Definition

- Let \( C \) be a plane curve. For a given angle \( \theta \) (with \( 0 \leq \theta \leq 180^\circ \)), a \( \theta \)-isoptic of \( C \) is the geometric locus of points in the plane through which pass a pair of tangents with an angle of \( \theta \) between them.
- The special case for which \( \theta = 90^\circ \) is called an orthoptic curve.
Orthoptics of conics

• The directrix of a parabola (always exists).
• The director circle of an ellipse (always exists).
• The director circle of a hyperbola (exists under a condition on the angle between the asymptotes).

Bisoptics of ellipses

\( E : x^2 + 4y^2 = 1 \)

\( \text{Opt}(E, 45 - 135) : (x^2 + y^2)^2 - \frac{7}{2}x^2 - \frac{13}{2}y^2 + \frac{41}{16} = 0 \)
Jordan curves

• A plane curve $C$ which is smooth, strictly convex, non self-intersecting, and closed is called a **Jordan curve**.

• **Theorem**: A Jordan curve divides the plane into three regions, namely the interior, the curve itself and the exterior.

• If the Jordan curve $C$ is strictly convex, then through an interior point, no tangent to $C$ passes, and through an exterior point passes one pair of tangents.

What happens for:
Non closed curves?
Non smooth curves?
Example 1: Isoptics of an astroid
parametric presentations

Example 2
105°-isoptic of a parabola
Orthoptic of an open quartic

\[ y = x^4 - x \]

Two approaches

- **Parametric method**
  - Define the input curve with a parametric presentation
  - Find a presentation for tangents vectors/lines
  - Find an expression for orthogonality of two tangents
  - Compute a parametric presentation of the isoptic
  - Compute an implicit equation by elimination

- **Implicit method**
  - Define the input curve as an algebraic equation
  - Compute partial derivatives at two hypothetical tangent points
  - Assume that the angle between the tangents is as required
  - Compute an implicit equation by elimination
Two approaches (comparison)

- **Parametric method**
  - Exact
  - Fast
  - Works only in some special cases

- **Implicit method**
  - Works in all cases when the degree is low
  - Computationally heavy from quartic cases (Gröbner bases)

Using Locus and LocusEquation commands

- Example: orthoptic of a closed Fermat curve

\[ x^{40} + y^{40} = 1 \]

With Mathematica.
Credit: Witold Mozgawa, Lublin

Numerical methods (DP-N)

Floor, entrance to an old synagogue, Budapest
Orthoptic of a quartic using LocusEquation

• Please see https://www.geogebra.org/m/J7tNfrMX

References


Thank you for your vision and audition!

And, of course, for your attention 😊

DPK: A new approach to isoptics - CADGME 2018