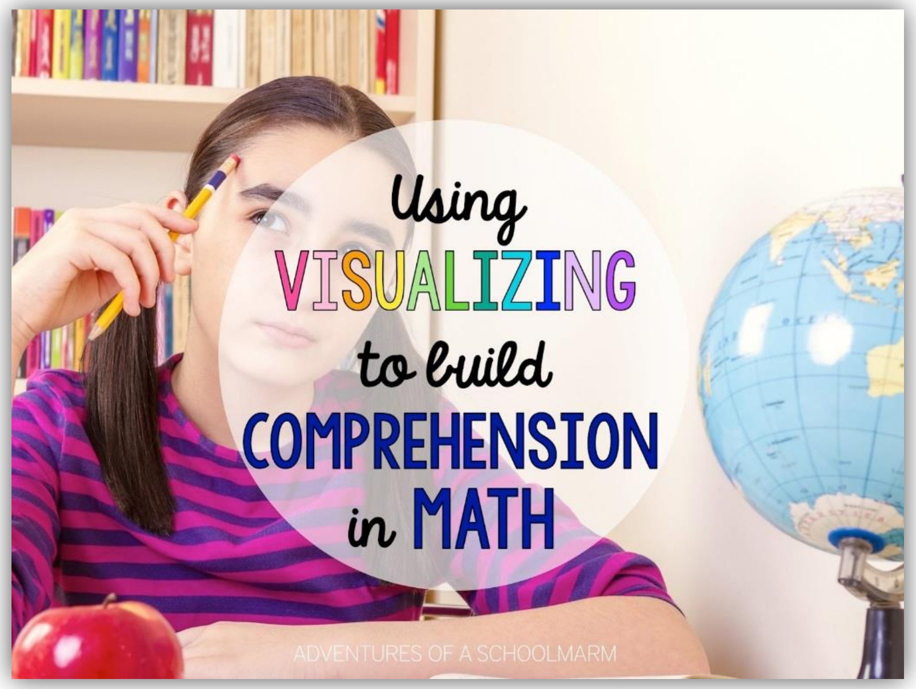

Does the Use of Iconic Planar Representations Affect Learner's Visualisation in Solid Geometry Problem Solving?

CADGME 2018
7th Conference on Applied Mathematics

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Visualisation

„Visualisation in mathematics is the kind of reasoning activity based on the use of visual or spatial elements, either mental or physical, performed to solve problems or prove properties.“ (Gutiérrez, 1996)

- In general, visualisation is a mental process which results in an identified product: visual image.

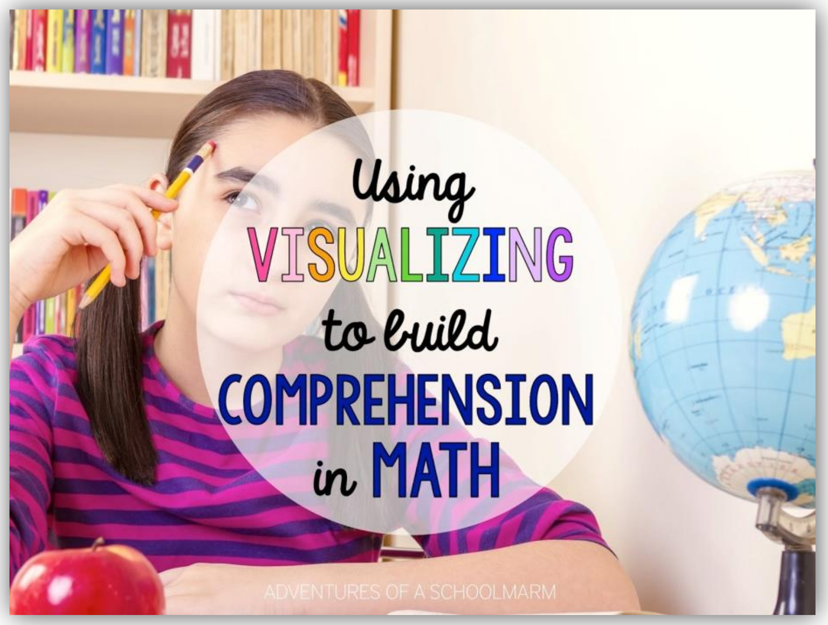
- Visualisation is integrated by four main elements: mental images, external representations, processes of visualization, and abilities of visualization.

- If one visualises a problem it means understanding of the problem in terms of a visual mental representation.

Motivations

- ❑ Teachers → should lead students to a deeper understanding of the base of mathematical problem.

- ❑ Mathematics teaching should be focused on “teaching for understanding”.



- ❑ It often happens that the geometric knowledge of the three-dimensional space is limited to the knowledge of some basic solids.

- ❑ Iconic planar representations instead of real or virtual models during the lesson.

Pedagogical research

RESEARCH SAMPLE:

- 93 students aged 19 at University of Constatine the Philosopher in Nitra in the Slovak Republic, 1st grade students.

RESEARCH WAS DIVIDED INTO 2 PARTS:

1. Presmeg‘s visualisation test - Mathematical processing instrument (MPI)
2. Atypical problem called *The problem of cube layering*
   - the key to the solution: visualisation of hidden spatial objects

OBJECTIVES OF THE RESEARCH WERE:

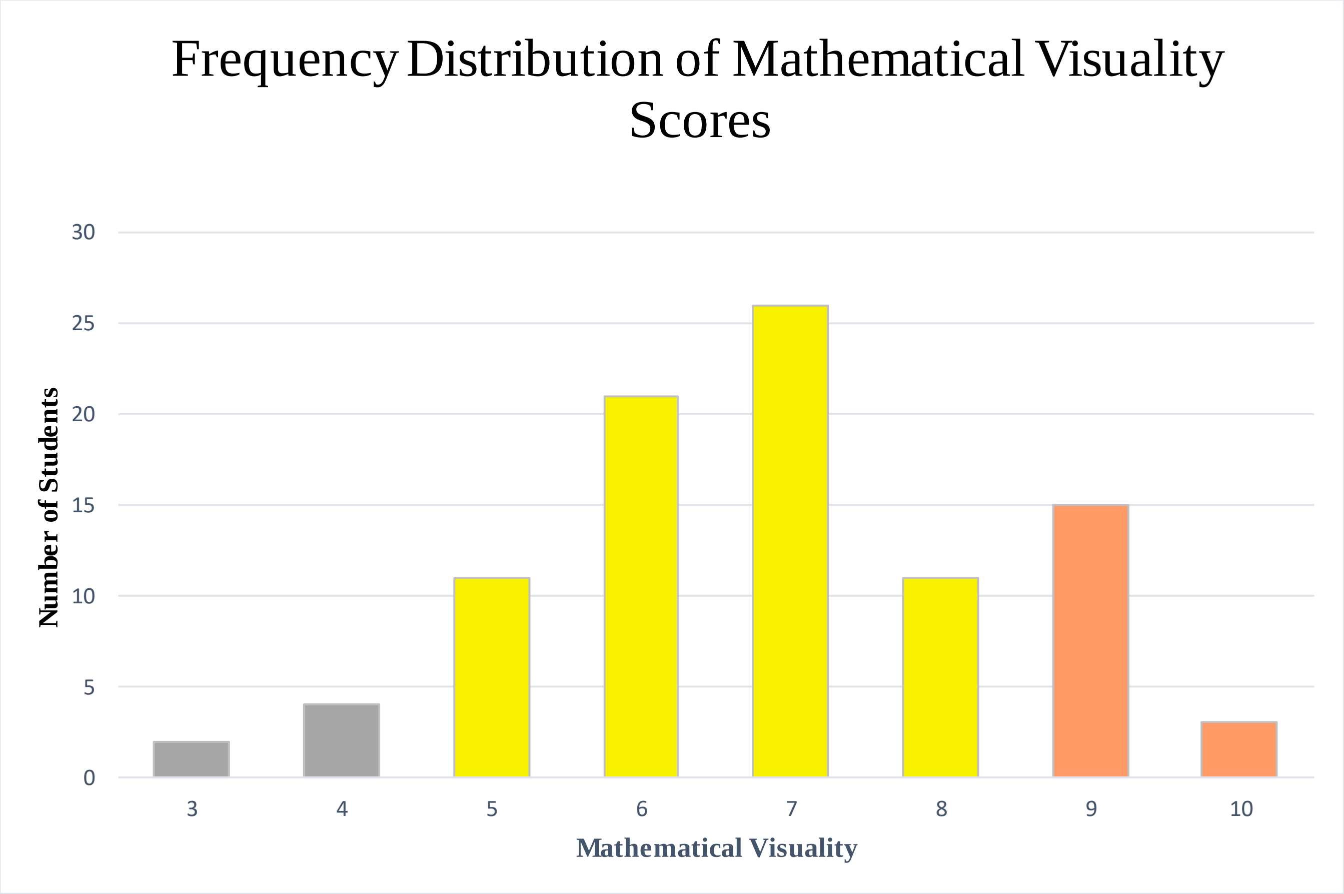
- to detect the students`s strategy of visualisation during the solving problem
- to find out the correlation between external representations and score
- to elaborate a planning proposal that focuses on representations in three dimensions, favoring the use of dynamic geometry software (GeoGebra) - to eliminate students‘ difficulties

Mathematical processing instrument (MPI)

- MPI measures **preference** for visualisation, not ability of visualisation.

