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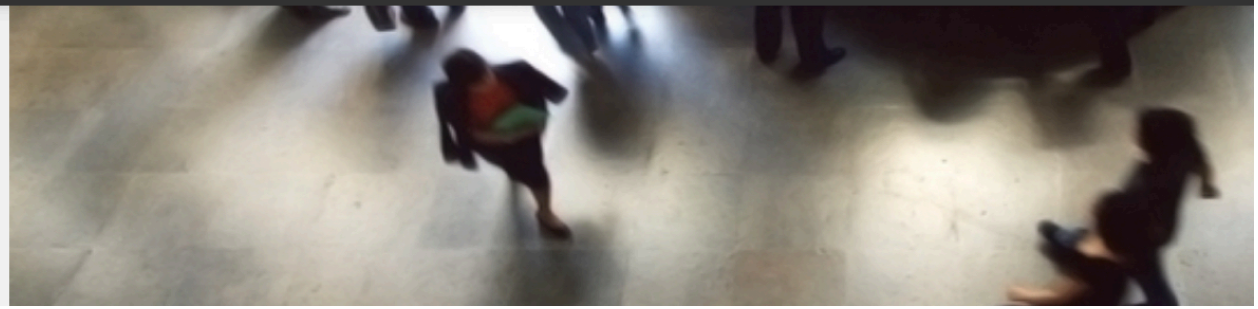

MATHEMATICS TEACHERS WORKING IN COLLABORATION AND USING TECHNOLOGY

Ornella Robutti

DIPARTIMENTO
DI MATEMATICA
GIUSEPPE PEANO
UNIVERSITÀ DI TORINO

I thank very much the organisers for the invitation.

I am particularly happy to speak to you today

I specially thank my colleagues who take part in this project with me: Arzarello, Bini, Carante, Prodromou, Trinchero

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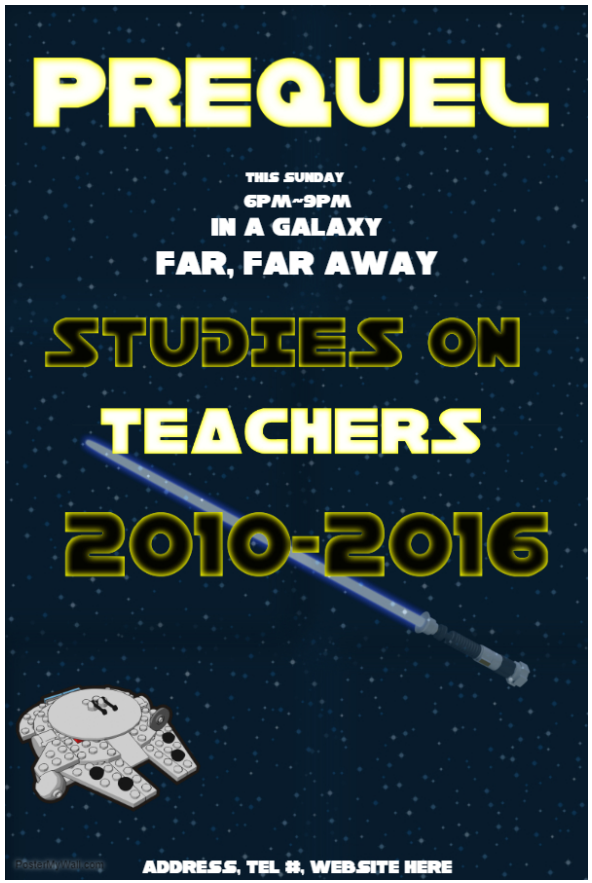

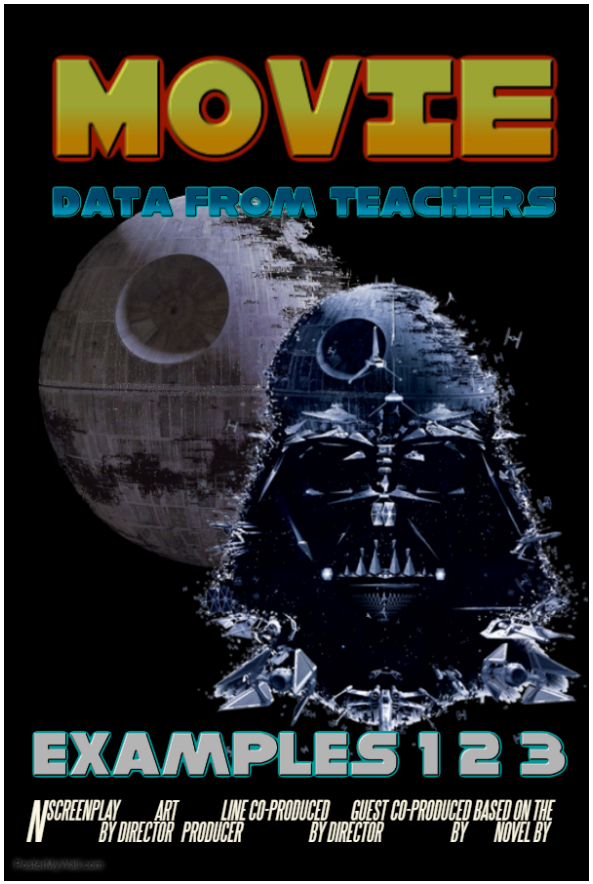

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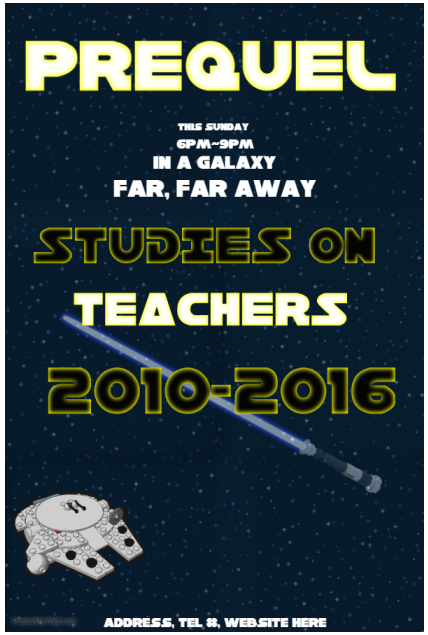
1. PREQUEL: STUDIES ON TEACHERS 2010-2016
2. MOVIE
3. SEQUEL



STUDIES ON TEACHERS

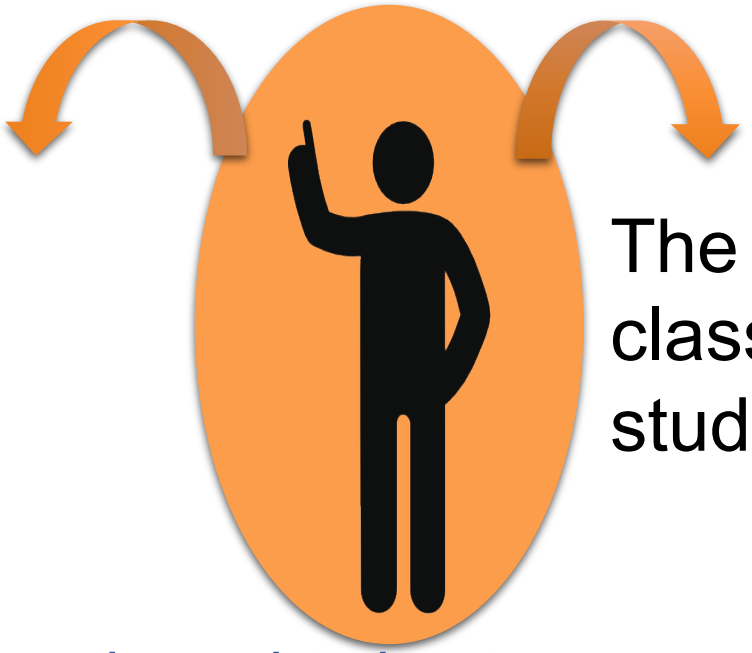
My interests on teachers come from a long time ago (about 30 years), in the sense that I have been engaged in teachers' professional development courses, especially on the use of technology and the teaching practices.

I began to study teachers as a research topic in 2010, after PME congress of 2009, participating at some presentations on this theme.

Moreover, since I have been involved in many national and local teachers' education programmes, I was interested in developing a theoretical frame to describe teachers while working together and with researchers in their professional development.

DEMANDS FROM RESEARCH

The teacher as a member of a community of professionals



The teacher in the classroom with her students

the advent of this era has brought about a re-conceptualisation of the relationship between the teacher and the researcher, arguing that in most of the international research studies, the question is not *what* is taught in classrooms, but *how* it is taught.

THE ERA OF THE TEACHER (Sfard, 2005)

DEMANDS FROM THE INSTITUTIONS

NEW NATIONAL CURRICULA: 2010 SECONDARY & 2012 PRIMARY SCHOOL (MINISTRY)

- Teaching for competences
- Conjecturing and arguing
- Problem solving
- Laboratory of mathematics

- New topics: statistics and probability, modeling, differential equations, spatial Cartesian geometry

NATIONAL ASSESSMENT (2008 – INVALSI)

- Theoretical frame of reference for mathematics: processes and topics
- National tests at grade 2, 5, 8, 10, and (next year) 13

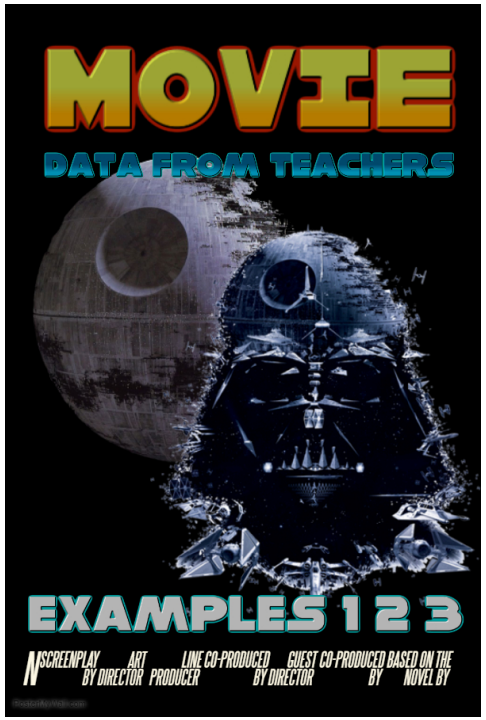
ANSWERS OF RESEARCHERS

Meta-Didactical Transposition framework
(Arzarello, Robutti, Malara, Garuti, Cusi, Sabena, Martignone, National Seminar in Didactics of Mathematics – Italy, 2012)

TO ANALYSE AND TO DESCRIBE TEACHERS' PRAXEOLOGIES WHEN INVOLVED IN PROGRAMMES OF PROFESSIONAL DEVELOPMENT, IN CONTACT WITH RESEARCHERS.

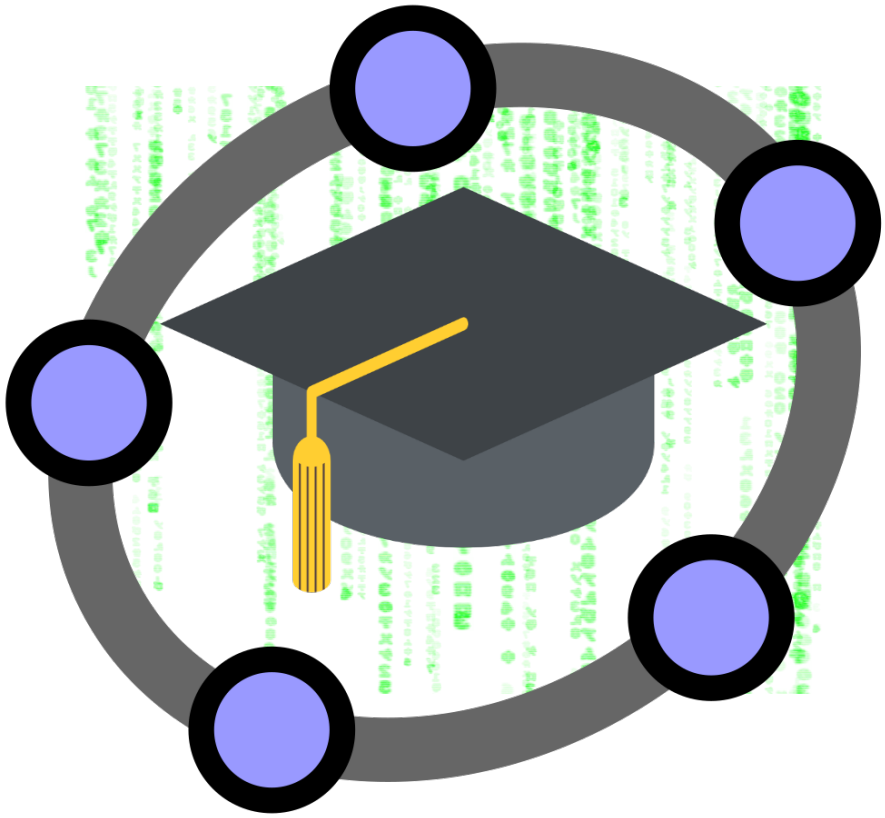
CONTENTS

1. PREQUEL
2. MOVIE: DATA FROM TEACHERS – EXAMPLE 1
3. SEQUEL



EXAMPLE 1 - PLS

PROFESSIONAL DEVELOPMENT OF SECONDARY SCHOOL TEACHERS ON MATHEMATICS ACTIVITIES WITH GEOGEBRA



A TEACHER: RICCARDO

Riccardo is a teacher of secondary school not so used in giving students activities to be solved with GeoGebra: he usually prefer to show solutions on the interactive whiteboard. And he does not know well geometry from a theoretical viewpoint.

