Amsterdam University of Applied Sciences

Interactive Virtual Math

Designing a digital tool for reasoning with covariation graphs: didactical considerations and classroom experience

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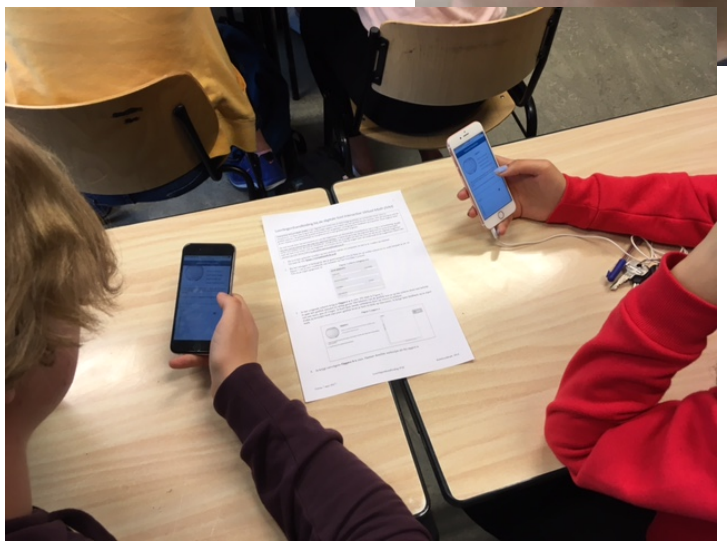


CADGME 7
26-29 juni 2018, Coimbra

About the IVM-project

- Mathematical thinking and reasoning and the add of technology
- History and aims of Interactive Virtual Math (IVM)
- Design Based Research (2016-2018): 5 phases
    1. preliminary study
    2. prototyping and scrum
    3. trial-out with individual students
    4. trial-out in the classroom
    5. evaluation and dissemination
- Focus of the talk results phases 4, 5 and follow up

Dynamic graphs and covariational reasoning

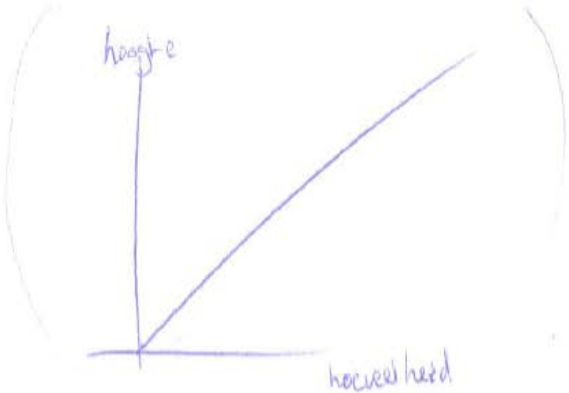
(e.g. Thompson, Carlson, Oehrtman)

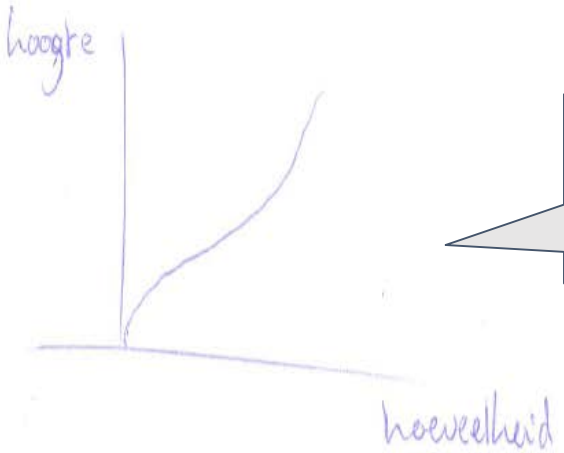
Examples

- the speed variating with time or
- the height of water in a bottle variating with volume

Imagine a bowl is steadily being filled with water.
Sketch a graph of the water height in the bowl
as a function of the amount of water in the bowl.

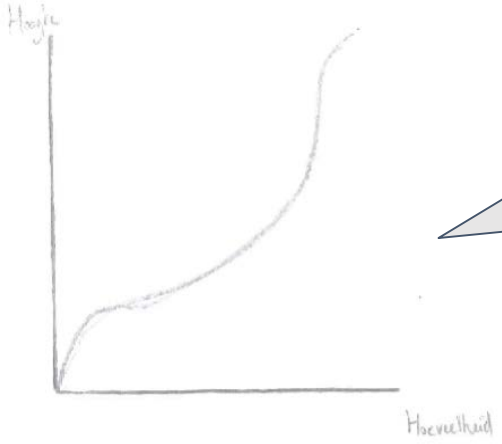
students (14-15 years old)



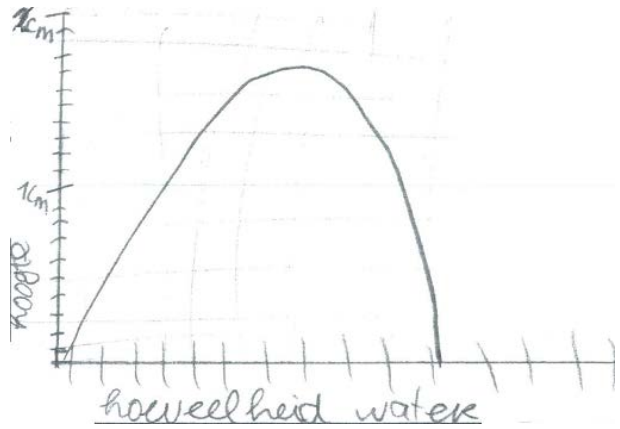


Correct

One third of the students



Influence from the form?