- Construct validity and reliability were tested and judged to be satisfactory in schools in Cambridge, England and in Durban, South Africa

- Students completed the problems from the section A (six problems) – every problem could be solved by visual and nonvisual methods



Problem of cube layering

We designed the following problem with questions divided into four levels: *Imagine that you have an unlimited number of small cubes (all the same size* $1\times1\times1$*) in different colours. Then, imagine you will build bigger cubes from these small cubes by wrapping layers (like in an onion or Russian nesting dolls) such that each layer has a different colour.*

Next, imagine the layers of small cubes and try to answer the following questions:

1. ***Imagine a cube*** C_1 ***of the size*** $3\times3\times3$ ***(each layer has a different colour).***
   a) *How many layers does the cube have?*

   b) *Sketch an outline how the small cubes of the outer layer* ***touch the faces*** *of the small cubes of the previous layers face. How many cubes of the outer layer have a face touching the inner layer faces?*

   c) *Sketch an outline how the small cubes of the outer layer* ***touch*** *the small cubes of the previous layer* ***along the edges*** *without touching any of the faces of the inner cubes. How many cubes of the outer layer touch the inner cubes along the edges?*

   d) *Sketch an outline how the small cubes of the outer layer* ***touch*** *the small cubes of the previous layer* ***at the vertices*** *without touching any of the edges of the inner cubes. How many cubes of the outer layer touch the inner cubes at the vertices?*

   e) *How many small cubes are there in total?*

Problem of cube layering

2. Imagine a cube C_2 that has one more layer than the previous cube C_1 (each layer has a different colour).

a) *How many small cubes did you add to the previous cube?*

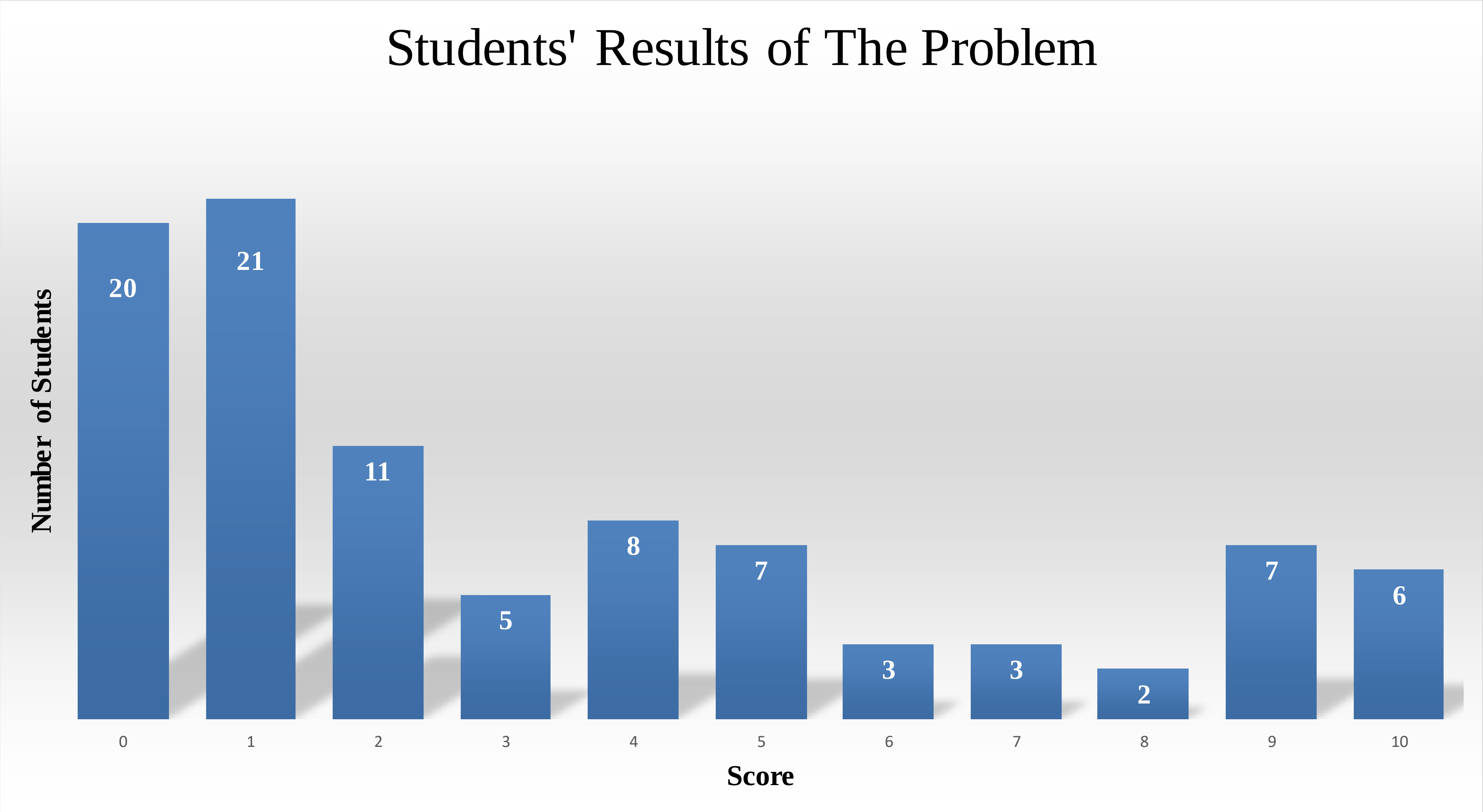
b) *Sketch an outline how the small cubes of the outer layer* ***touch the faces*** *of the small cubes of the previous layers face. How many cubes of the outer layer have a face touching the inner layer faces?*

c) *Sketch an outline how the small cubes of the outer layer* ***touch*** *the small cubes of the previous layer* ***along the edges*** *without touching any of the faces of the inner cubes. How many cubes of the outer layer touch the inner cubes along the edges?*

