

MoMaTrE (Math Trails and AAR)

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<http://www.recio.tk>

- Erasmus+ Program: Mobile Math Trails in Europe



Augmented reality (to explore math objects in a real world context)

<https://www.facebook.com/geogebra/videos/10155662725038232/>



TASKS

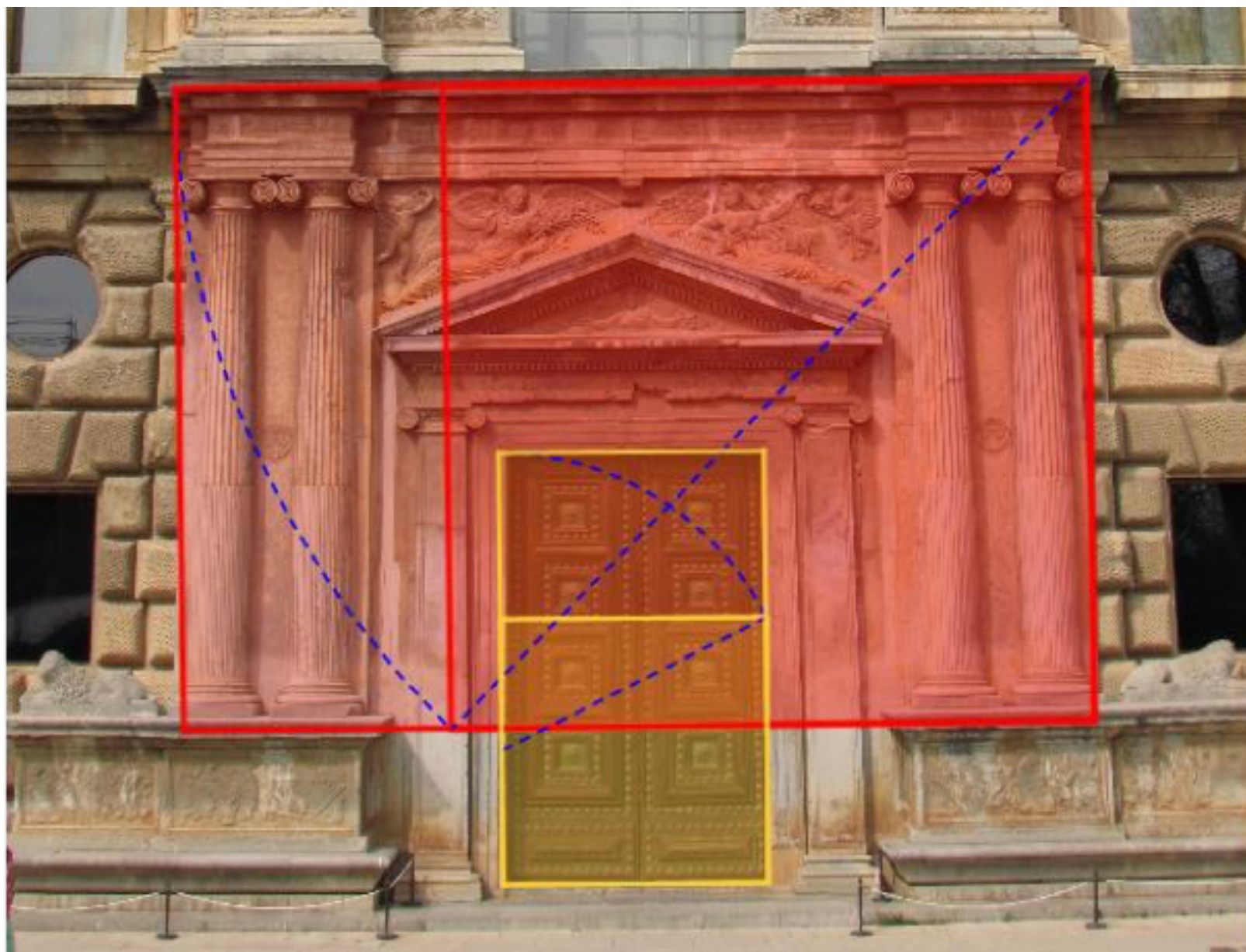
- DESIGNED WITH THE HELP OF A MOBILE PHONE
(AR; AI)
- DEVELOPED WITH THE HELP OF THE MOBILE
PHONE