RICCARDO AT THE BEGINNING

Interview (at the beginning of the educational programme): he has used GeoGebra in his classrooms only on some sporadic occasions and “in a even less than basic way”:

> “With grade 13 students, I showed them how to draw graphs of functions with GeoGebra, whereas, with grade 10 students I made something of statistics, but with grade 9 students, I haven’t gone yet [to the digital laboratory], but I intend to show them something of geometry with GeoGebra. The training course can help me in this sense.”

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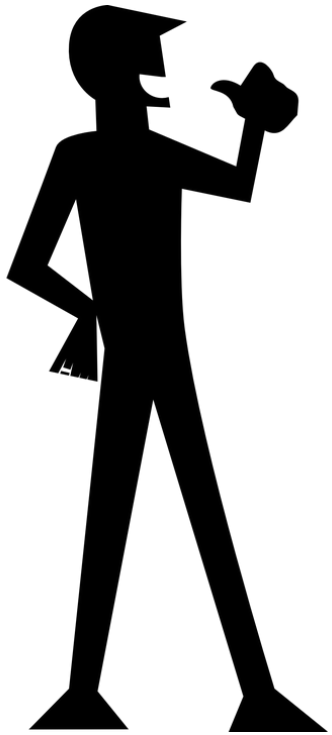
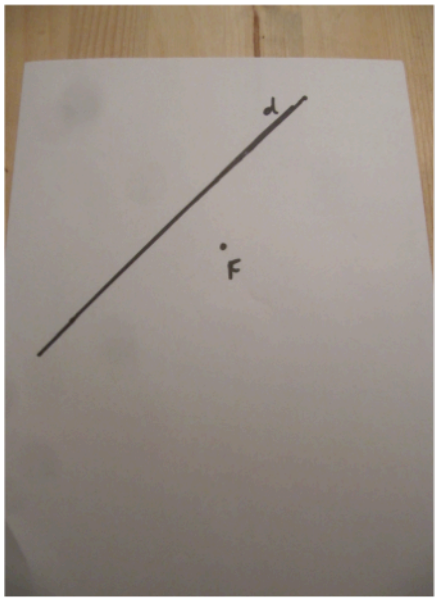
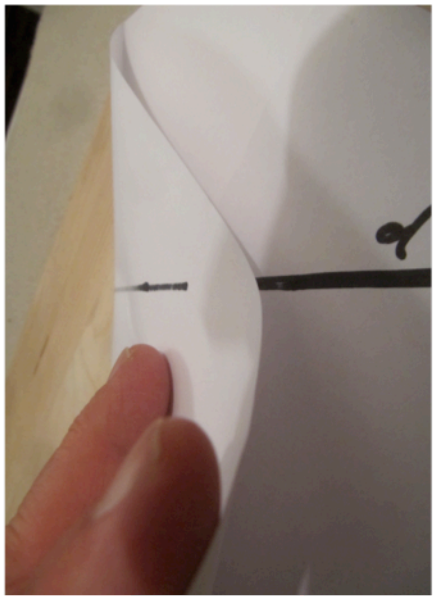


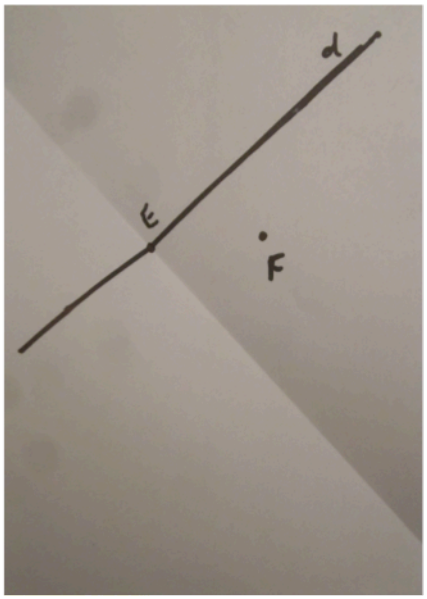
“With grade 13 students, **I showed** them **how to draw** graphs of functions with GeoGebra, whereas, with grade 10 students **I made something of statistics**, but with grade 9 students, I haven’t gone yet [to the digital laboratory], but **I intend to show them** something of geometry with GeoGebra. The training course can help me in this sense.”

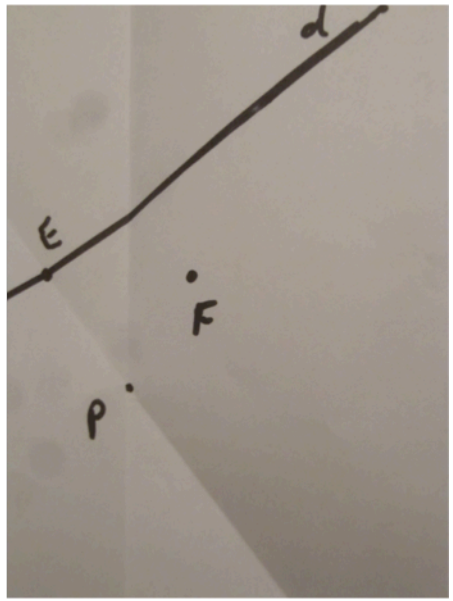
RICCARDO PARTICIPATES TO THE PARABOLA ACTIVITY – folding the paper

A point and a line – folding on the vertex – folding in another point



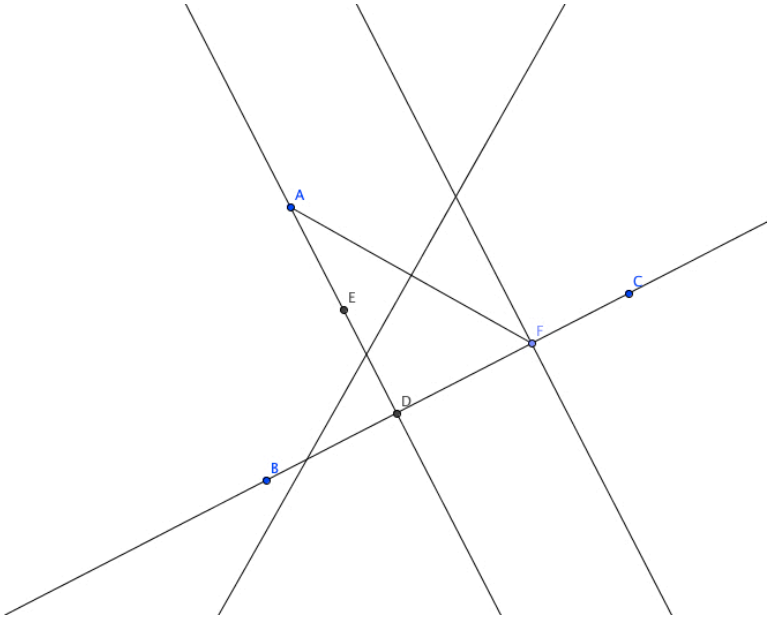


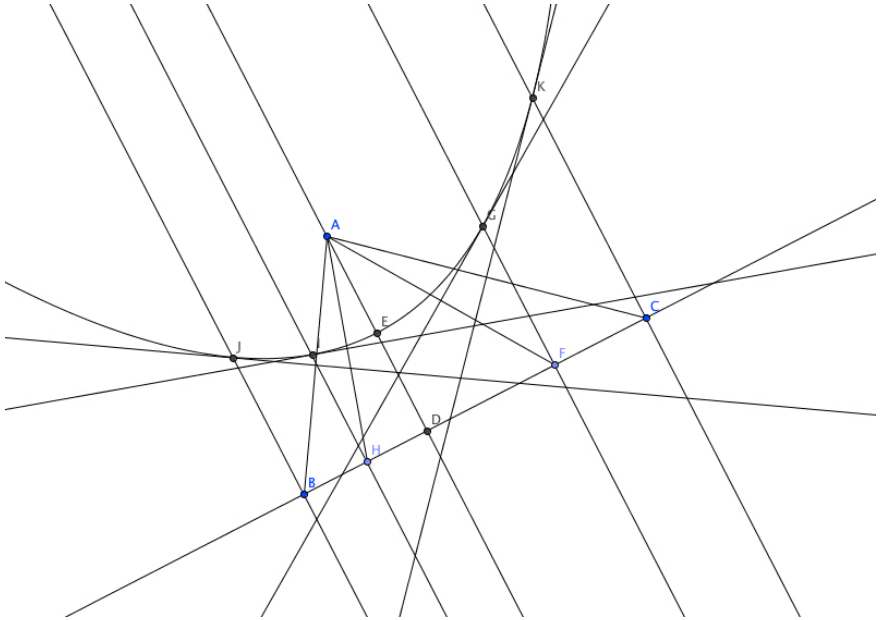




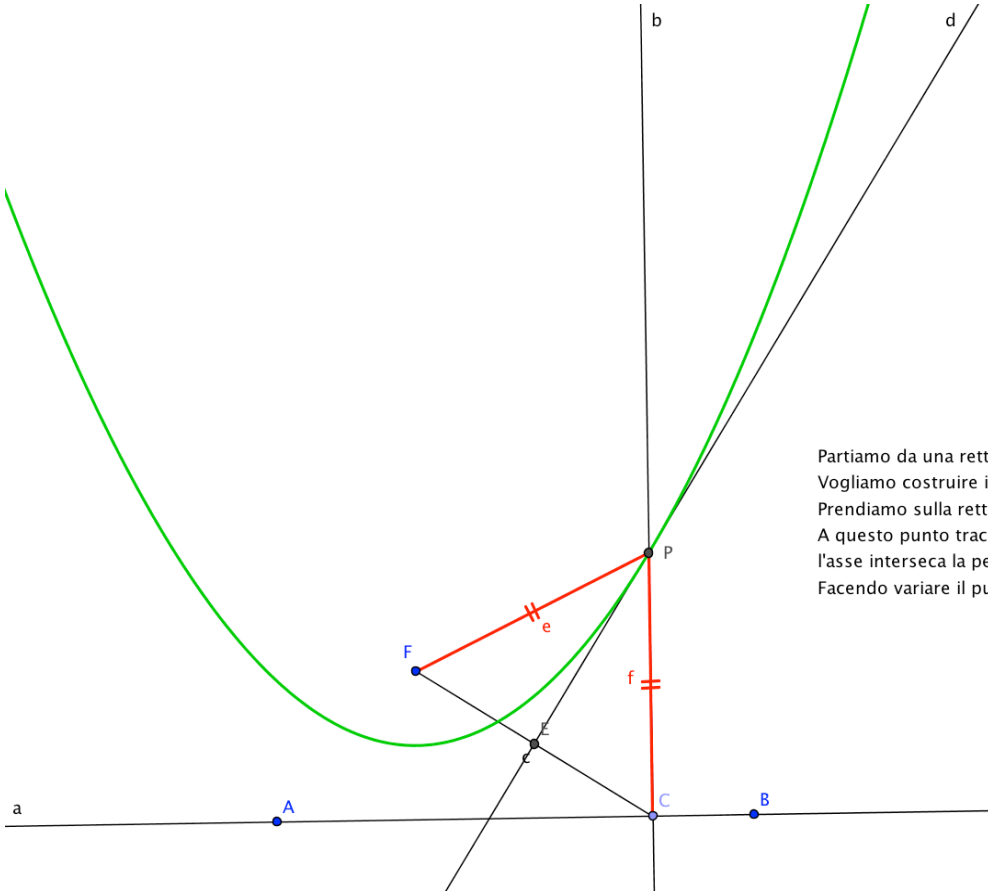
RICCARDO PARTICIPATES TO THE PARABOLA ACTIVITY – construction in GeoGebra, with the same steps

Perpendicular line in the focus and in other points & axis of the segment





RICCARDO PARTICIPATES TO THE PARABOLA ACTIVITY – Dynamic use of trace and locus to visualise the parabola in GeoGebra

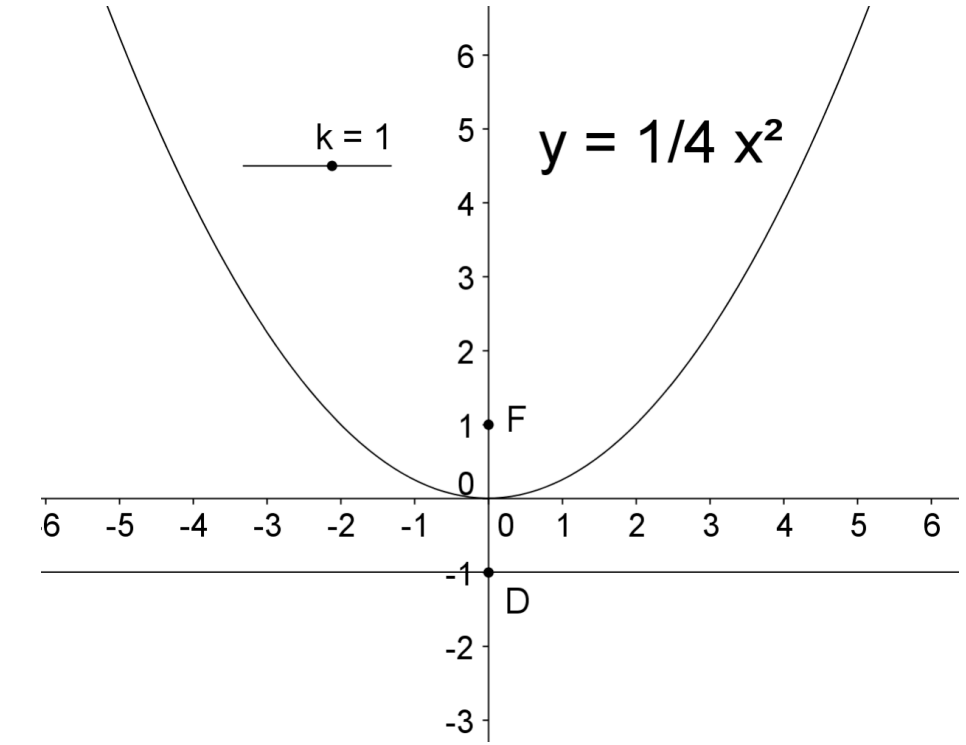


Partiamo da una retta a qualsiasi e da un punto F.
Vogliamo costruire il luogo dei punto equidistanti da F e da a.
Prendiamo sulla retta a un punto C mobile e tracciamo la perpendicolare ad a per C.
A questo punto tracciamo il segmento CF e il suo asse d
l'asse interseca la perpendicolare nel punto P e tale punto è equidistante dal punto F e dalla retta a (i triangoli FEP e PEC sono cong
Facendo variare il punto C sulla retta a il punto P descrive il luogo dei punti equidistanti da F e da a, cioè la parabola.