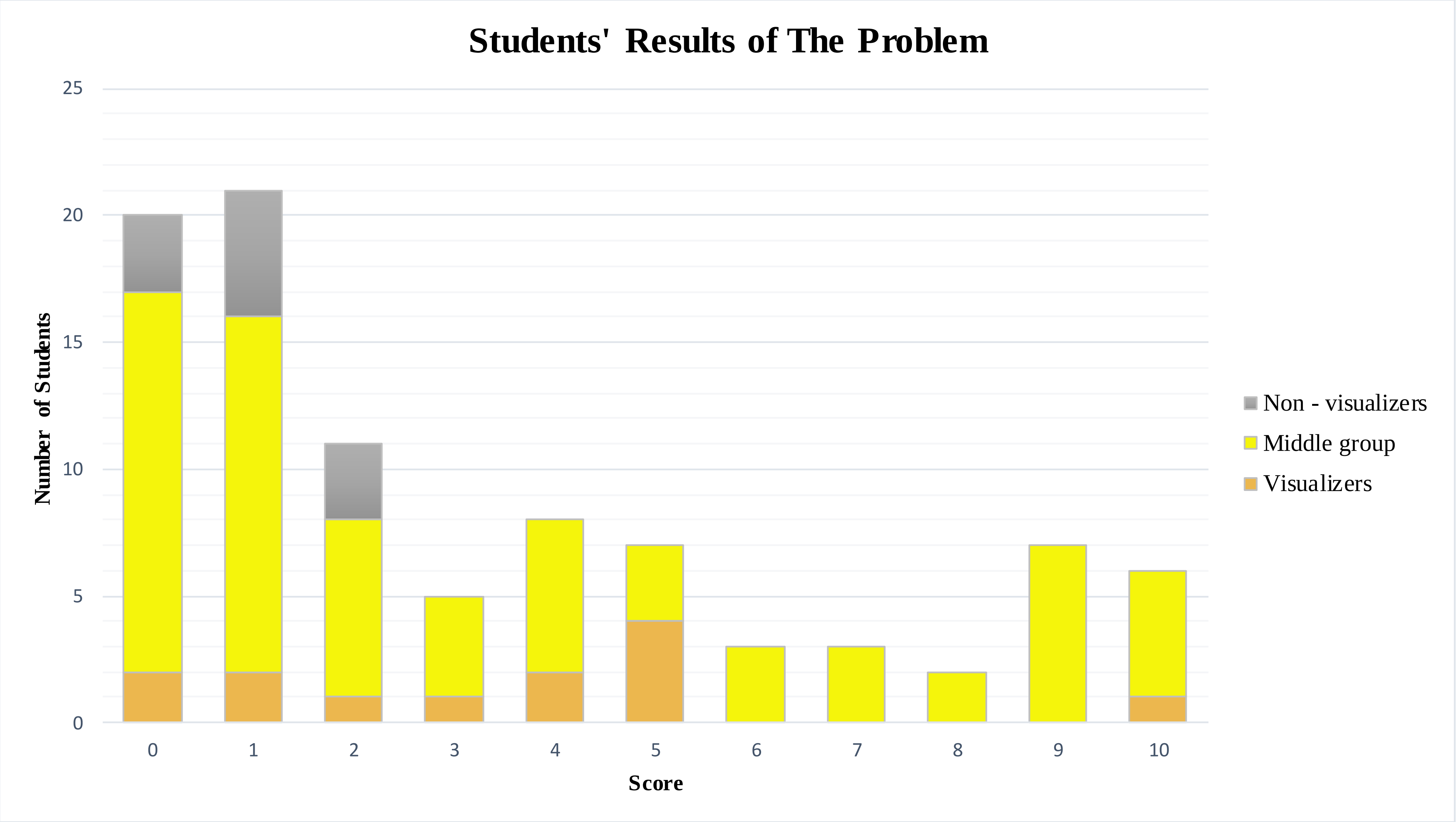
d) *Sketch an outline how the small cubes of the outer layer* ***touch*** *the small cubes of the previous layer* ***at the vertices*** *without touching any of the edges of the inner cubes. How many cubes of the outer layer touch the inner cubes at the vertices?*

e) *How many small cubes are there in total?*

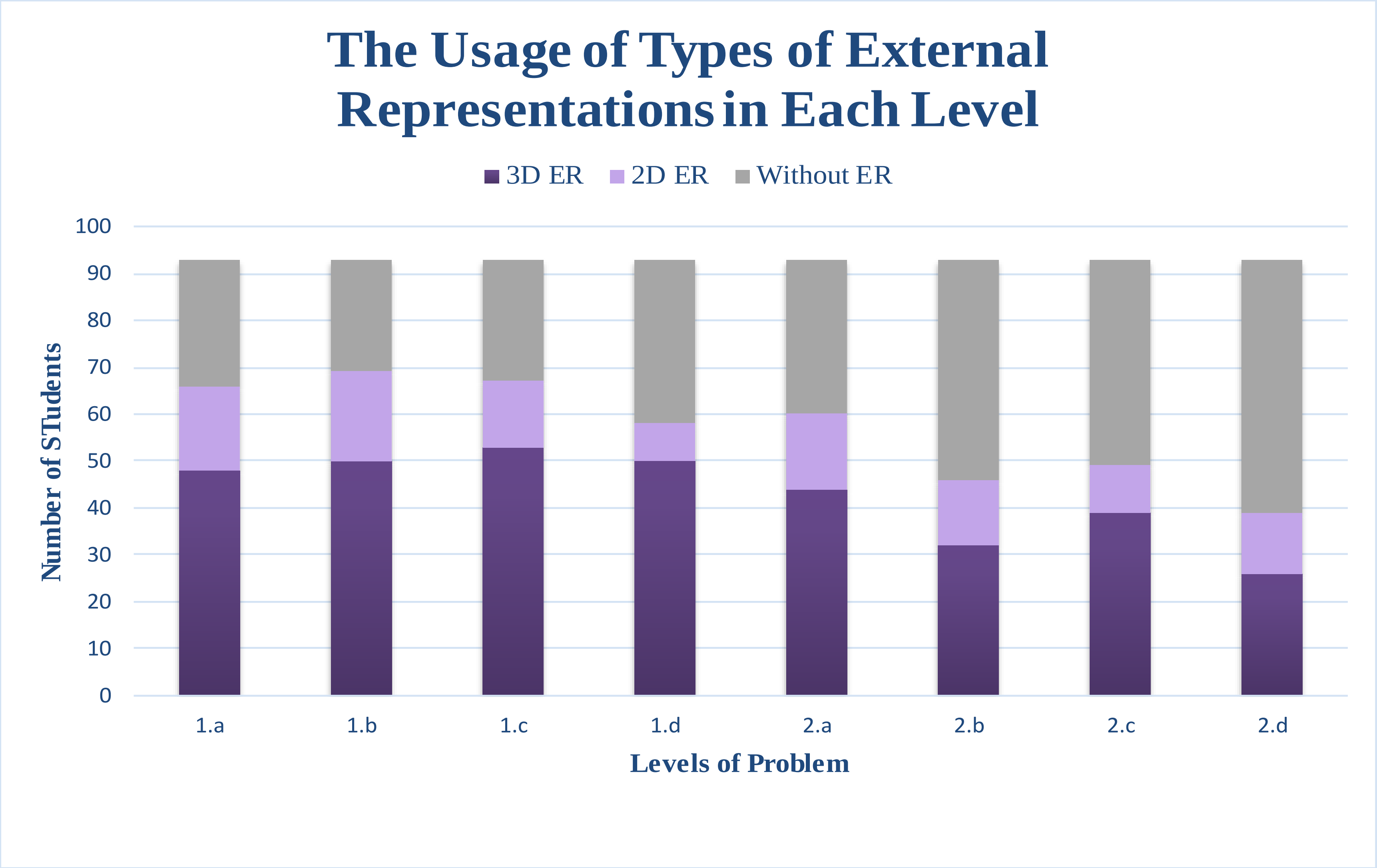
Students‘ Results



Students' Results

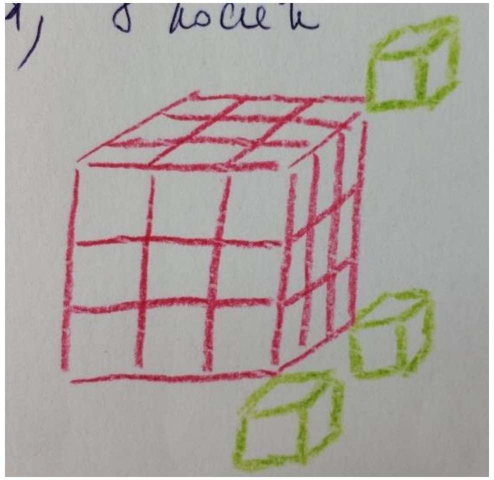


Types of External Representations

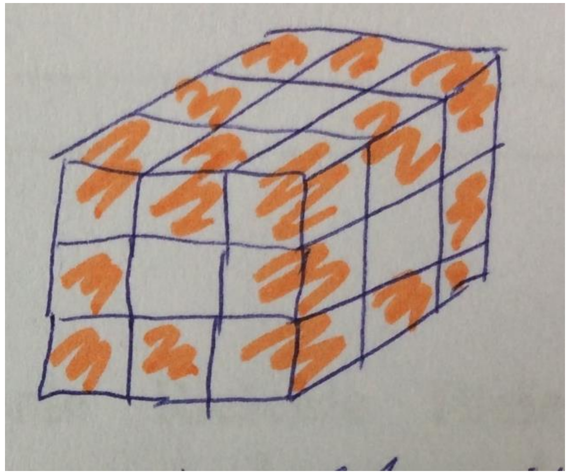


Types of Ex. Rep.