Dynamic graphs and covariational reasoning

(e.g. Thompson, Carlson, Oehrtman)

Modeling a dynamic situation into a graph:

- It requires imagining how the output values of a function are changing while imagining changes in the input values
  (covariation= relationship between two variables that vary simultaneously)
- It requires an external representation of this mental image/model

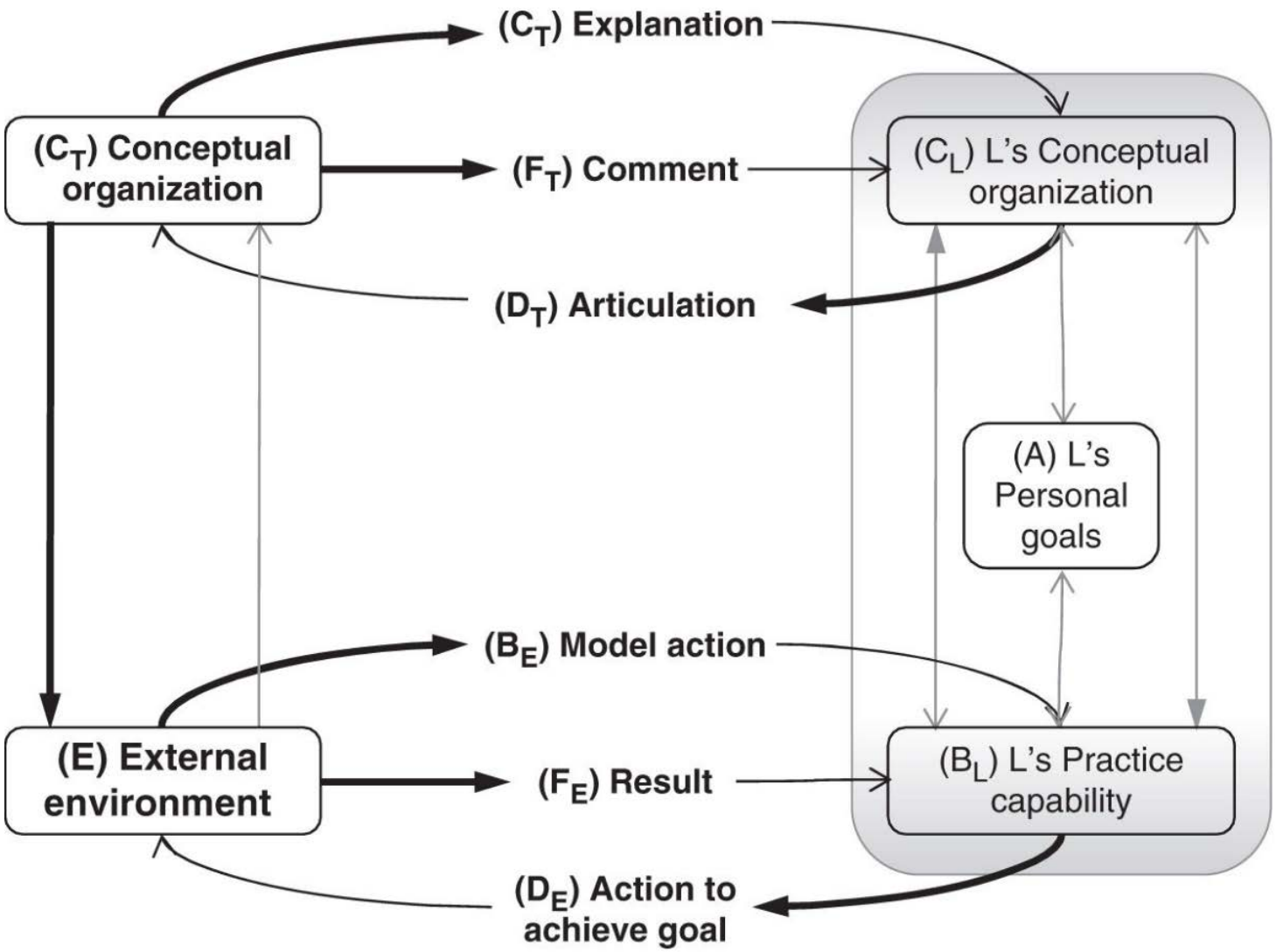
Students's difficulties with dynamic graphs

- tendency to view functions in terms of symbolic manipulations rather than as relationships of dependency