RICCARDO PARTICIPATE TO THE PARABOLA ACTIVITY – Algebraic aspects in GeoGebra with sliders

$y = ax^2$

$a = 1/4k$



RICCARDO AFTER THE PARABOLA ACTIVITY

“I used the video-projector and I started showing the functionalities because they [the students] didn’t know how to do. I had some problems too, so we helped each other. By the way, I surfed the Internet to try to understand how to do some things. Someone suggested me: “Try with YouTube”, and we found several videos […] In that way, basically they followed [the activity], they worked in groups and, […] I induced them to get to understand that they were actually constructing the parabola.”

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RICCARDO IN THE INTERVIEW

“I believe it was a positive experience because, let me say… from the next year on, the first time I can use the digital laboratory, I will do it. In fact, with grade 9 students, I found a video on YouTube […] we studied the first Euclid’s theorem and we proved it practically in a particular case, but then, since we could move [the points in the construction], we proved it in a practical way”.

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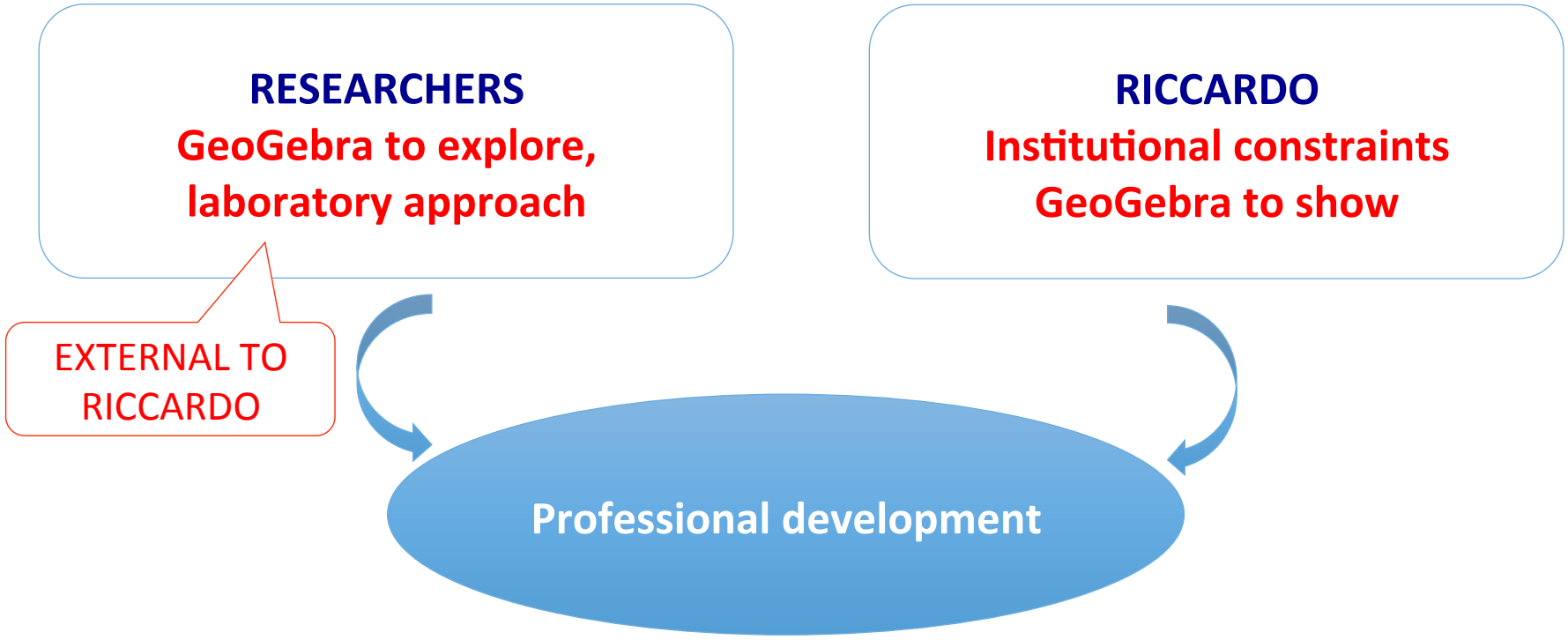
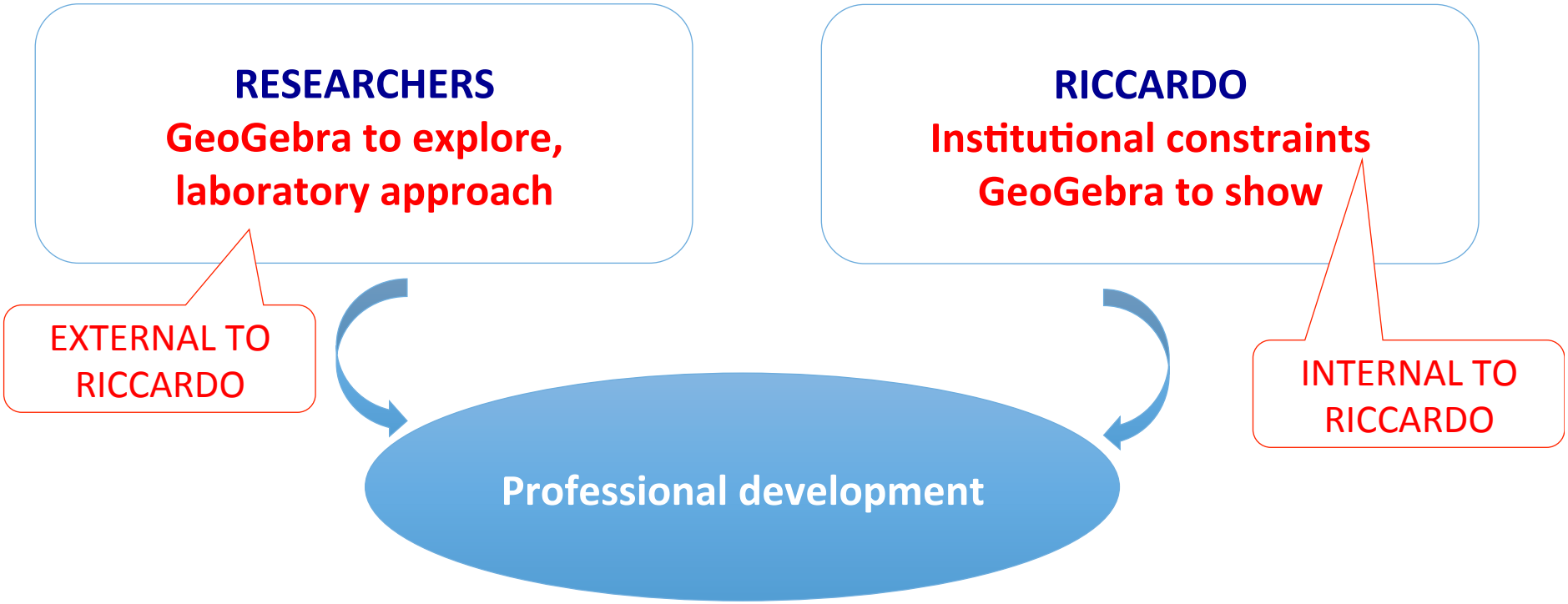

RESEARCHERS
GeoGebra to explore, laboratory approach
RICCARDO
Institutional constraints
GeoGebra to show
EXTERNAL TO RICCARDO
Professional development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INTERNAL TO RICCARDO
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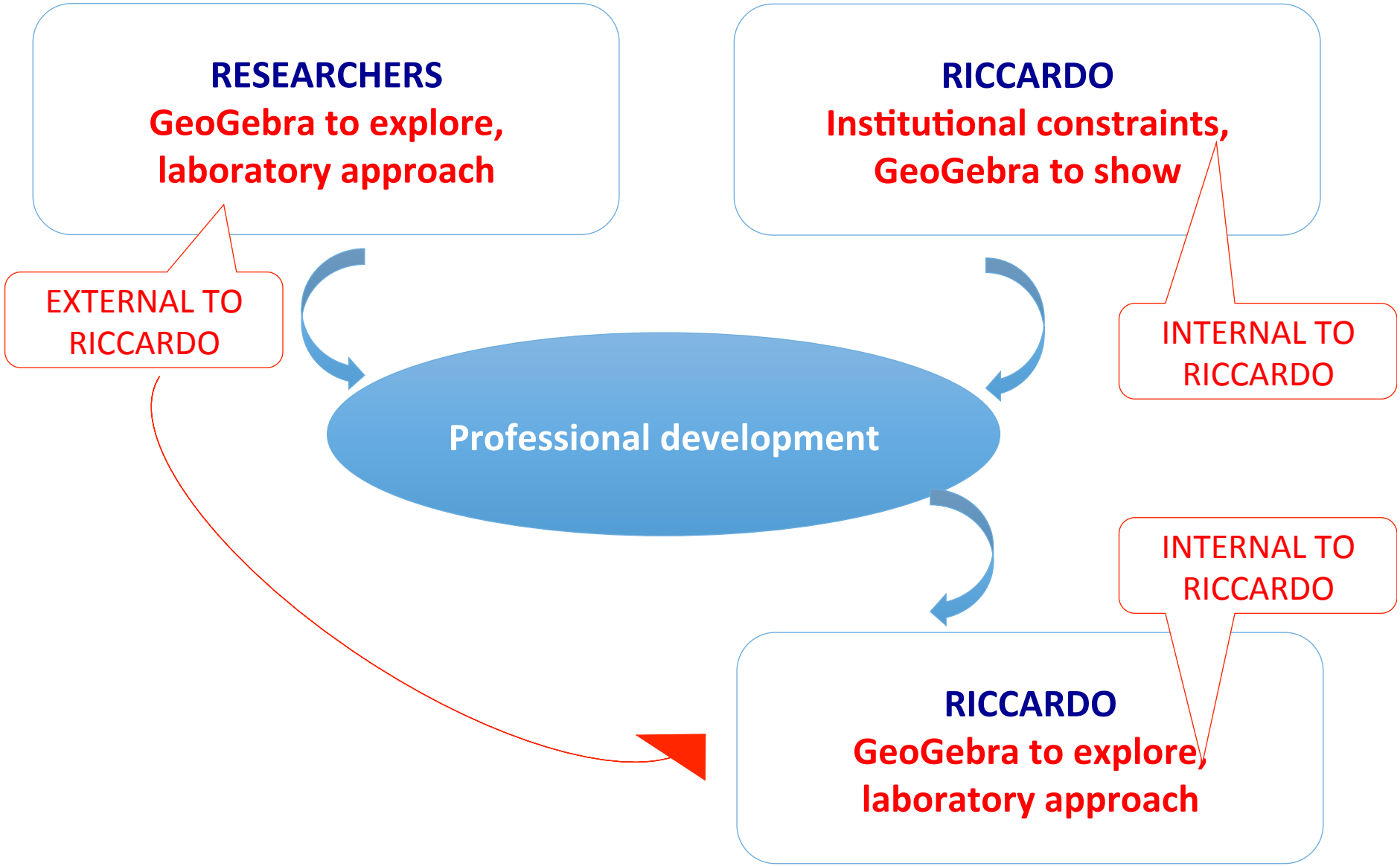
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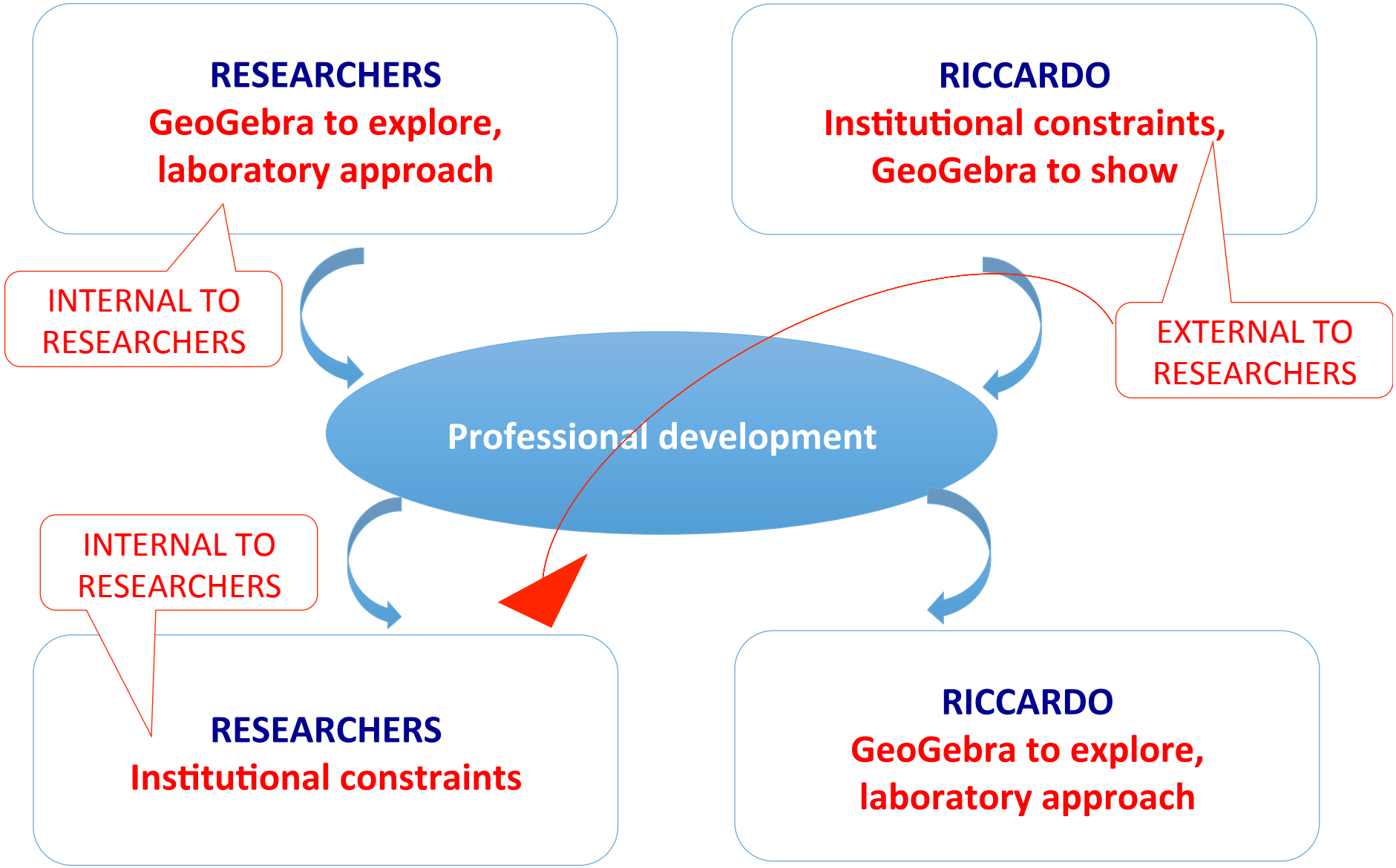
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INTERNAL TO RESEARCHERS
EXTERNAL TO RESEARCHERS
Professional development
INTERNAL TO RESEARCHERS
RESEARCHERS
Institutional constraints
RICCARDO
GeoGebra to explore, laboratory approach