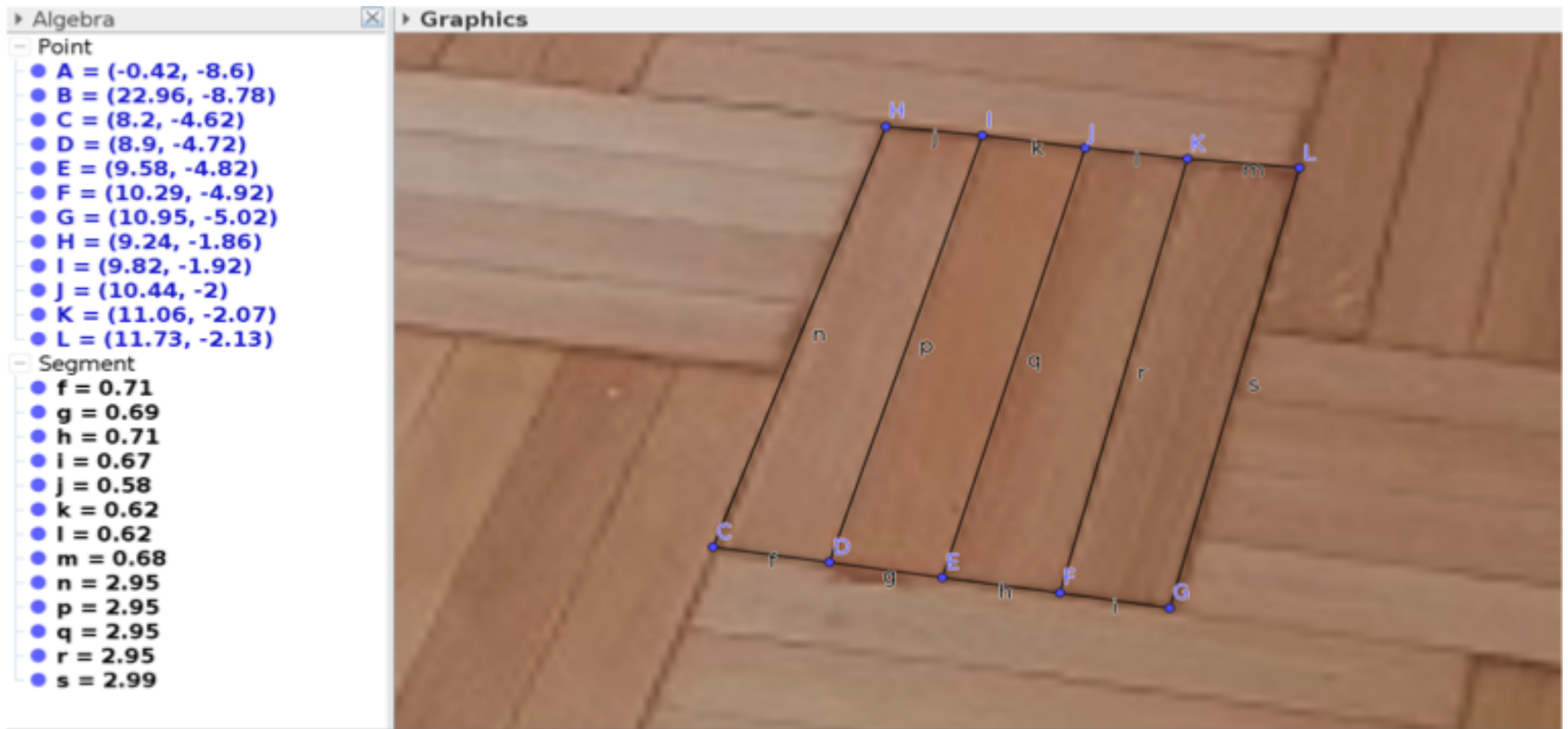
A new scenery

- **Real world** (Smart phones, robotic vision, sensors) – mathematical layer
 - 1) Pre-processed, associated by geo-positioning, markers;
 - 2) Measures {image — Hough transform — recognizing geometric elements } — input GeoGebra — systematic and automatic derivation of geometric properties — generalizing, proving, discovering;
- **Automatically augmented geometric reality**