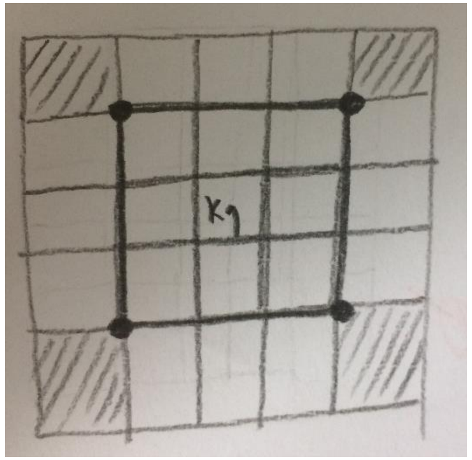
3-Dimensional External Representation of Spatial Relations



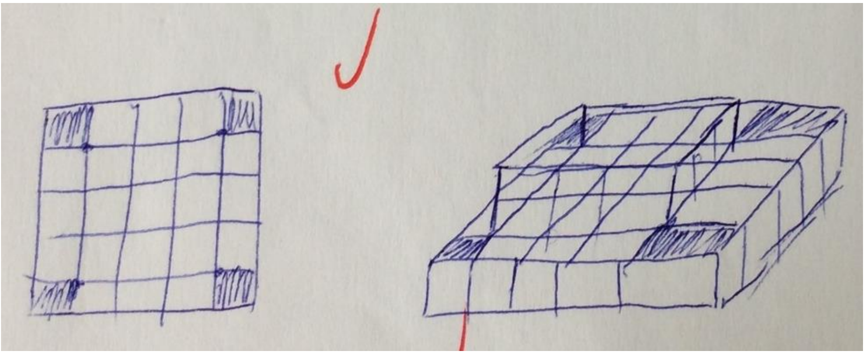
3-Dimensional External Representation of the Cube Model



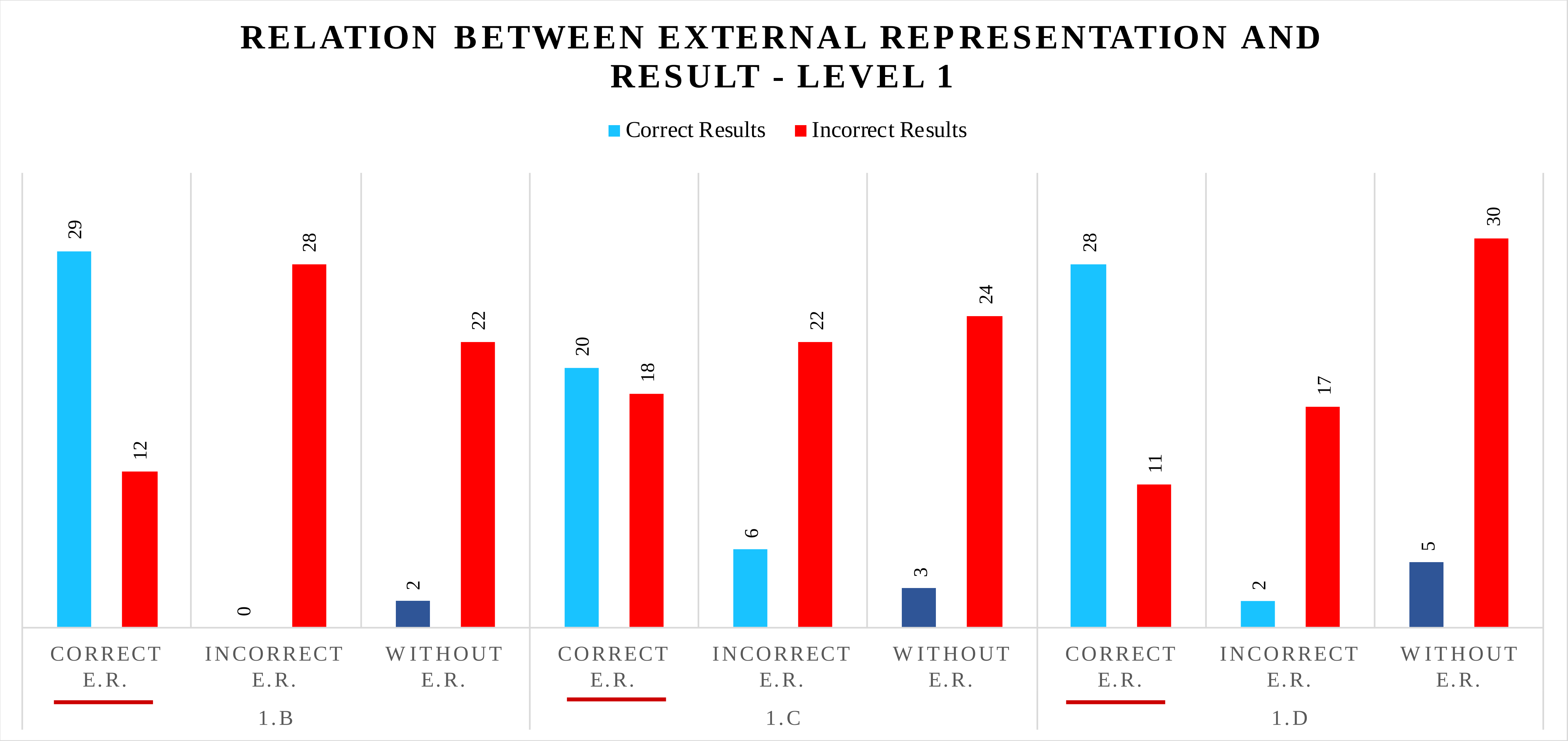
2-Dimensional External Representation



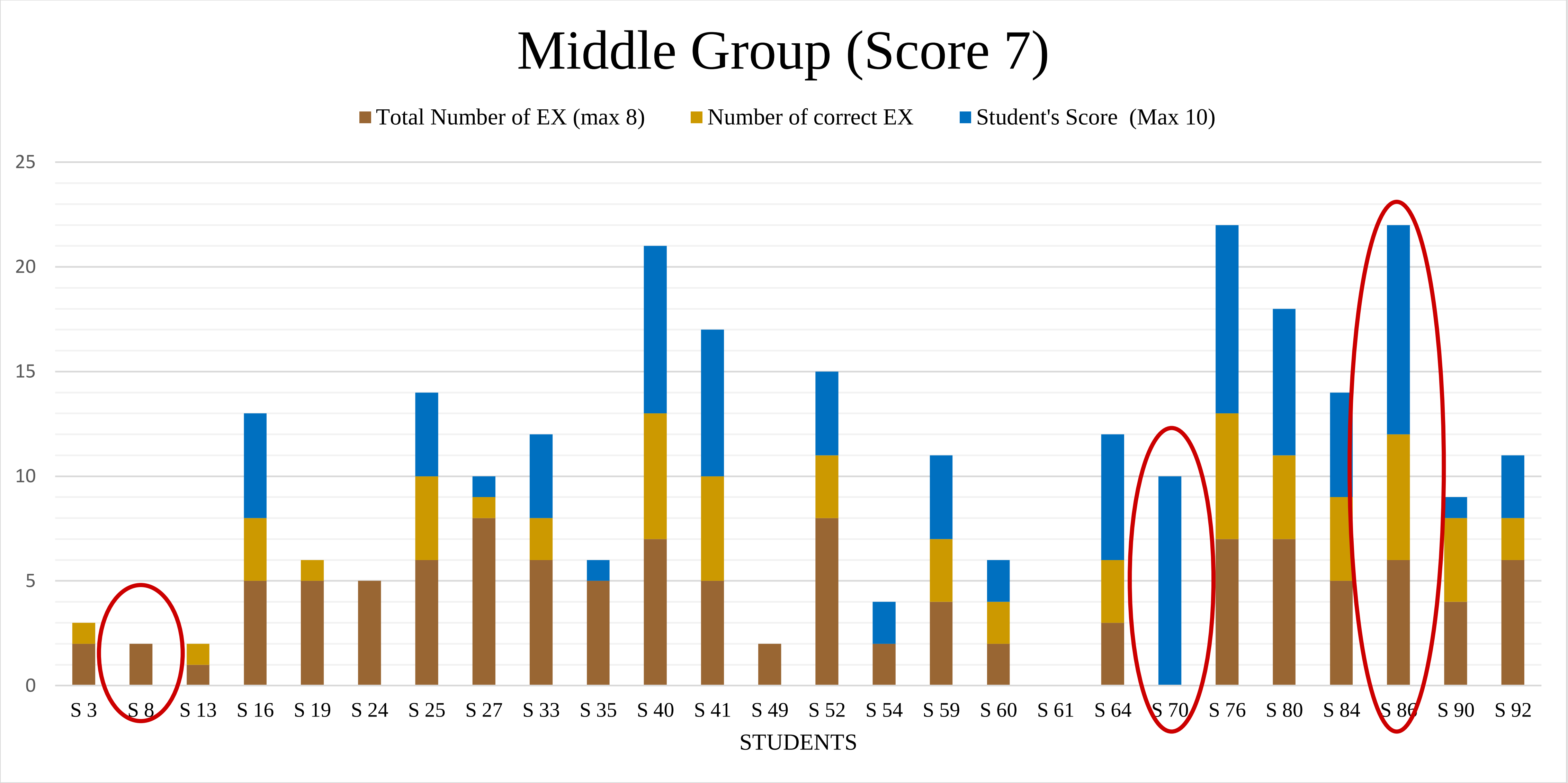
External Representation as combinations of 3 and 2-Dimensional pictures