PRAXEOLOGY (Chevallard, 1999)

A **praxeology** can be **mathematical** (if referred to a mathematical task to be solved by the students), or **didactical** (if referred to the teachers activity when engaged in a lesson).

COMPONENTS of the praxeology

2 PRACTICAL:

- **Task**
- **Technique** to solve the task

2 THEORETICAL that validate the use of that technique:

- **Justification** of the technique
- **Theory** that supports justification

META–DIDACTICAL PRAXEOLOGY (Arzarello et al., 2014)

REFERRING TO THE TEACHERS' IN THEIR PROFESSIONAL DEVELOPMENT:

COMPONENTS of the praxeology

2 PRACTICAL:

- **Task**
- **Technique** to solve the task

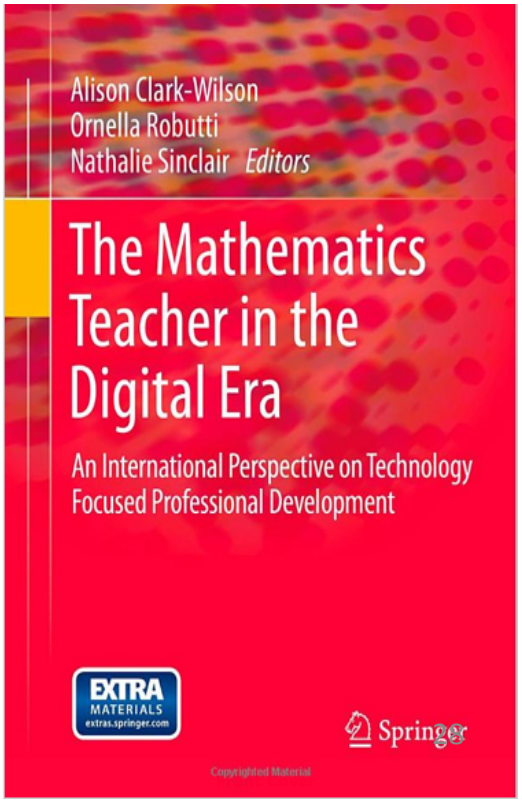
2 THEORETICAL that validate the use of that technique:

- **Justification** of the technique
- **Theory** that supports justification

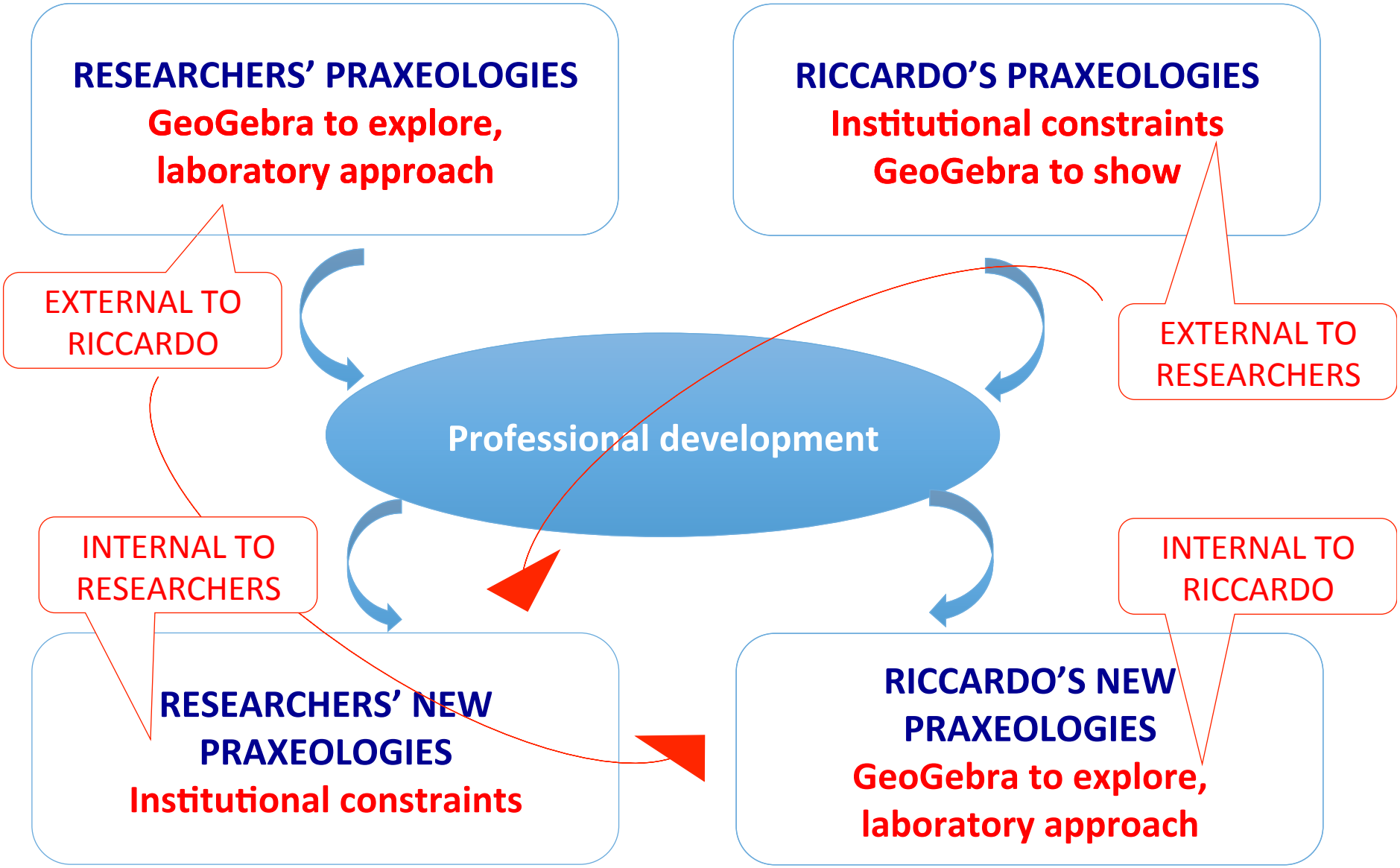
THE ERA OF THE TEACHER: Meta-Didactical Transposition framework

- Arzarello, Robutti, Malara, Cusi, Sabena, Martignone, Garuti, 2014
- Aldon, Arzarello, Robutti, Malara, Cusi, Sabena, Martignone, Garuti, Seury-Lavergne, PME 2013
- Clark-Wilson, Cusi, Goos, Robutti, Thomas, PME 2014

The META-DIDACTICAL TRANSPOSITION: a chance to observe evolution in teachers' and researchers' praxeologies towards shared praxeologies.



PRAXEOLOGIES COMPONENTS



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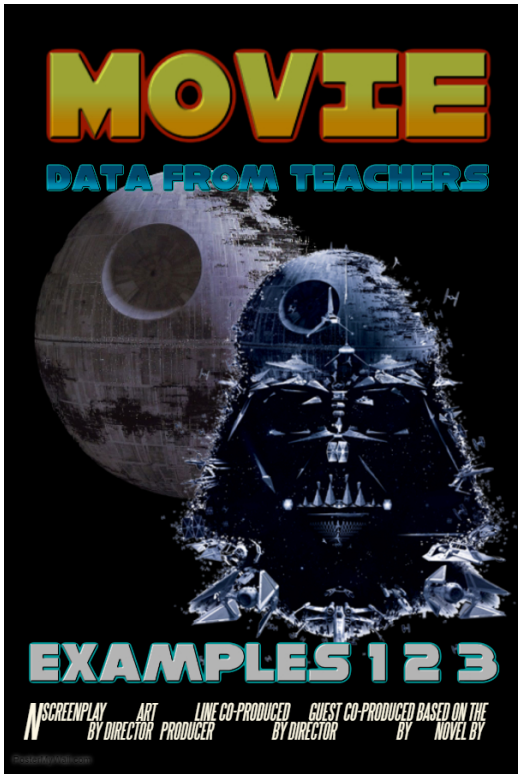


EXAMPLE 2

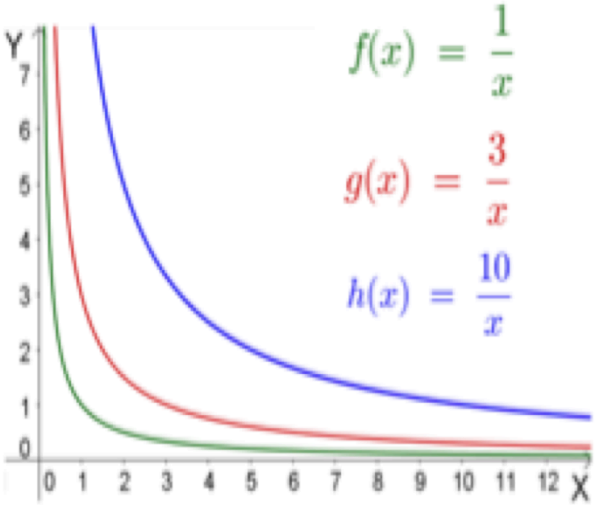
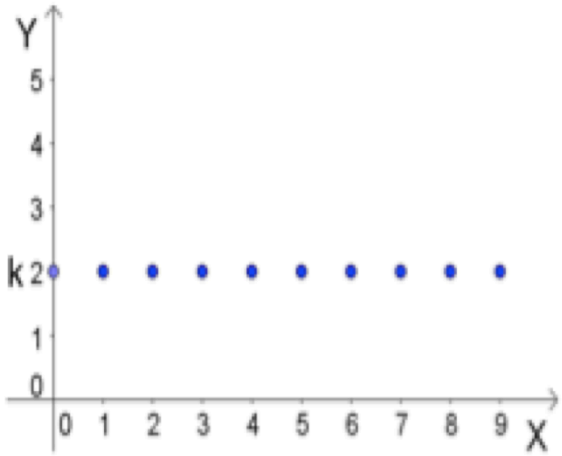
PROFESSIONAL DEVELOPMENT OF SECONDARY SCHOOL TEACHERS IN MERLO PROJECT – *STATIC* APPROACH

MERLO PROJECT: Meaning Equivalence Reusable Learning Objects

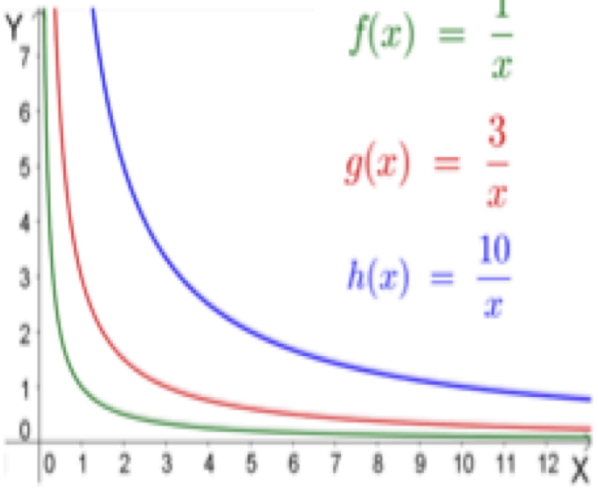
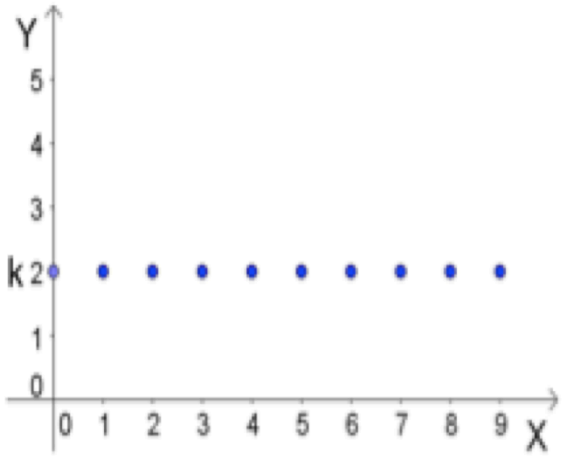
MERLO is a project of RESEARCH & TEACHER EDUCATION involving different countries