Ongoing work with F. Botana (U. Vigo) y Z. Kovács (University College
Diocese of Linz), A. Martínez Sevilla (U. Granada), Pilar Vélez-Melón (U.
Nebrija)







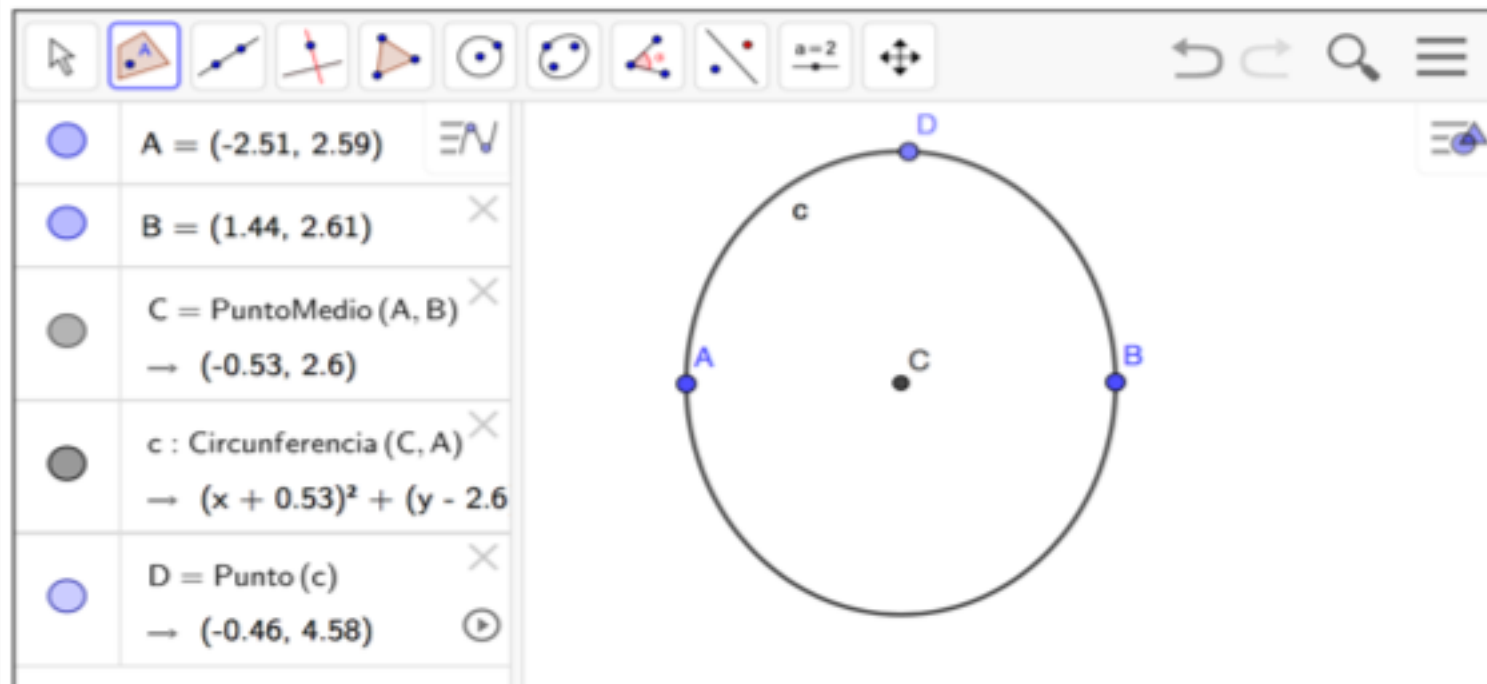
Imaginary detection of reality with an internal representation of a parquet floor being photographed by a smartphone and translated into GeoGebra

Our tool: Combinatorics with geometric elements,
automated application of the automated
reasoning tools developed by Botana, Kovacs,
Recio, Velez.