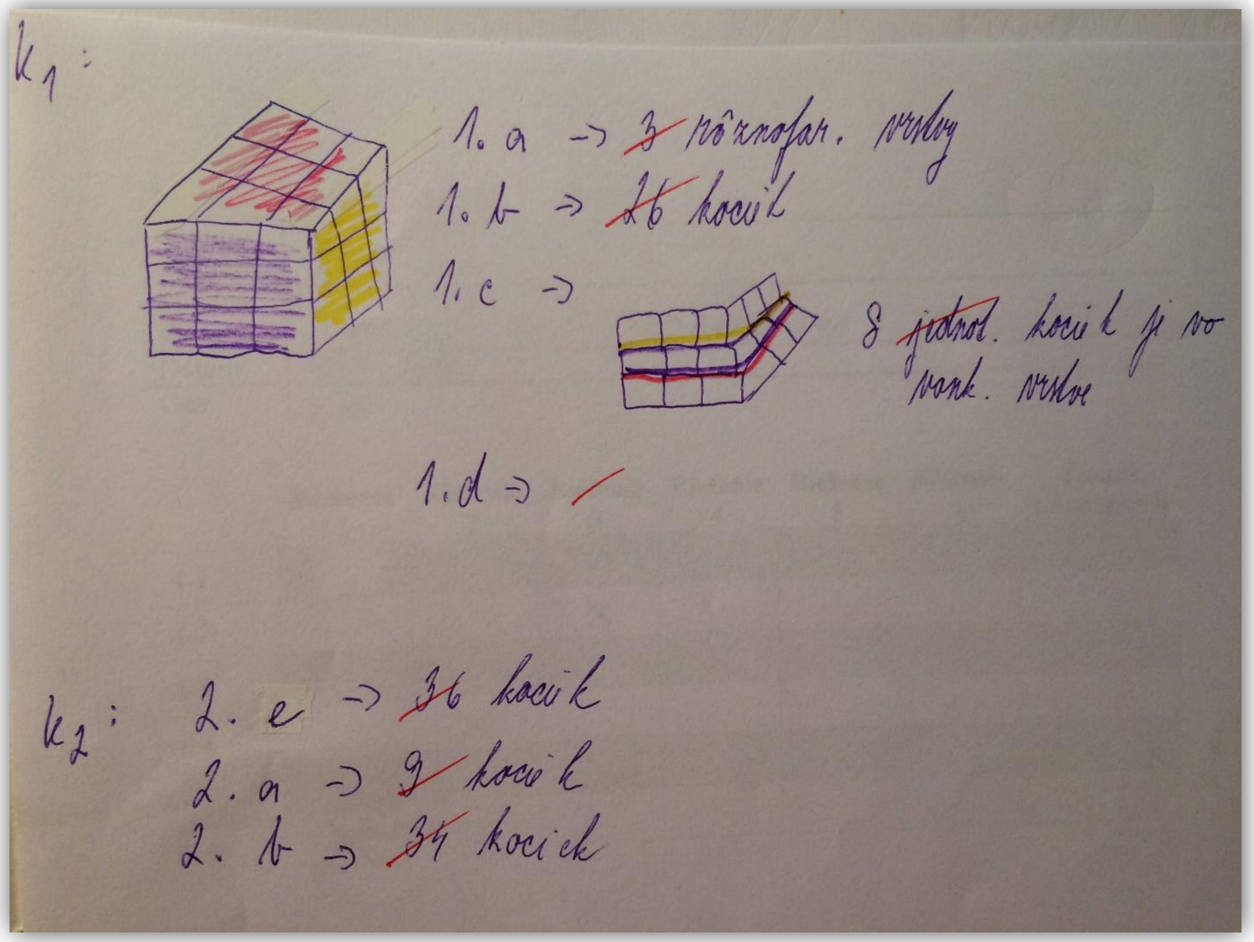
Correlation between ER and Score:



Correlation between ER and Score:



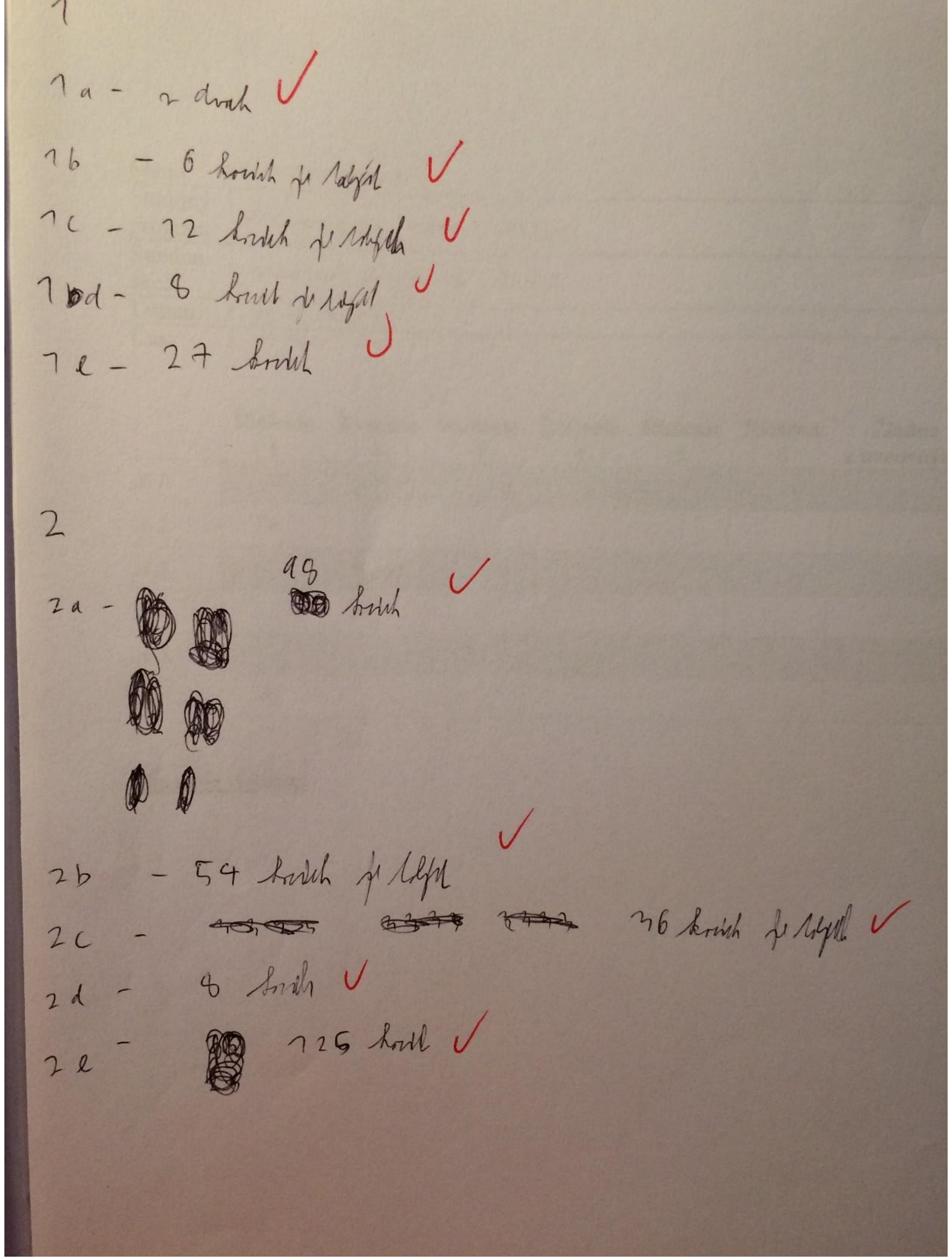
Student 8:



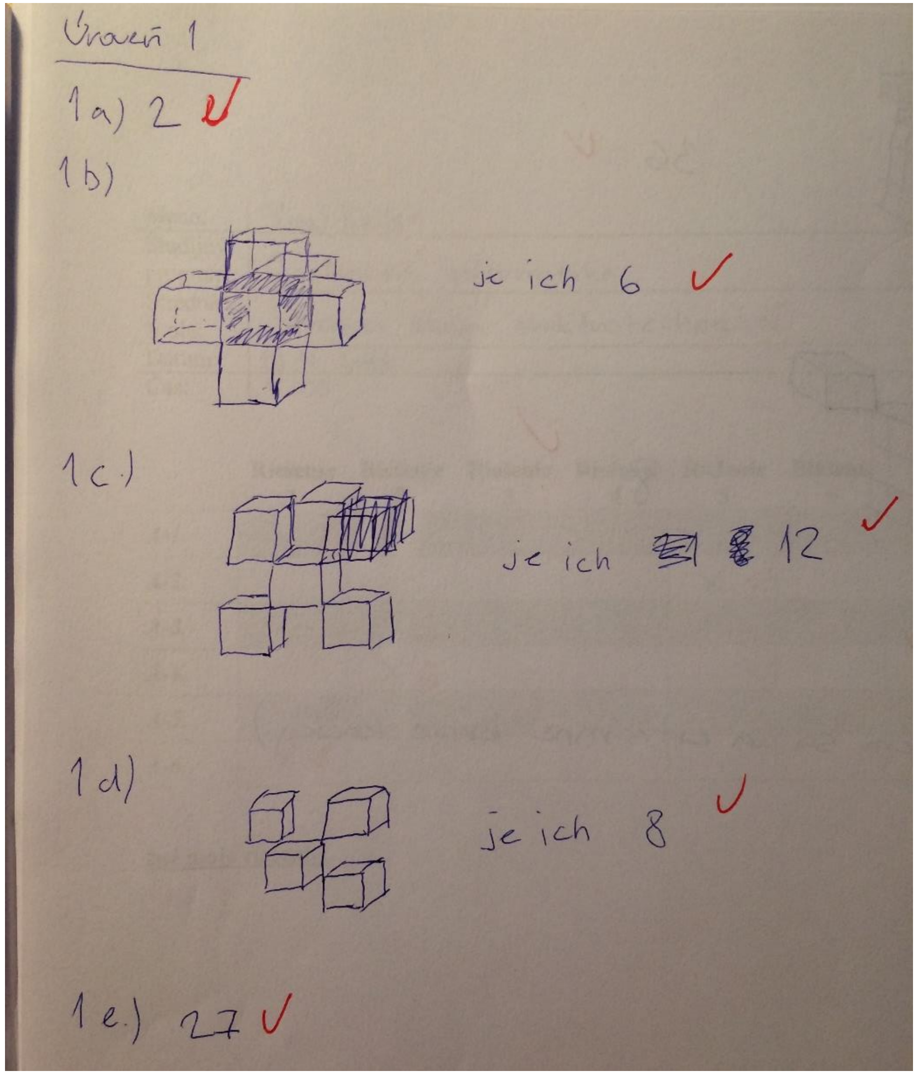
- 2/8 ER
- 0/8 correct ER
- Score: 0/10

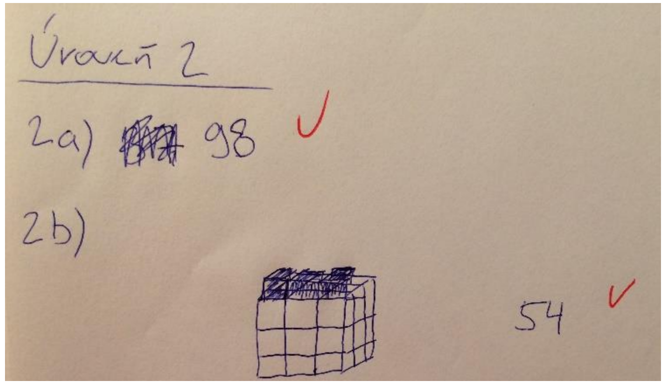
Student 70:

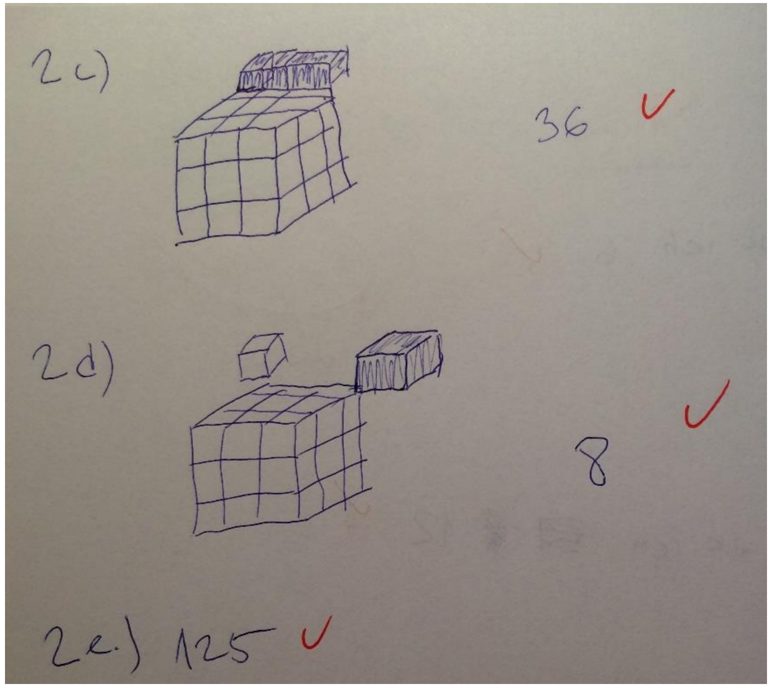
- 0/8 ER
- Score: 10/10



Student 86:





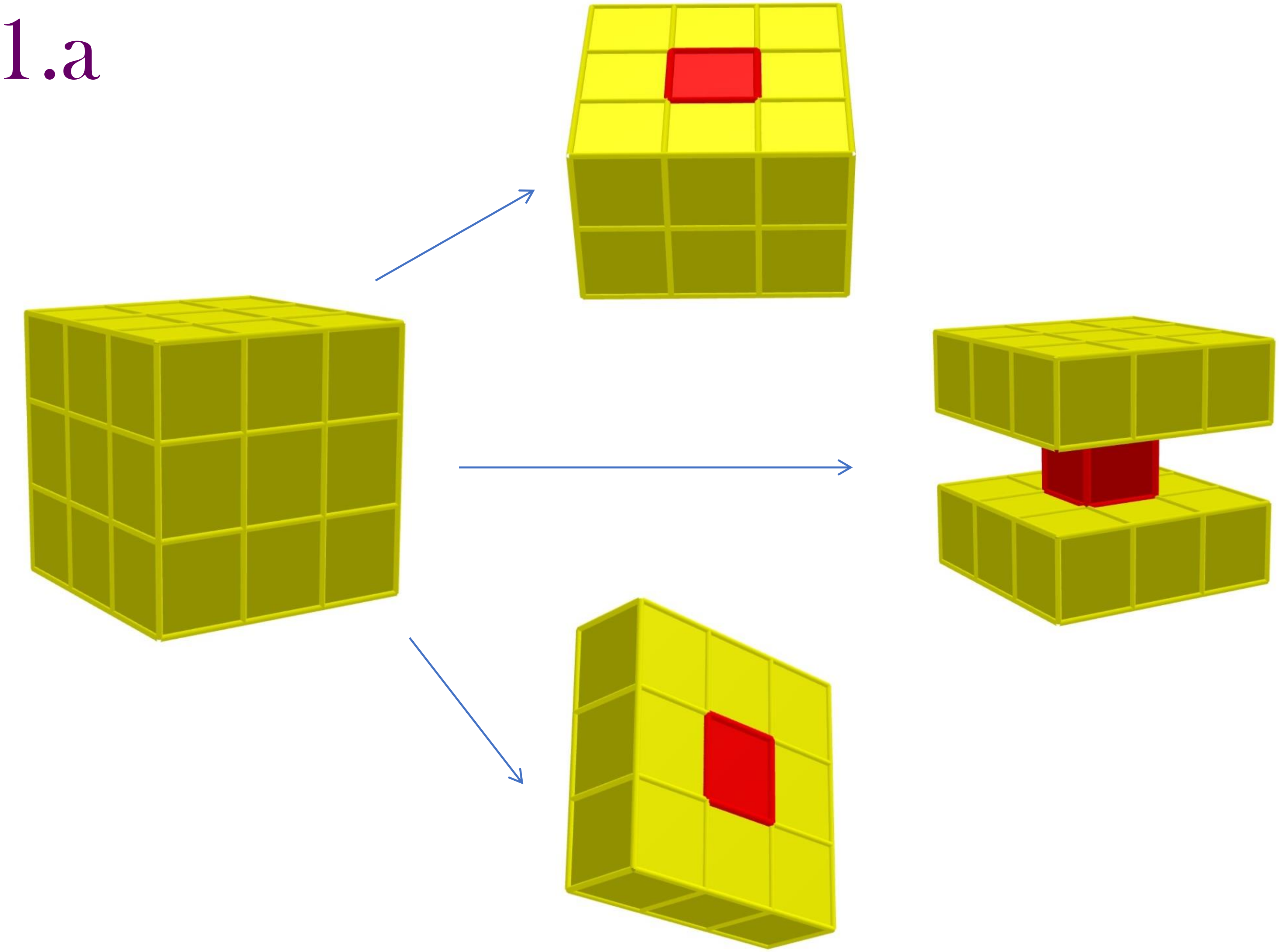


- 8/8 ER
- 8/8 correct ER
- Score: 10/10

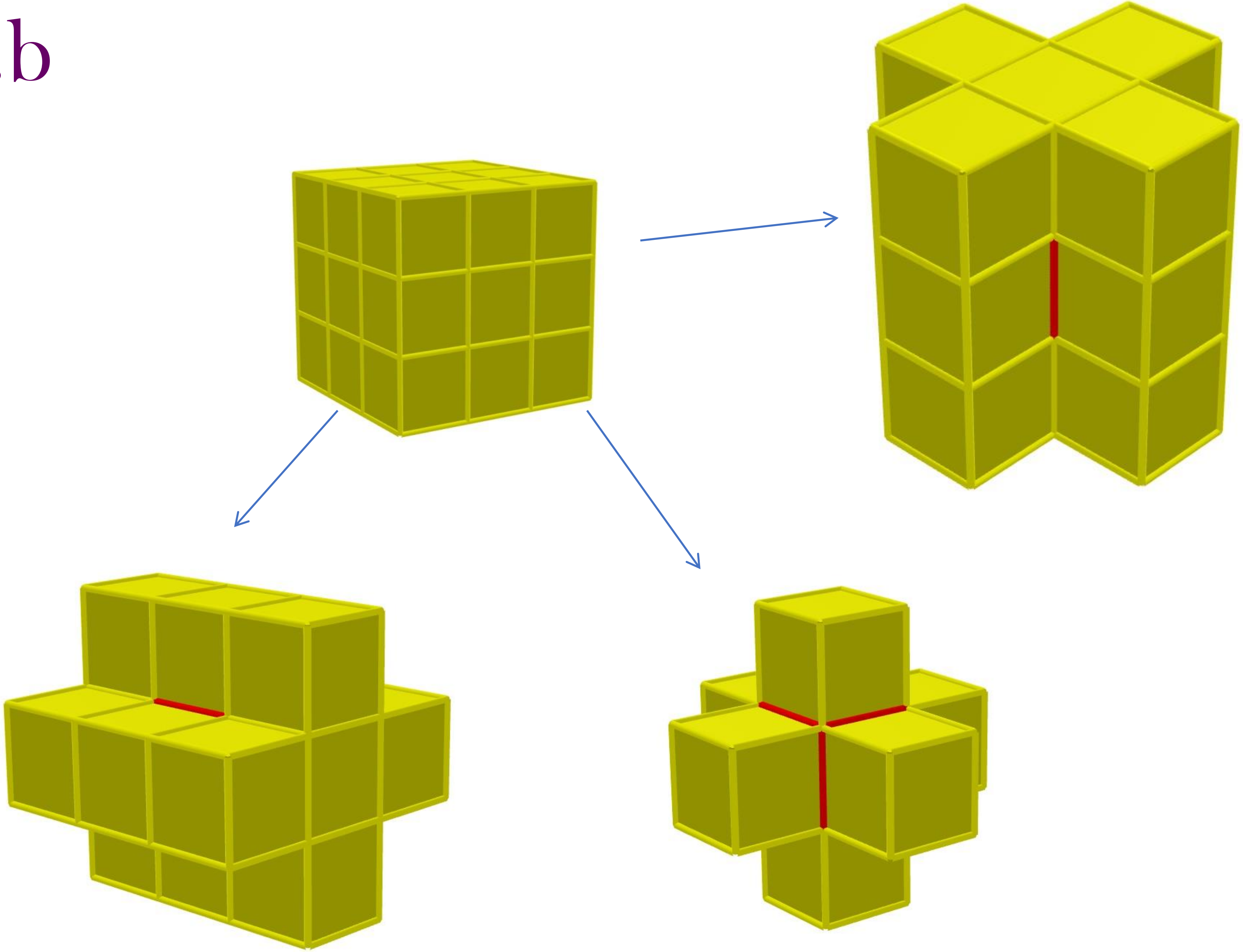
Using GeoGebra as a tool to create to correct mental image

- Findings of various research: students have many difficulties in the work and representation of the three – dimensional models
- Classic 3D models such as a cube, a sphere, etc. do not allow us to look into the centre of them.
- Teachers use iconic planar representations of geometrical bodies → difficulties in creating of a figure in three dimensions
- Modern information and communication technologies → different ways of representation.
- We offer a way how the teacher can easily demonstrate the problem via GeoGebra software of Level (1) and Level (2).