Pedagogical directions for constructing the tool

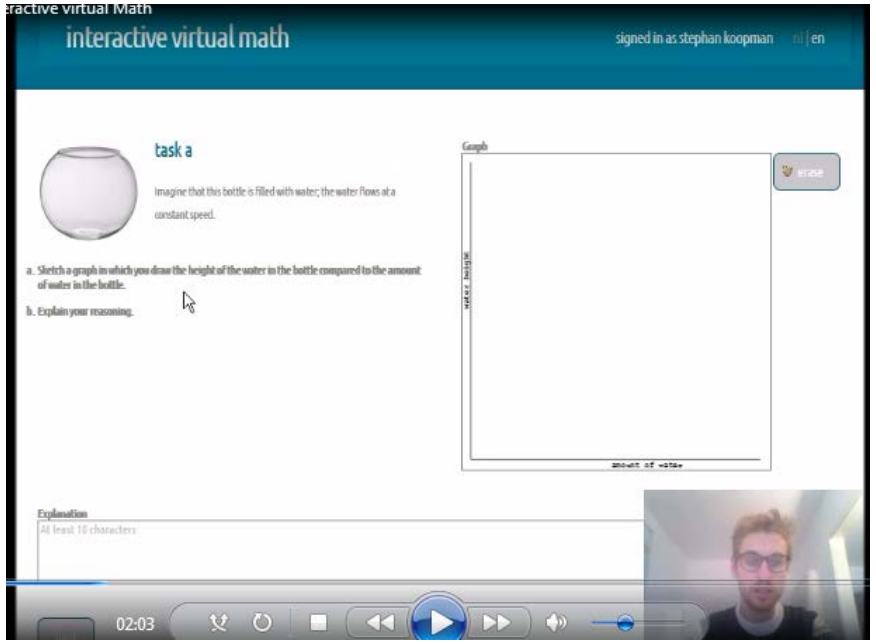
- engage in mental activity to visualize a situation and construct relevant quantitative relationships prior to determining formulas
- focus on quantities and generalizations about relationships, connections between situations, and dynamic phenomena

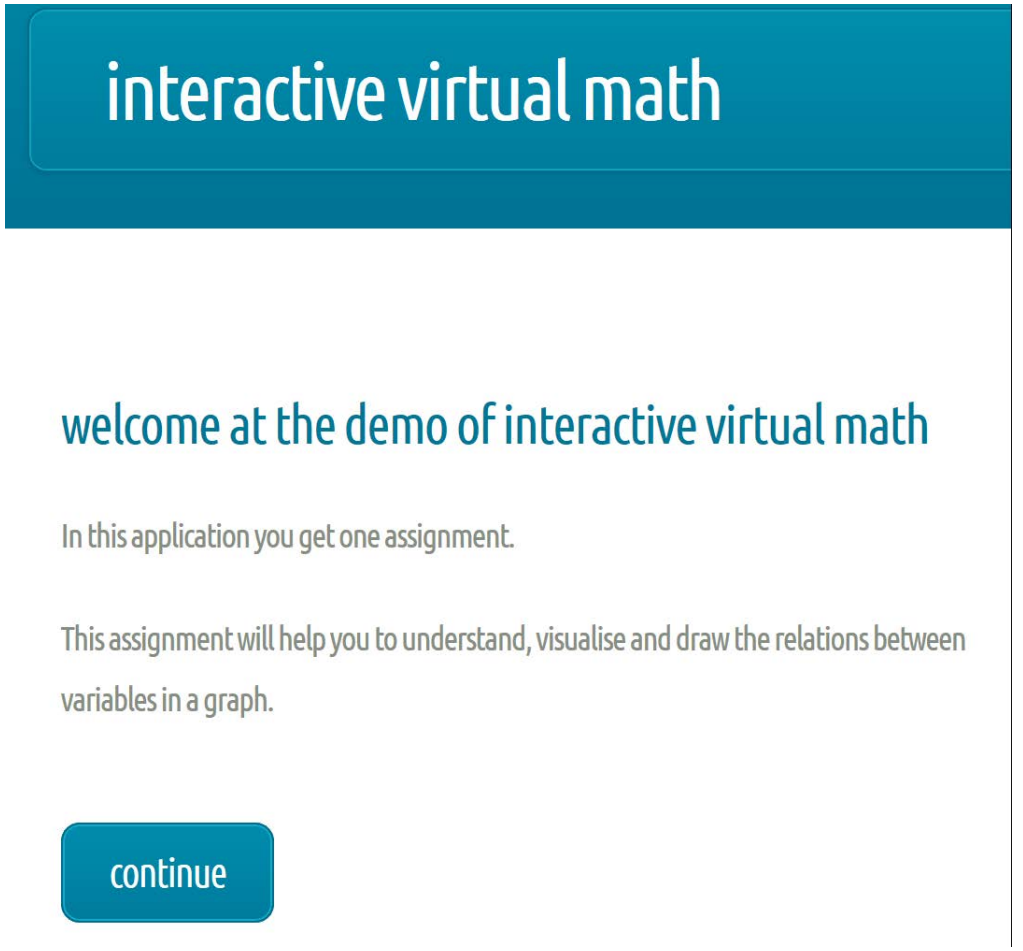
Learning as an active process (Laurillard, 2013)