1. **Canada** (Shafrir, Etkind, Sinclair)
2. **Israel** (Kenett)
3. **Italy** (Robutti, Arzarello, Carante, Bini, Trinchero)
4. **Australia** (Prodromou)

Students' version: TASK

	A [ ]	B [ ]
1. Mark the statements (at least two) that share the same mathematical meaning. 2. Write the reasons that guided your choice.	$y = \frac{k}{x}$ k constant	Two inversely proportional quantities X and Y have fixed product
C [ ]	**D [ ]**	**E [ ]**
	$y = \frac{k}{x^2}$ k constant	

Students' version: SOLUTION

1. Mark the statements (at least two) that share the same mathematical meaning. 2. Write the reasons that guided your choice.	A [X] $y = \frac{k}{x}$ k constant	B [X] Two inversely proportional quantities X and Y have fixed product
C [X] 	D [ ] $y = \frac{k}{x^2}$ k constant	E [ ] 

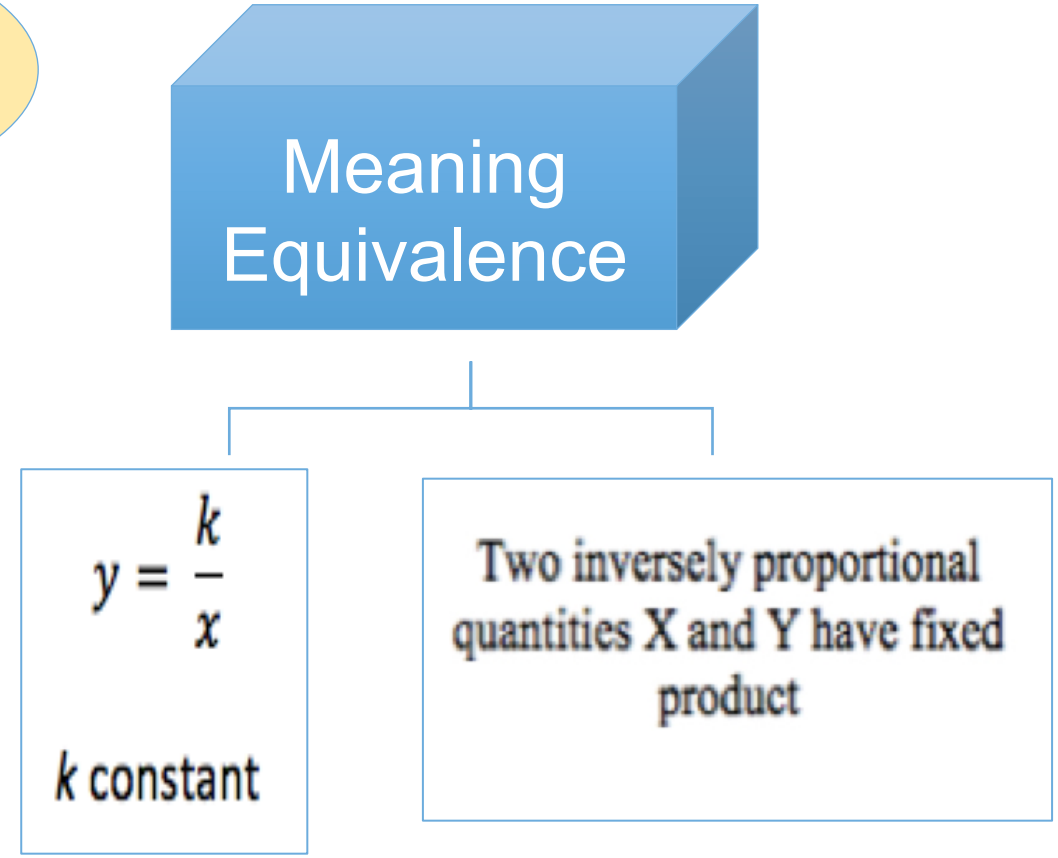
RESEARCH QUESTION 1

WHICH ARE THE TEACHERS' PRAXEOLOGIES (OR THEIR COMPONENTS) IN MERLO ITEM DESIGN THAT BECOME SHARED AMONG TEACHERS AND RESEARCHERS IN A PROFESSIONAL DEVELOPMENT PROGRAMME?

Paola Carante PhD thesis

MERLO item design: 1ST CRITERION

THEORETICAL COMPONENTS OF THE PRAXEOLOGY



Conversion
Duval, R. (2006)

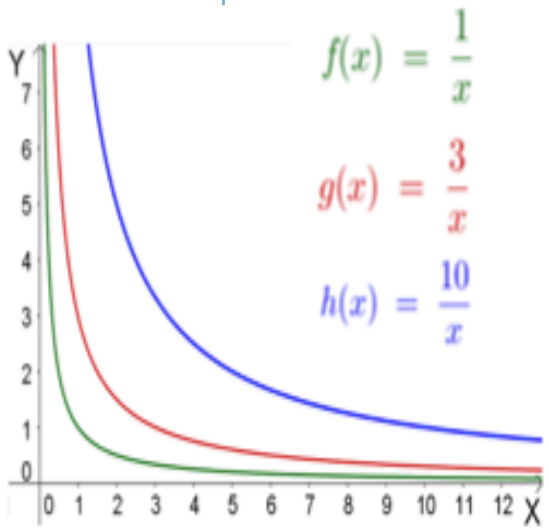
MERLO item design: 1ST CRITERION

THEORETICAL COMPONENTS OF THE PRAXEOLOGY

Meaning Equivalence

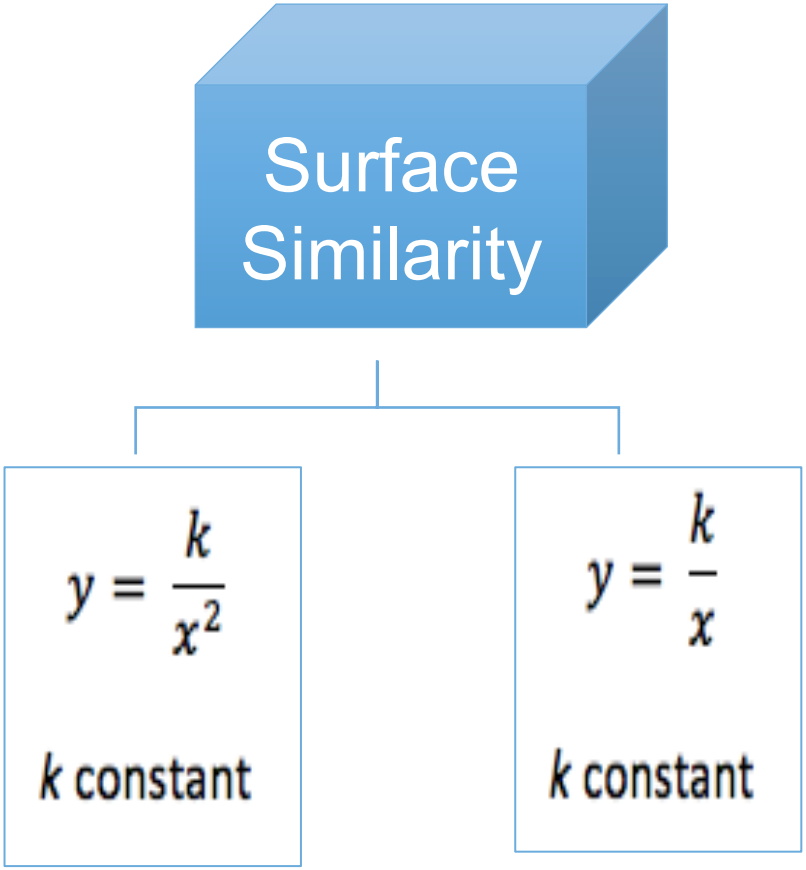
$$y = \frac{k}{x}$$

k constant



Conversion
Duval, R. (2006)

MERLO item design: 2ND CRITERION



THEORETICAL COMPONENTS OF THE PRAXEOLOGY

Designer's version

1. Mark the statements (at least two) that share the same mathematical meaning.
2. Write the reasons that guided your choice.

A [X]

Meaning equivalence

$$y = \frac{k}{x}$$

k constant

Shared Meaning
Inverse proportionality

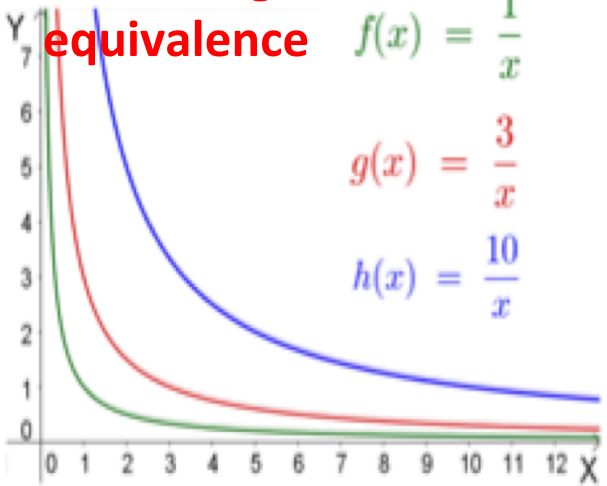
B [X]

Two inversely proportional quantities X and Y have fixed product

Meaning equivalence

C [X]

Meaning equivalence



D [ ]

Surface Similarity

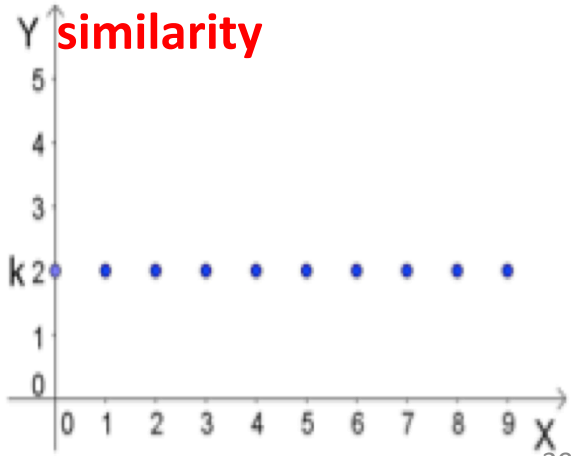
$$y = \frac{k}{x^2}$$

k constant

Distractor

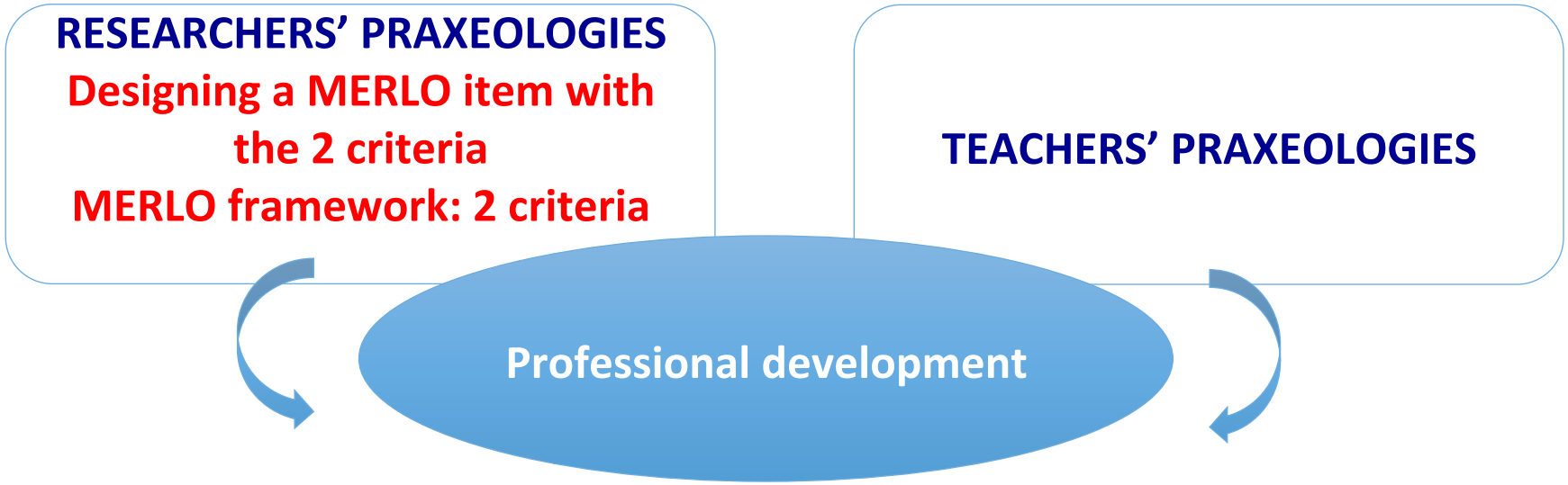
E [ ]