- <http://htmlpreview.github.io/?https://github.com/kovzol/ag/blob/master/automated-geometer.html>

Welcome to the Automated Geometer!

Let us consider this initial input construction (you may freely edit the construction or upload another one as well; only the visible points will be observed):



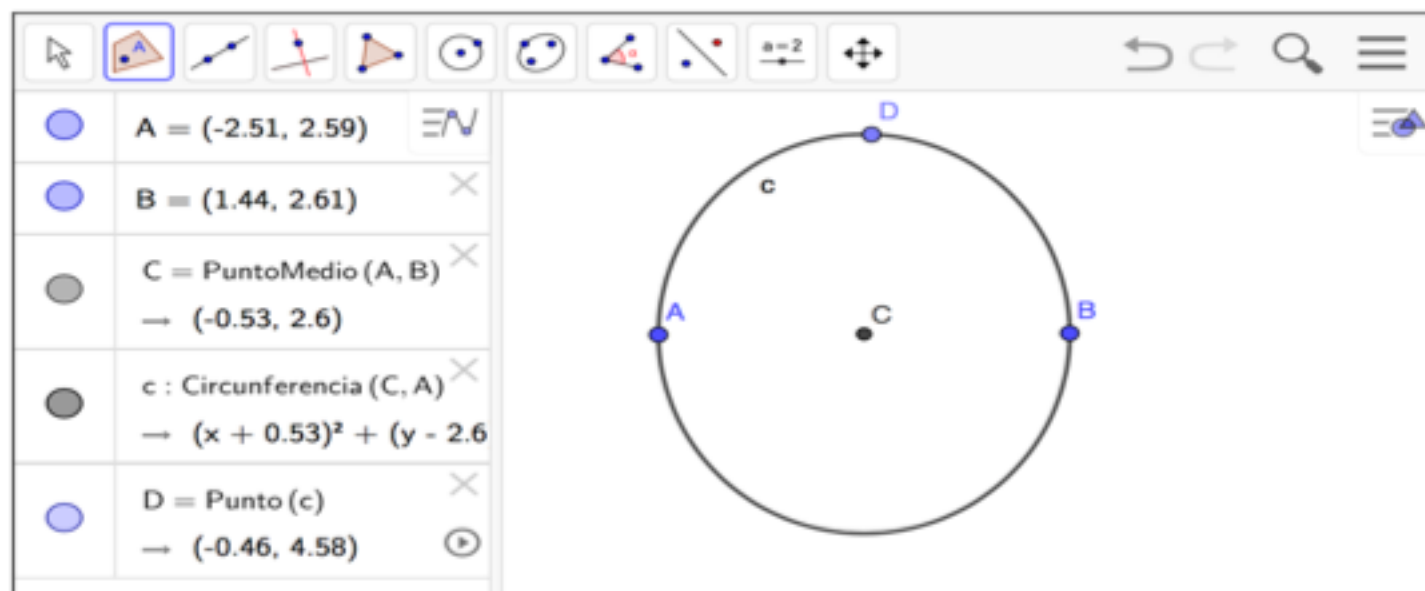
Select relations to check:

- Collinearity of three points
- Equality of distances between two points
- Perpendicularity of segments defined by two points**
- Parallelism of segments defined by two points

Start discovery

Welcome to the Automated Geometer!