The digital tool Interactive Virtual Math

https://virtualmath.hva.nl (tool)
https://youtu.be/lc7mNUcZ8CQ (tutorial)
https://virtualmath.hva.nl/admin (site teacher)





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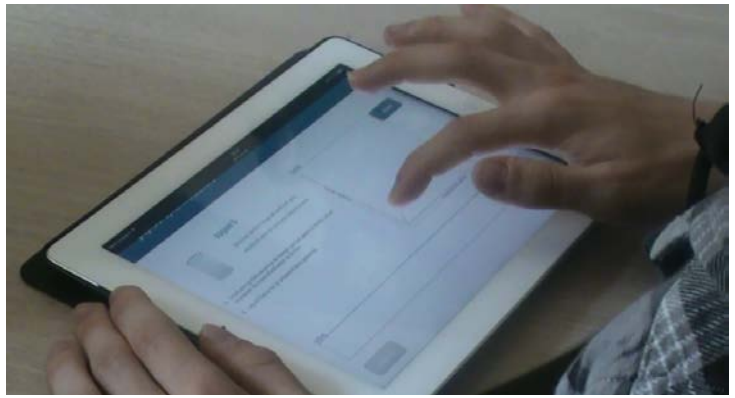
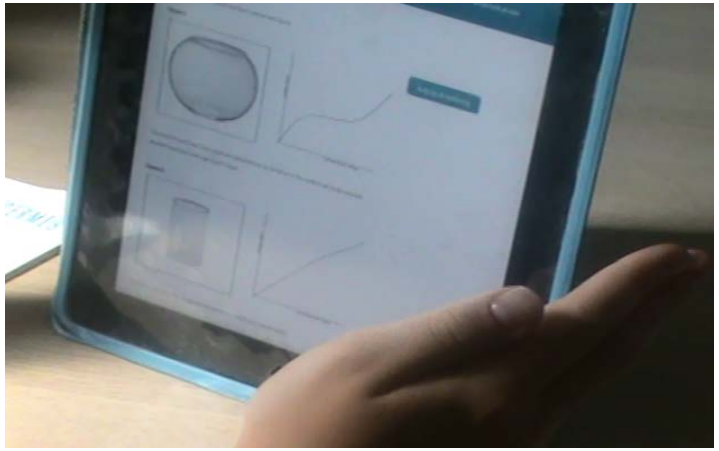
Design principles

- Students are encouraged to imagine two variables changing simultaneously (Task)
- encourage to externalize their concept image graphically and verbally (Vinner, 1983) (self-construction)
- Challenge student to keep improving the construction through the use of cognitive conflict (contrast) and feedback (reward)
- Students can go back and forth (flow)
- Help to visualize the change of the quantitaties in relation with each other using concrete materials/situations Help 1
- encourage students to relate a certain quantity in the concrete situation with a dot in the Cartesian graph by requesting students to place the dots themselves Help 2
- It provides data about students' processes that teacher can use (at classroom level)- logbook

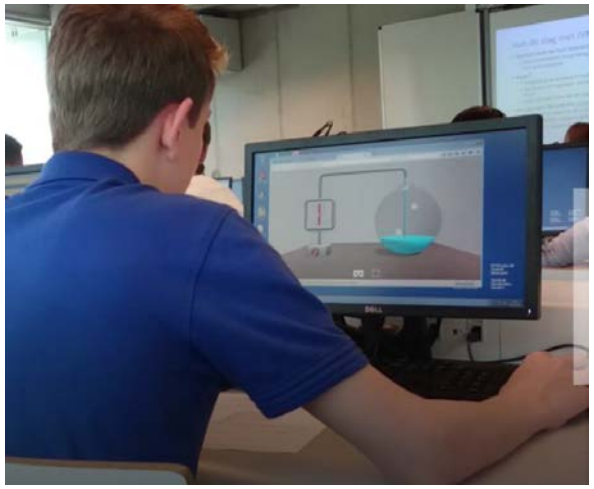
1 Self-construction



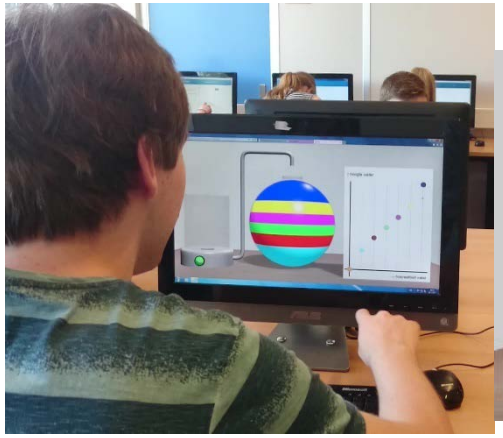
2) Contrast (provoke cognitive conflict)