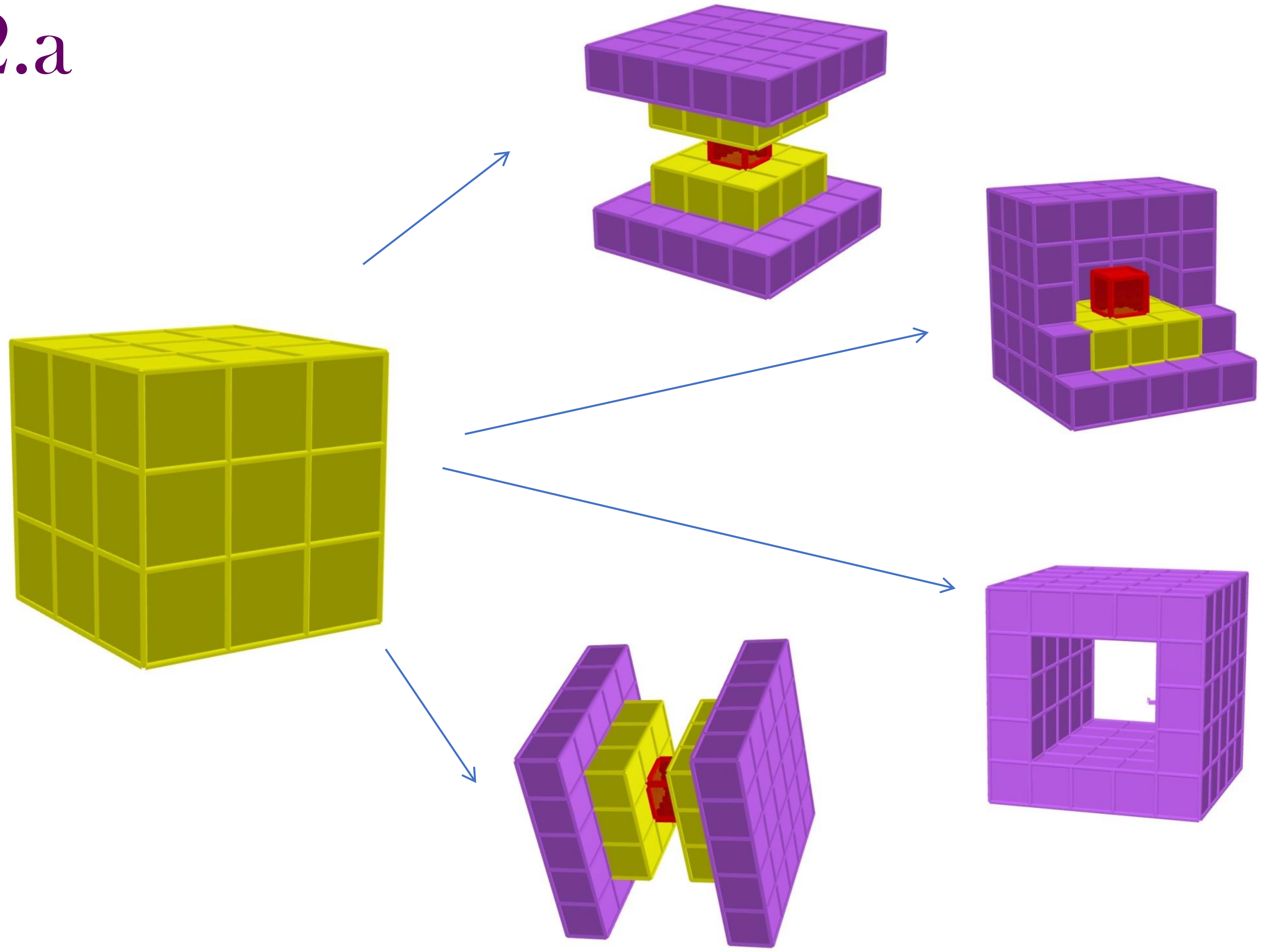
Level 1.a



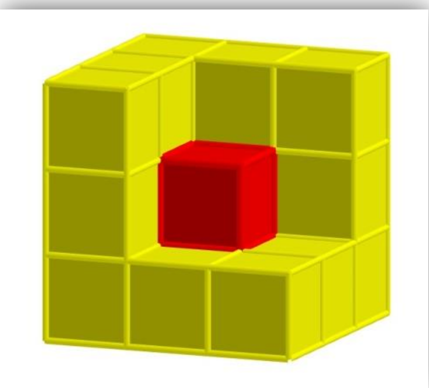
Level 1.b



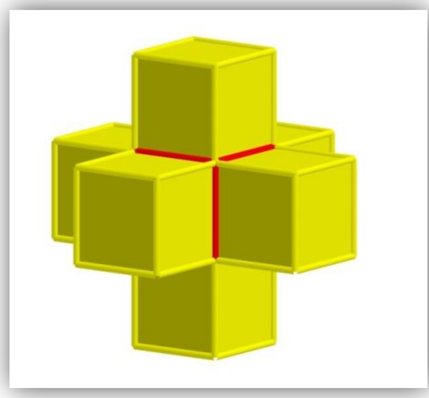
Level 2.a



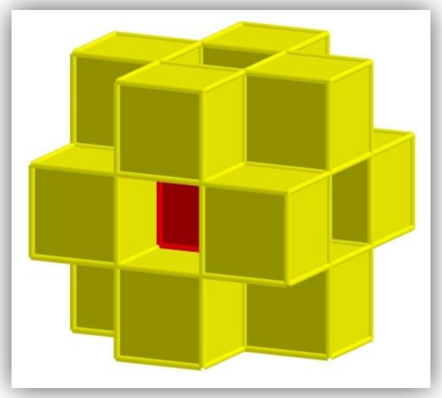
Solution of Level 1



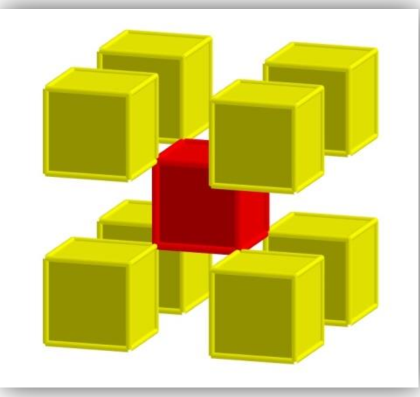
1.a



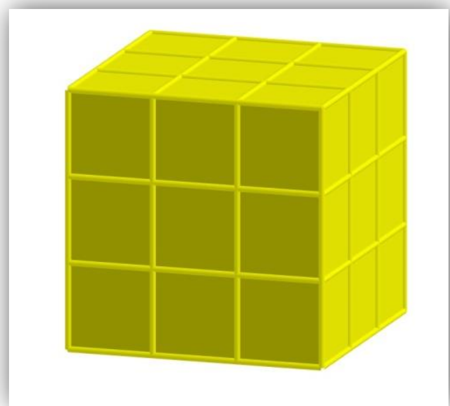
1.b



1.c

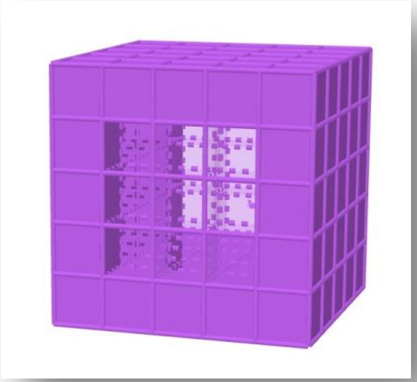


1.d

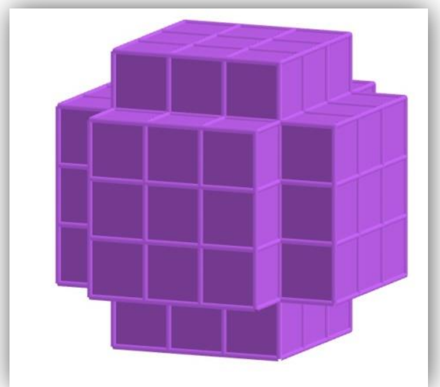


1.e

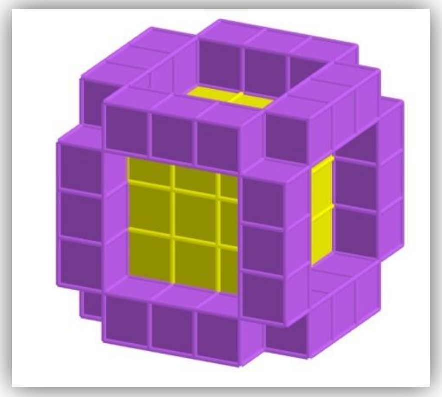
Solution of Level 2



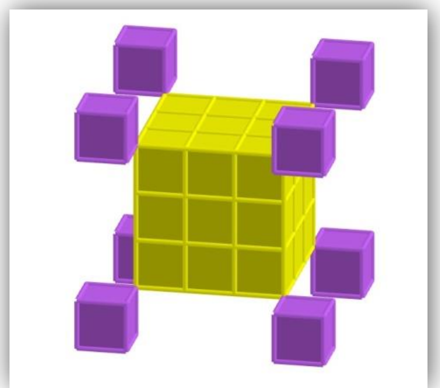
2.a



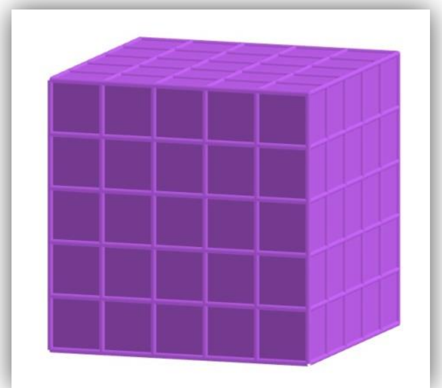
2.b



2.c



2.d



2.e

Conclusion

- Pedagogical research - we designed a solid geometry problem with hidden objects.
- It could be seen that the students had difficulties regarding visualisation of three-dimensional geometric bodies, especially drawing them.
- Dynamic geometry softwares support correct mental imagery in solid geometry problem solving.
- Benefits of using GeoGebra software in this problem solving are the following:
  - ✓ precise and ordered construction of 3D objects
  - ✓ transparency of surfaces
  - ✓ colour differentiation of every layer
  - ✓ manipulation with cubes
  - ✓ demonstration of a correct image of the situation.

THANK YOU FOR YOUR ATTENTION