Let us consider this initial input construction (you may freely edit the construction or upload another one as well; only the visible points will be observed):



Select relations to check:

- ☐ Collinearity of three points
- ☐ Equality of distances between two points
- ☒ Perpendicularity of segments defined by two points
- ☐ Parallelism of segments defined by two points

Now the following theorems can be obtained:

1. $AD \perp BD$

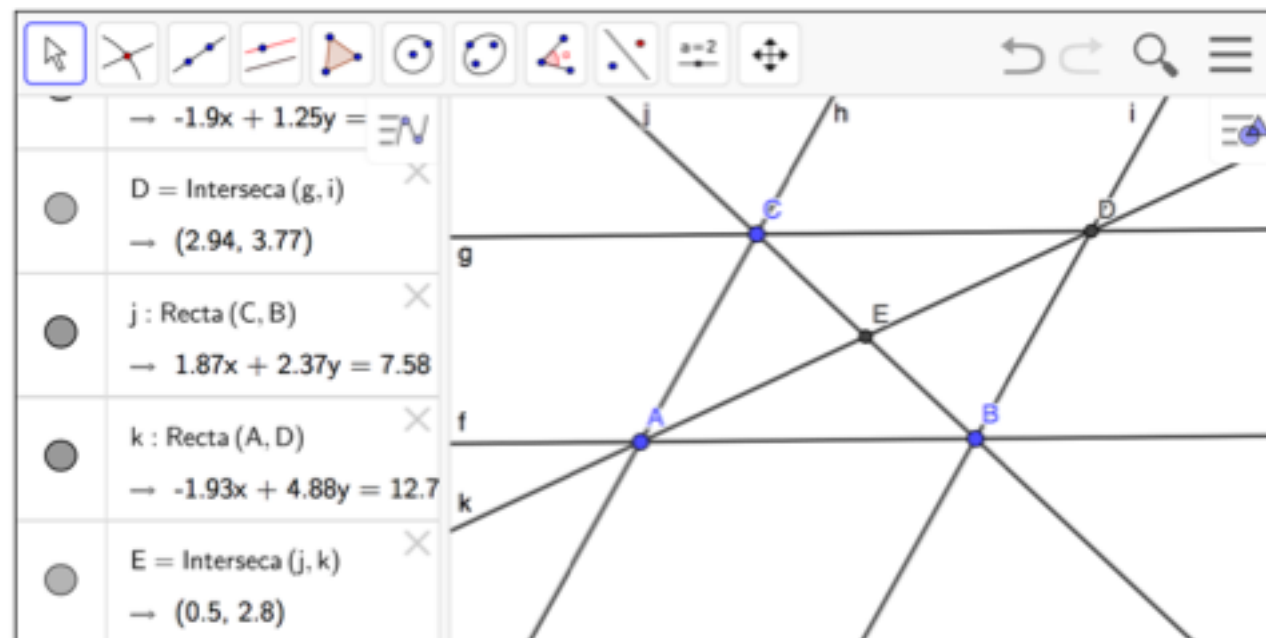
Finished, found 1 theorems among 15 possible statements.

Elapsed time: 0h 0m 1s

Please reload the page if you want to start a new experiment.

Welcome to the Automated Geometer!

Let us consider this initial input construction (you may freely edit the construction or upload another one as well; only the visible points will be observed):



Select relations to check:

- ☐ Collinearity of three points
- ☐ Equality of distances between two points
- ☐ Perpendicularity of segments defined by two points
- ☐ Parallelism of segments defined by two points

Now the following theorems can be obtained:

1. $E \in AD$
2. $E \in BC$
3. $AB = CD$
4. $AC = BD$
5. $AE = DE$
6. $BE = CE$

Finished, found 6 theorems among 55 possible statements.

Elapsed time: 0h 0m 3s

Please reload the page if you want to start a new experiment.

Welcome to the Automated Geometer!