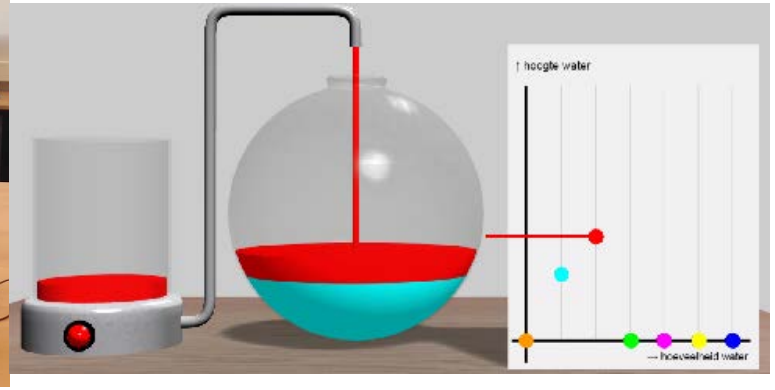


3) Help 1



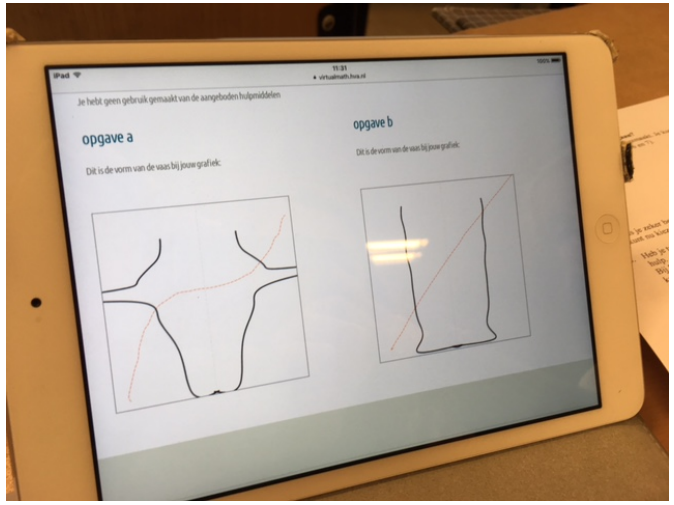
4) Help 2





5) Improve or submit

6) Reward

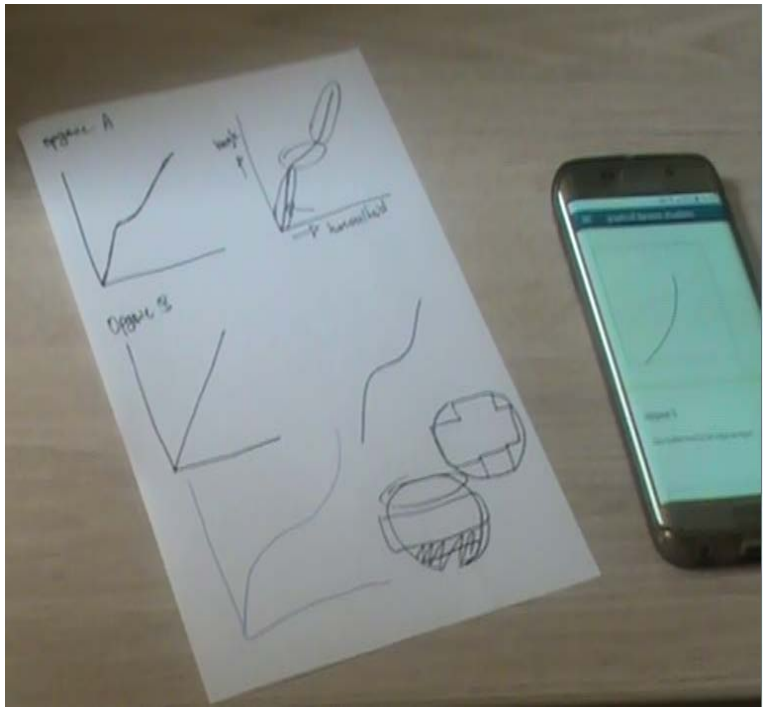


Trial-out with individual students(Phase 3)

Palha & Koopman (2017)

What made you improve your graph?