No meaning eq., no similarity



Distractor

PRAXEOLOGIES COMPONENTS



PRAXEOLOGIES COMPONENTS

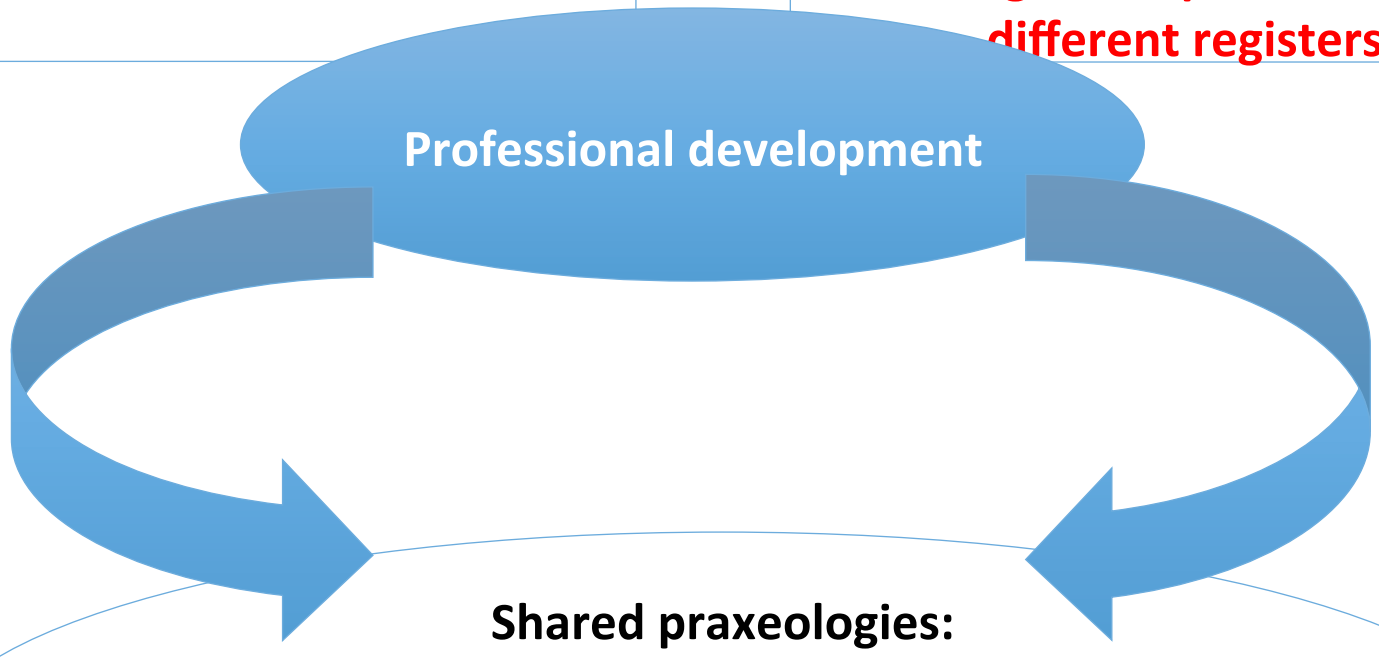
RESEARCHERS' PRAXEOLOGIES
Designing a MERLO item with the 2 criteria
MERLO framework: 2 criteria

TEACHERS' PRAXEOLOGIES
Choosing a question from national assessment,
Selecting five representations in different registers

PRAXEOLOGIES COMPONENTS

RESEARCHERS' PRAXEOLOGIES
Designing a MERLO item with the 2 criteria
MERLO framework

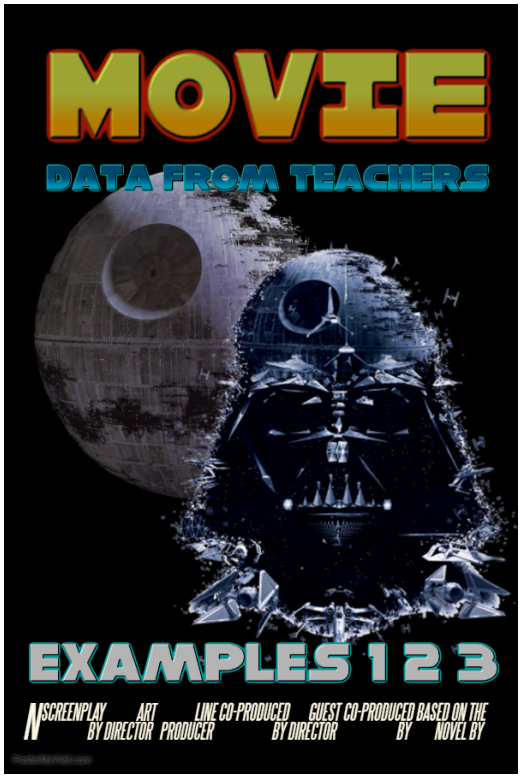
TEACHERS' PRAXEOLOGIES
Choosing a question from national assessment,
Selecting five representations in different registers



Shared praxeologies:
Choosing a question from national assessment, Selecting five representations in different registers, Designing MERLO items with the 2 criteria, using the framework

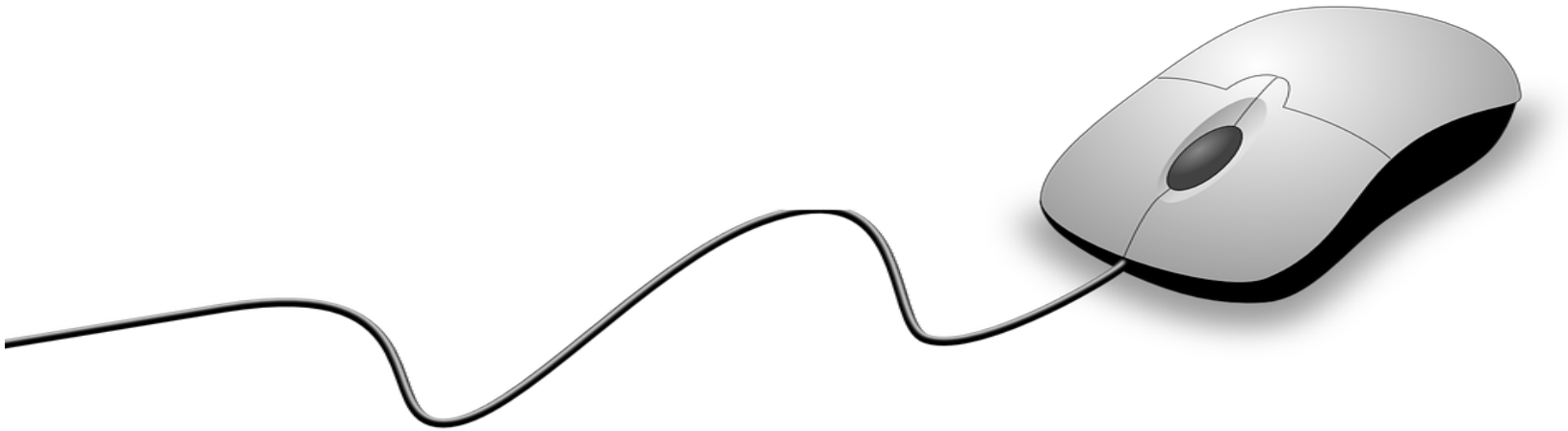
CONTENTS

1. PREQUEL
2. MOVIE: DATA FROM TEACHERS – EXAMPLE 3
3. SEQUEL



EXAMPLE 3

PROFESSIONAL DEVELOPMENT OF SECONDARY SCHOOL TEACHERS IN MERLO PROJECT – *DYNAMIC* APPROACH

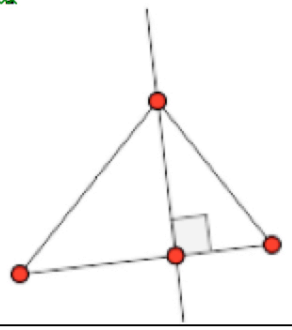
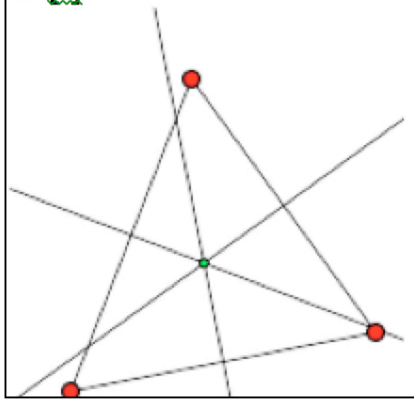
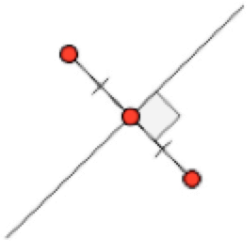


RESEARCH QUESTION 1 & 2

WHICH ARE THE TEACHERS' PRAXEOLOGIES (OR THEIR COMPONENTS) IN MERLO ITEM DESIGN THAT BECOME SHARED AMONG TEACHERS AND RESEARCHERS IN A PROFESSIONAL DEVELOPMENT PROGRAMME?

WHAT IS THE LEARNING OF TEACHERS WHILE WORKING IN COMMUNITY?

MERLO IN GEOMETRY (Robutti & Sinclair, 2017)

1. Mark the statements (at least 2) that share the same mathematical meaning. 2. Write the reasons that guided you in the choice.	A [ ] A perpendicular bisector of a segment AB is a line perpendicular to and passing through the midpoint of AB.	B [ ] 
C [ ] 	D [ ] The perpendicular bisector of A and B is the locus of all points equidistant to the given points A and B.	E [ ] 

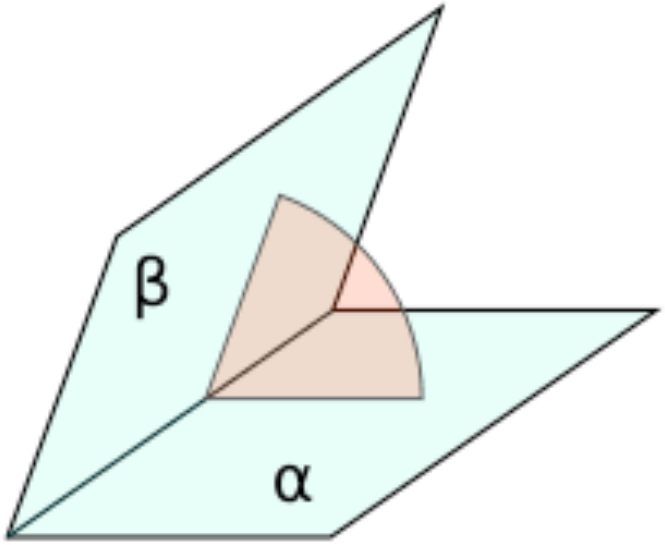
DESIGN:

- **Only linguistic and graphical registers**
- **A coordination of language and diagram**
- **Diagrams imply dimensional deconstruction**
- **Special notation: the sign for a right angle and for congruent segments.**

DYNAMIC MERLO ITEMS IN GEOMETRY WITH GEOGEBRA (Teachers' professional development, 2018, ONE GROUP)

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- **Choosing the concept in the institutional reference**
- **Designing the options in a static way**
- **Building the item in GG**
- **Re-designing the item**



NATIONAL CURRICULUM:

- **extension to space of some of the themes of plane geometry,**
- **reciprocal positions of lines and planes in space,**
- **parallelism and the perpendicularity,**
- **properties of the main geometric solids**

DIHEDRAL ANGLE: Angle between two planes (α, β, green) in a third plane (pink) which cuts the line of intersection at right angles

ONE GROUP: DIHEDRAL ANGLE

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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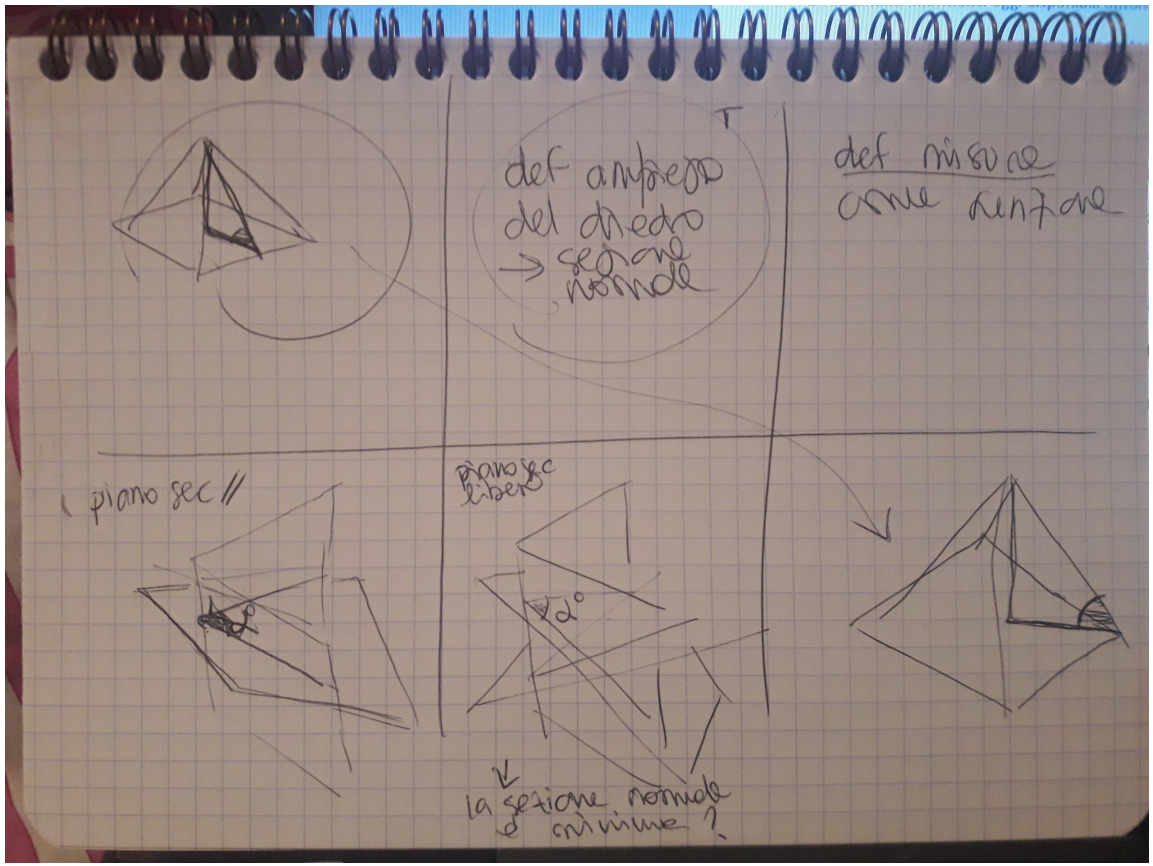