Let us consider this initial input construction (you may freely edit the construction or upload another one as well; only the visible points will be observed):



	$C = (2, 4)$	
	$D = \text{PuntoMedio}(A, B)$ $\rightarrow (0.5, 1)$	
	$E = \text{PuntoMedio}(B, C)$ $\rightarrow (3, 2.5)$	
	$F = \text{PuntoMedio}(C, A)$ $\rightarrow (-0.5, 2.5)$	
	$G = \text{PuntoMedio}(D, E)$ $\rightarrow (1.75, 1.75)$	



Select relations to check:

Collinearity of three points
Equality of distances between two points
Perpendicularity of segments defined by two points
Parallelism of segments defined by two points

Start discovery



Select relations to check:

☐ Collinearity of three points
☐ Equality of distances between two points
☐ Perpendicularity of segments defined by two points
☐ Parallelism of segments defined by two points

Now the following theorems can be obtained:

- | | | | | | | | | | |
|---------------|---------------|---------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 1. $D \in AB$ | 6. $AD = BD$ | 11. $BE = CE$ | 16. $DG = EG$ | 21. $AC \perp CF$ | 26. $AD \perp EF$ | 31. $BC \perp BE$ | 36. $BE \perp DF$ | 41. $CF \perp DE$ | 46. $DG \perp EG$ |
| 2. $F \in AC$ | 7. $AD = EF$ | 12. $BE = DF$ | 17. $AB \perp AD$ | 22. $AC \perp DE$ | 27. $AF \perp CF$ | 32. $BC \perp CE$ | 37. $BF \perp BG$ | 42. $CF \perp DG$ | |
| 3. $E \in BC$ | 8. $AF = CF$ | 13. $BG = FG$ | 18. $AB \perp BD$ | 23. $AC \perp DG$ | 28. $AF \perp DE$ | 33. $BC \perp DF$ | 38. $BF \perp FG$ | 43. $CF \perp EG$ | |
| 4. $G \in BF$ | 9. $AF = DE$ | 14. $CE = DF$ | 19. $AB \perp EF$ | 24. $AC \perp EG$ | 29. $AF \perp DG$ | 34. $BD \perp EF$ | 39. $BG \perp FG$ | 44. $DE \perp DG$ | |
| 5. $G \in DE$ | 10. $BD = EF$ | 15. $CF = DE$ | 20. $AC \perp AF$ | 25. $AD \perp BD$ | 30. $AF \perp EG$ | 35. $BE \perp CE$ | 40. $CE \perp DF$ | 45. $DE \perp EG$ | |

Finished, found 46 theorems among 665 possible statements.

Elapsed time: 0h 0m 20s

Please reload the page if you want to start a new experiment.

Some recent references for these ideas:

- MARTÍNEZ-SEVILLA, Álvaro; RECIO, Tomás. GeoGebra y Realidad Aumentada en la Divulgación de la Matemática. III Foro Iberoamericano de Divulgación y Cultura Científica (Córdoba, Spain, 2017). <http://www.oei.es/historico/divulgacioncientifica/IMG/pdf/orales-3.pdf>
- F. Botana, Z. Kovács, A. Martínez-Sevilla, T. Recio. Automatically Augmented Reality with GeoGebra. In: Augmented Reality in Educational Settings. (Ed. Theodosia Prodromou). Sense Publishers. To appear.
- Carlos Ureña, Álvaro Martínez-Sevilla and Tomás Recio. Augmented Reality, Maths Walks and GeoGebra. Communication to the CADGME WG7 - Augmented and Virtual Reality in Mathematics Education Workshop. Coimbra, June 2018. <https://www.uc.pt/en/congressos/cadgme2018/generalinformation/workgroupshops>
- Francisco Botana, Zoltán Kovács and Tomas Recio: Towards an Automated Geometer. AISC 2018. Suzhou. <http://aisc2018.cc4cm.org>
- Francisco Botana, Zoltán Kovács and Tomas Recio: Automated Geometer, a web based discovery tool (extended abstract). ADG 2018 Nanning. <http://adg2018.cc4cm.org>
- KOVÁCS, Zoltan; RECIO, Tomás and VÉLEZ, M.Pilar. Using Automated Reasoning Tools in GeoGebra in the Teaching and Learning of Proving in Geometry. In: International Journal of Technology in Mathematics Education, to appear: <http://www.researchinformation.co.uk/time/contents/timecont.php>

Thanks!