(Karin, 13 years old)



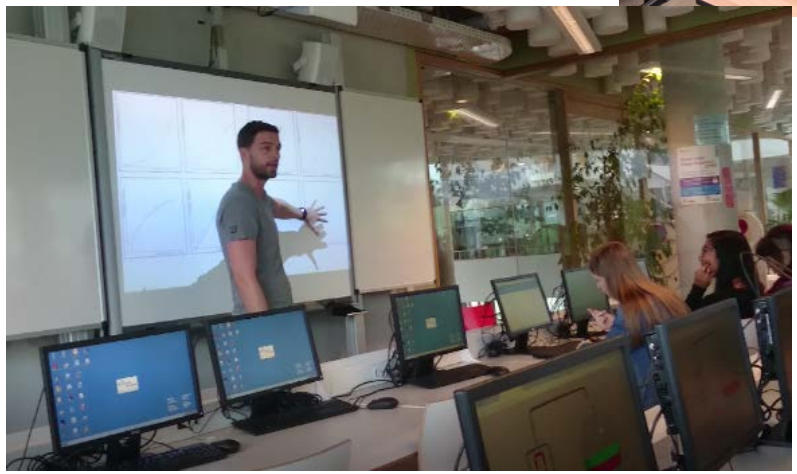
In film 1 Karin improve his construction because of the reward at the end

In film 2 Anna explains that she improved her graph because of help

Trial-out in the classroom (Phase 4)

Palha (2017), Palha & Koopman (2017)

- small scale experiment at secondary (3 classes) and tertiary education (1 class)
- students' responses to questionnaires (N=79)
- Teachers n=4 and students (n=6) interviews (these ones not yet analysed)



Did the tool help to create or improve the graph?

Fig. 3A Class DS, N=28, 11th grade

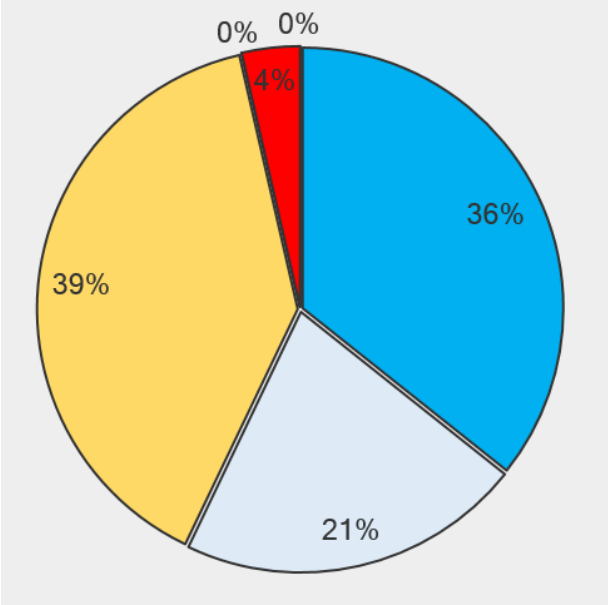
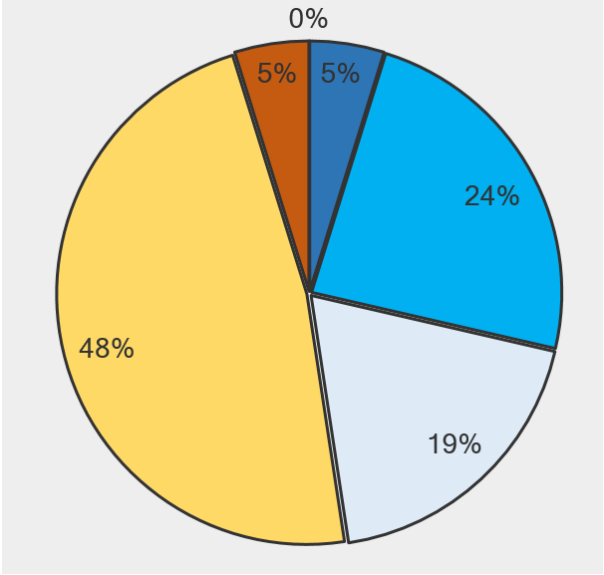


Fig. 3B Class FS (N=21, 10th grade pre-univ.)



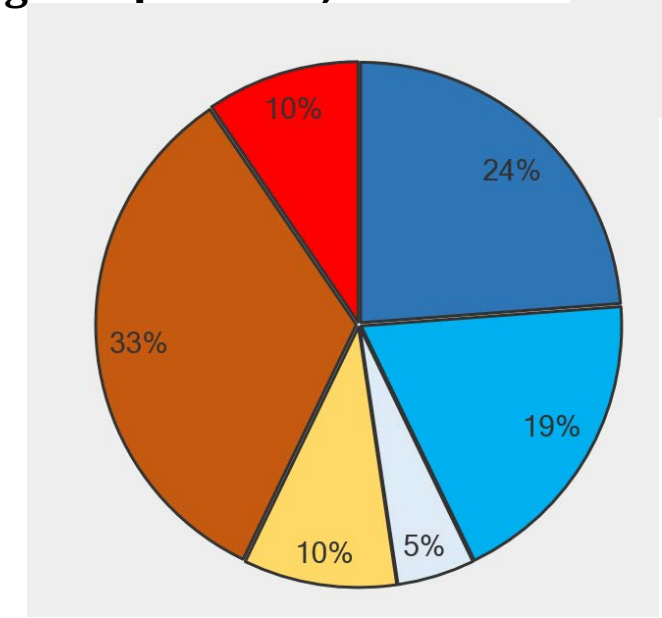


Fig. 3C Class RJ (N=21, 10th grade vocat.)