SOME OPTIONS

CHOOSING THE TARGET STATEMENT

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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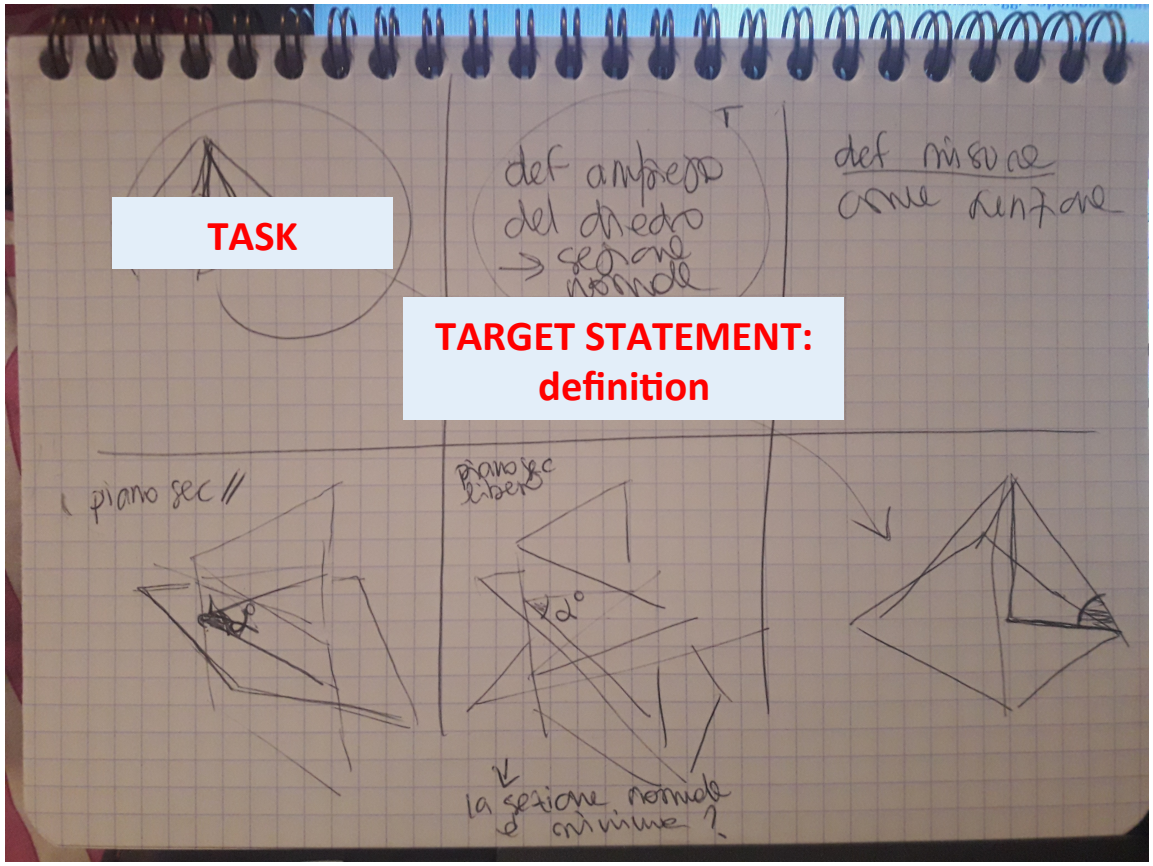

STRUCTURE OF MERLO ITEM

CHOOSING OTHER REPRESENTATIONS

STRUCTURE OF MERLO ITEM

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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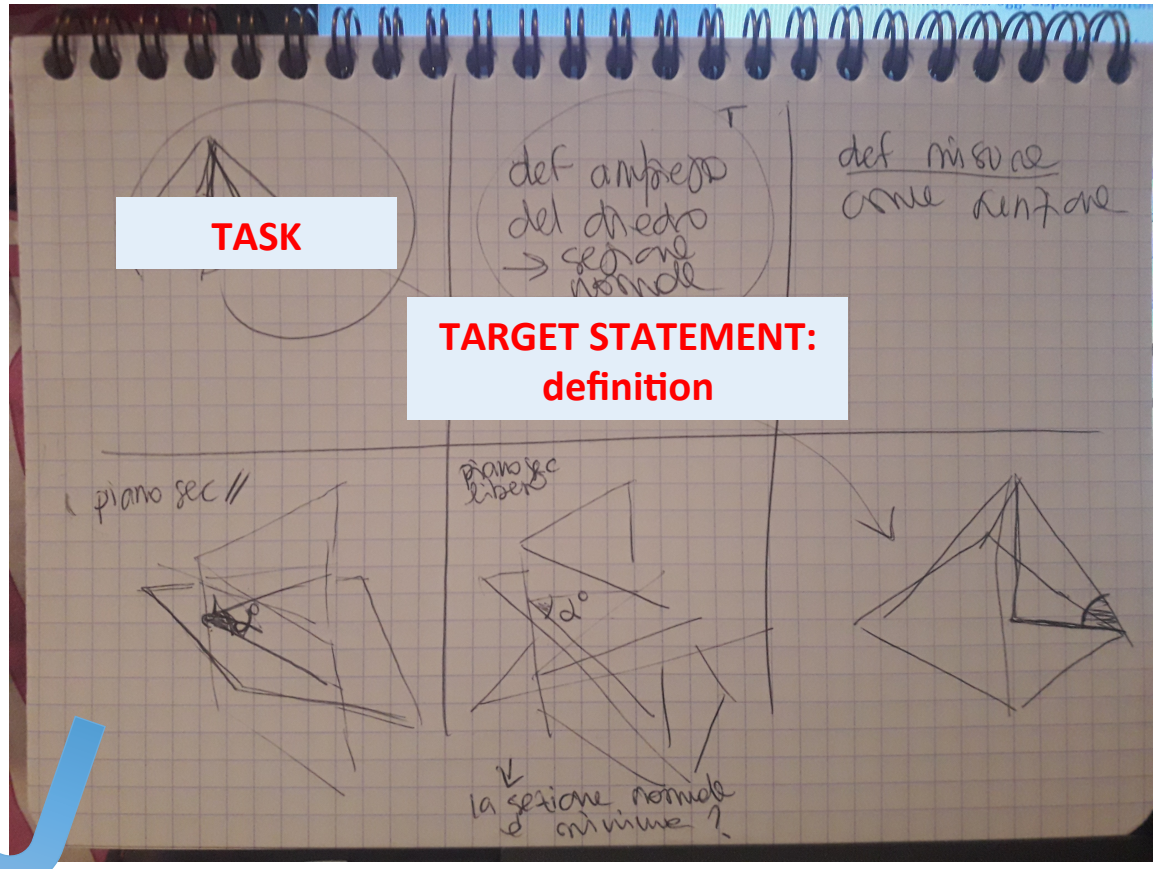

Normal planes intersecting the dihedral angle: the amplitude is fixed and matches the definition

CHOOSING OTHER REPRESENTATIONS

STRUCTURE OF MERLO ITEM

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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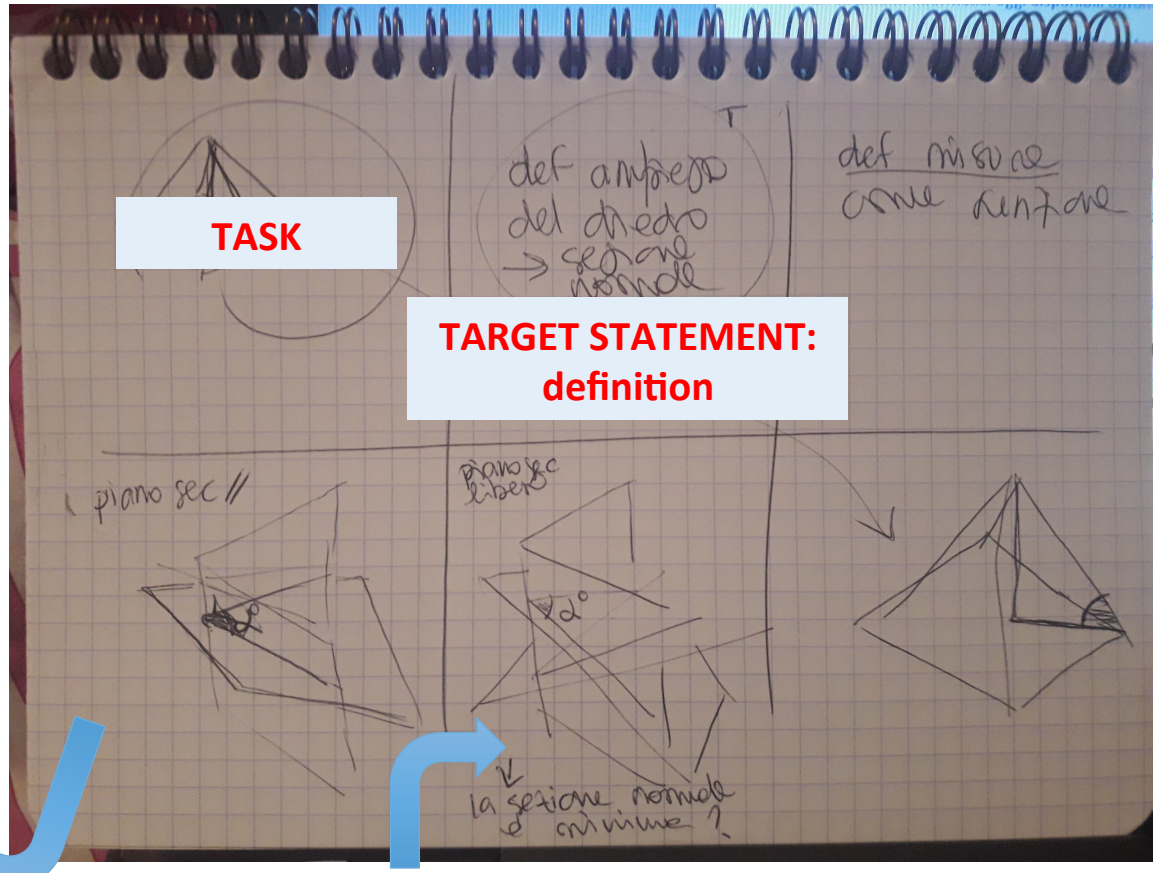

Normal planes intersecting the dihedral angle: the amplitude is fixed and matches the definition

Planes at varying inclinations intersecting the dihedral angle: the amplitude is not a fixed value

CHOOSING OTHER REPRESENTATIONS

STRUCTURE OF MERLO ITEM

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
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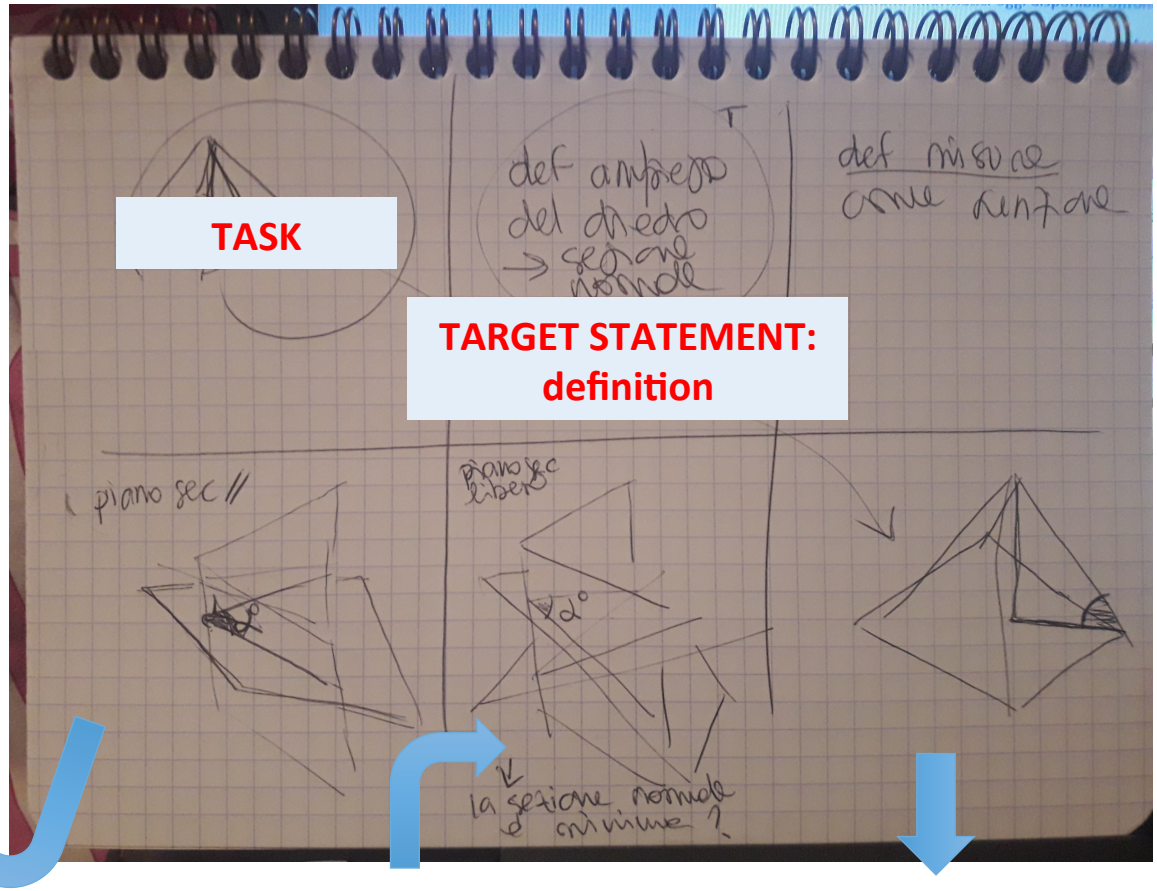