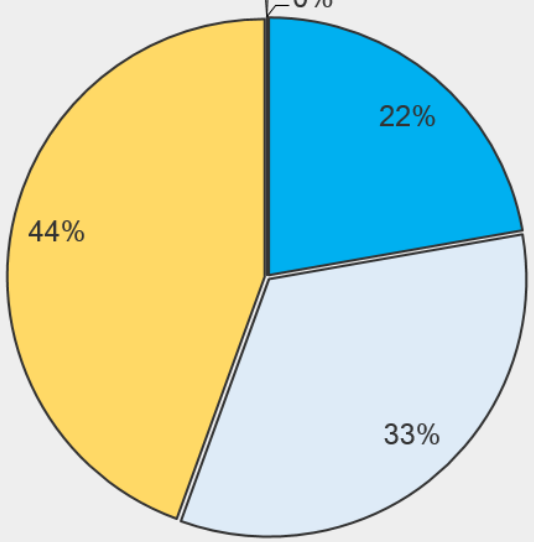
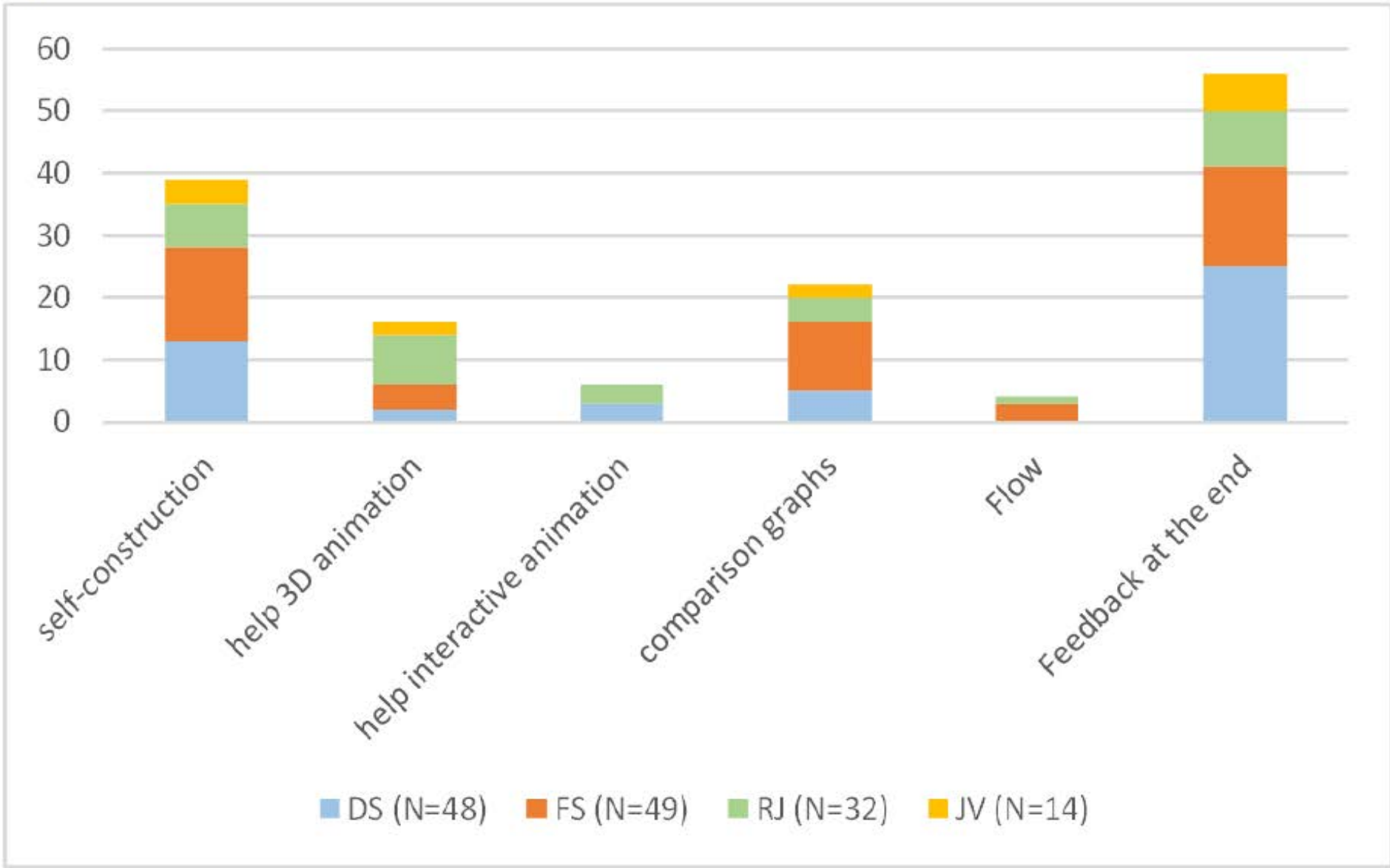


Fig. 3D Class JV (N=9, bachelor)

some reasons presented by the students who felt helped by the tool

- **visualization** (9) e.g. “At the animation, you saw at the bowl how the proportions between the amount of water and its height were“
- **drawing** (4)
- **one sees the result** (4) e.g. "Because if you got something that did not exactly look like what you thought, than you think about yourself "how should I do it to get what I wanted" such evaluations work well. And, in my case, it also makes me want to try out other lines and what's going on" . 'Because at the end he showed my drawn jar, I understood it better"
- **help to improve the form** (4)
- **It clarifies** (4)
- **Receiving explanation** (from the teacher) (4)

Fig. 5 - Tool features that students found to help them the most (Q10)

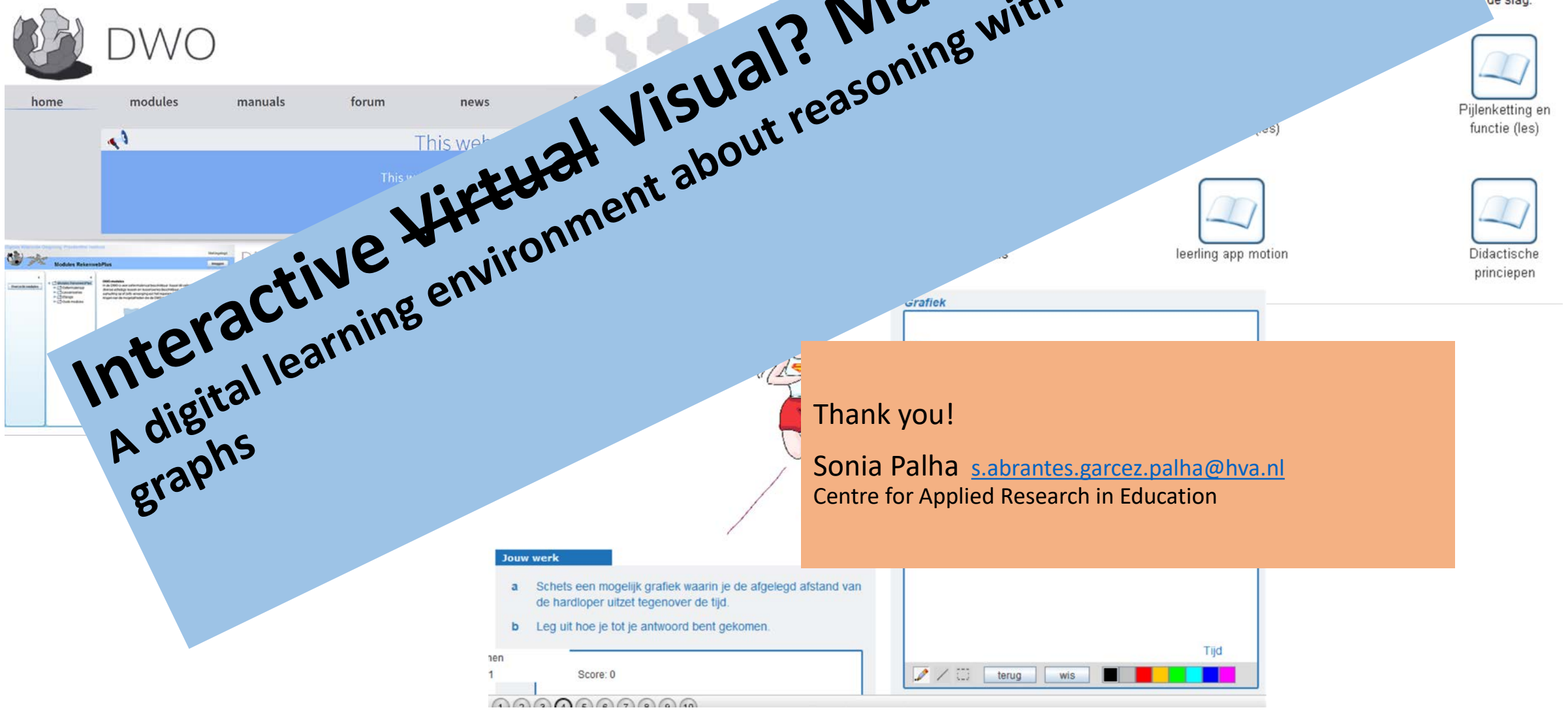


Main findings & Discussion

- the tool offers different support for students who already have some knowledge about graphs from dynamic events than from students who don't. In the last case it can happen that the tool doesn't help to construct the graph (which doesn’t mean that it doesn’t ad understanding)
- the reasons most pointed by the students with regard to the learning with the tool were: “one seeing the results" and "visualizing”
- Seeing the result of the form of the jar at the end and the self-construction graph were the most helping to students
- The help-features were not often mentioned. We find this surprising
- Students suggestions provided insightful ideas to improve the tool (which I would like to take in the discussion at CADGME)

Follow up

Interactive ~~Virtual~~ Visual? Math
A digital learning environment about reasoning with covariation graphs



Thank you!

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Some references

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