Normal planes intersecting the dihedral angle: the amplitude is fixed and matches the definition

Planes at varying inclinations intersecting the dihedral angle: the amplitude is not a fixed value

Distractor: the angle between edges and base of a pyramid is often mistaken for a dihedral angle

CHOOSING OTHER REPRESENTATIONS

STRUCTURE OF MERLO ITEM

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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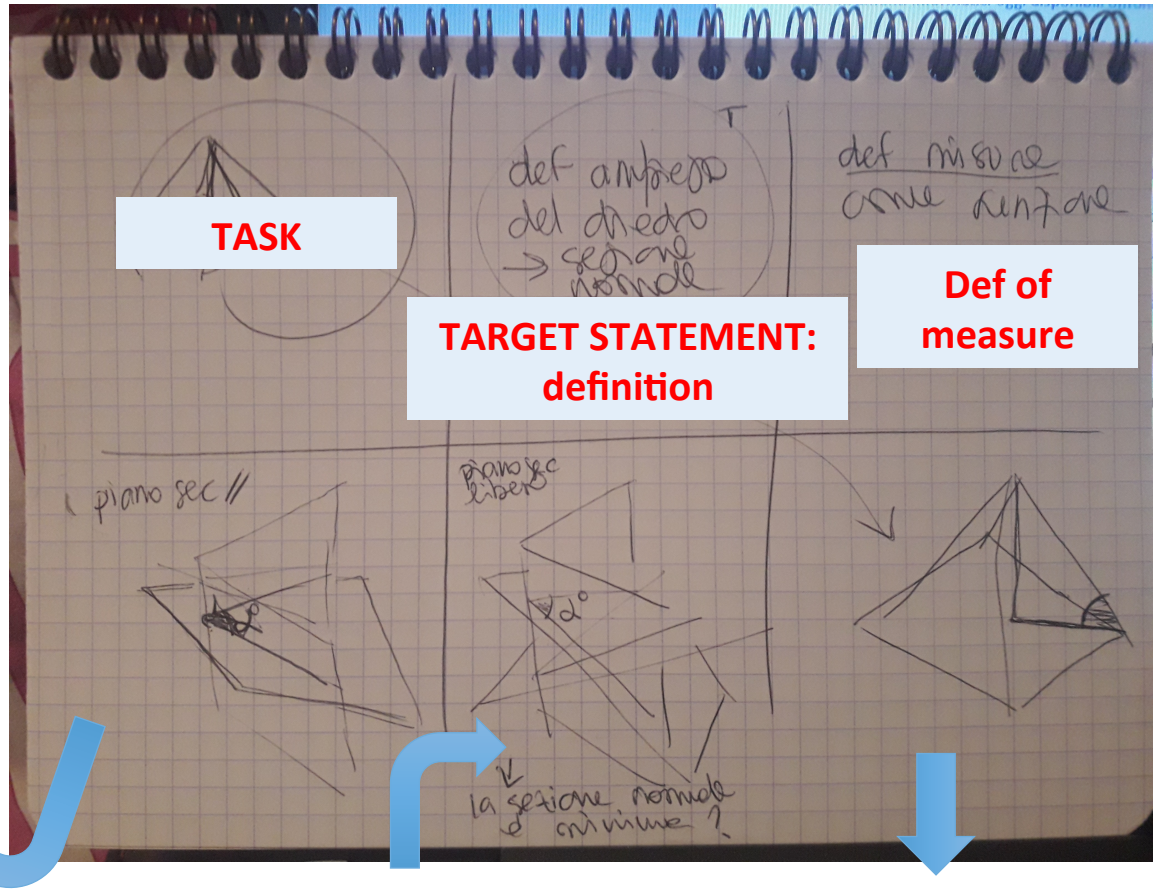


Normal planes intersecting the dihedral angle: the amplitude is fixed and matches the definition

Planes at varying inclinations intersecting the dihedral angle: the amplitude is not a fixed value

Distractor: the angle between edges and base of a pyramid is often mistaken for a dihedral angle

PRAXEOLOGIES COMPONENTS OF THE PREVIOUS EXPERIENCE OF PROFESSIONAL DEVELOPMENT

Shared praxeologies:

Choosing a question from national assessment, Selecting five representations in different registers, Designing MERLO items with the 2 criteria, using the framework

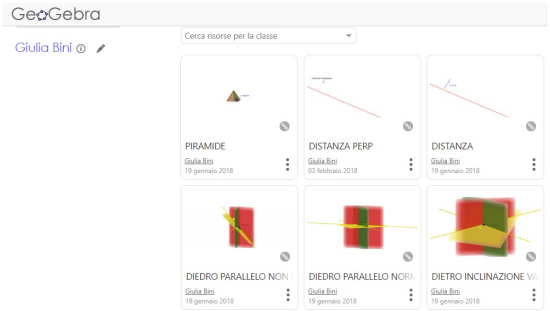
THE STARTING POINT FOR THESE TEACHERS IS THE ARRIVAL POINT OF THE PREVIOUS EXPERIENCE

IMPLEMENTING IN GEOGEBRA

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
- Designing the options in a static way
- **Building the item in GG**
- Re-designing the item

1 - Upload single dynamic objects to GeoGebratube



The problem of building and sharing MERLO items with dynamic Geogebra resources emerges and is solved with the help of one of the teachers of the team.

IMPLEMENTING IN GEOGEBRA

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
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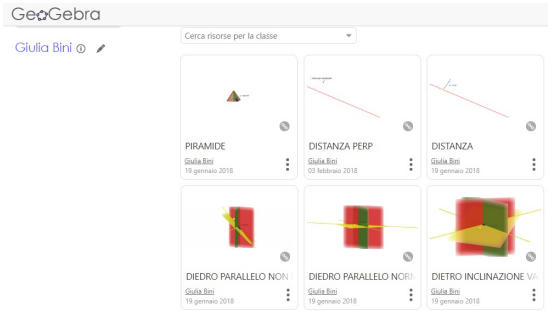
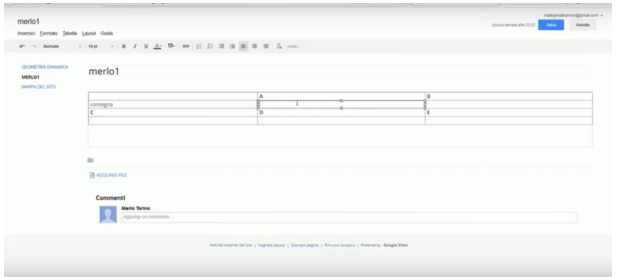


2 – Create the MERLO frame in a Google site



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IMPLEMENTING IN GEOGEBRA

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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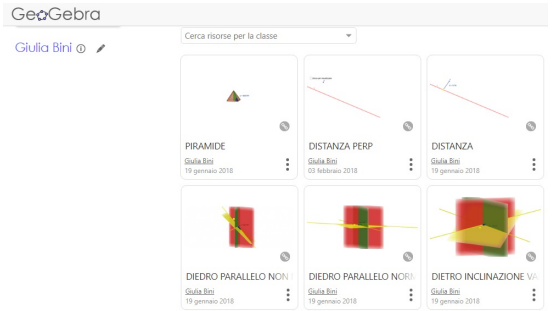
- Re-designing the item

One of the teachers as broker

1 - Upload single dynamic objects to GeoGebratube



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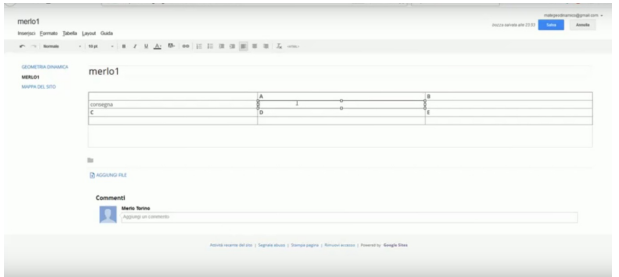
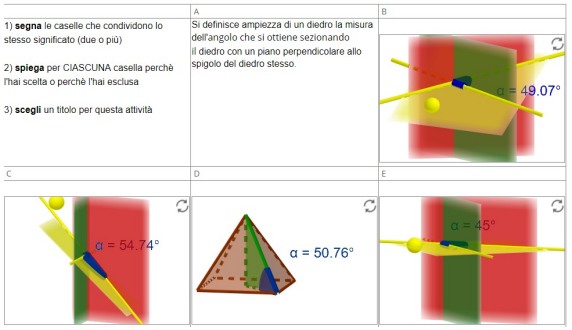


3 - Embed the dynamic objects using the HTLM interface



PRAXEOLOGIES COMPONENTS

RESEARCHERS' PRAXEOLOGIES
Previous shared praxeologies on the design of MERLO items
Observation of teachers' and students' processes

TEACHERS' PRAXEOLOGIES
Previous shared praxeologies on the design of MERLO items

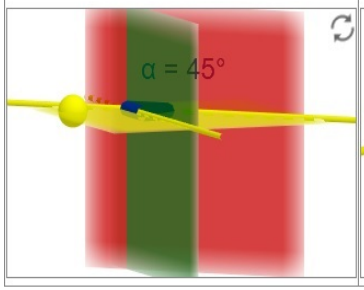
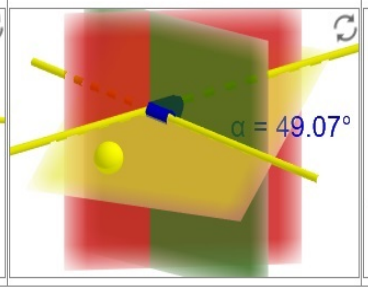
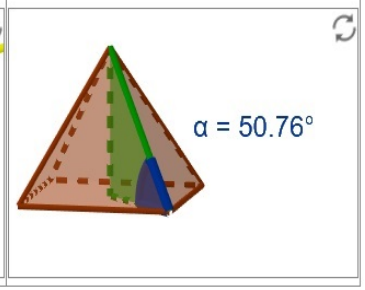
Shared praxeologies:

- **Building the item in GeoGebra**

FIRST DRAFT OF MERLO ITEM BY TEACHERS

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
- Designing the options in a static way
- **Building the item in GG**
- Re-designing the item

	A	B
1) **mark** the boxes that share the same meaning (two or more) 2) **explain** for EACH box why you chose it or why you excluded it 3) **choose** a title for this activity	The amplitude of a dihedral is defined as the measurement of the angle obtained by cutting the dihedral with a plane perpendicular to the edge of the dihedral.	A measure is defined as a function that assigns a real number to certain subsets of a given set to make quantitative the notion of their extension
C	D	E
		

A. definition of amplitude of a dihedral angle (TS)
B. definition of measure
C. 3D dynamic file with normal planes
D. 3D dynamic file with planes at varying inclinations
E. 3D dynamic file with pyramid

NEW KIND OF TASK BY RESEARCHERS

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
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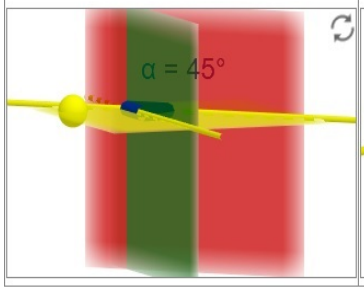
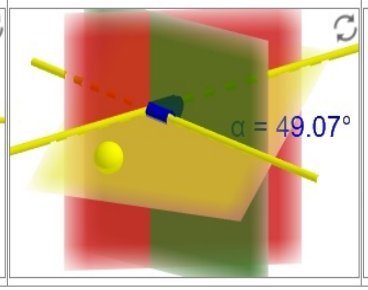
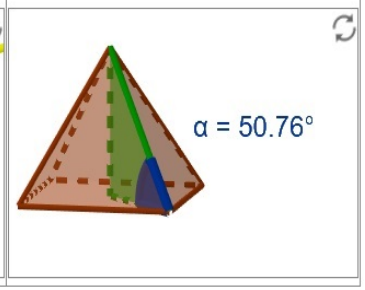
- **Building the item in GG**
- Re-designing the item

NEW TASK:
- FOR EACH BOX EXPLAIN WHY
- CHOOSE A TITLE

	B
e of a dihedral is defined as the t of the angle obtained by edral with a plane r to the edge of the dihedral.	A measure is defined as a function that assigns a real number to certain subsets of a given set to make quantitative the notion of their extension
	E
	
	

A. definition of amplitude of a dihedral angle (TS)
B. definition of measure
C. 3D dynamic file with normal planes
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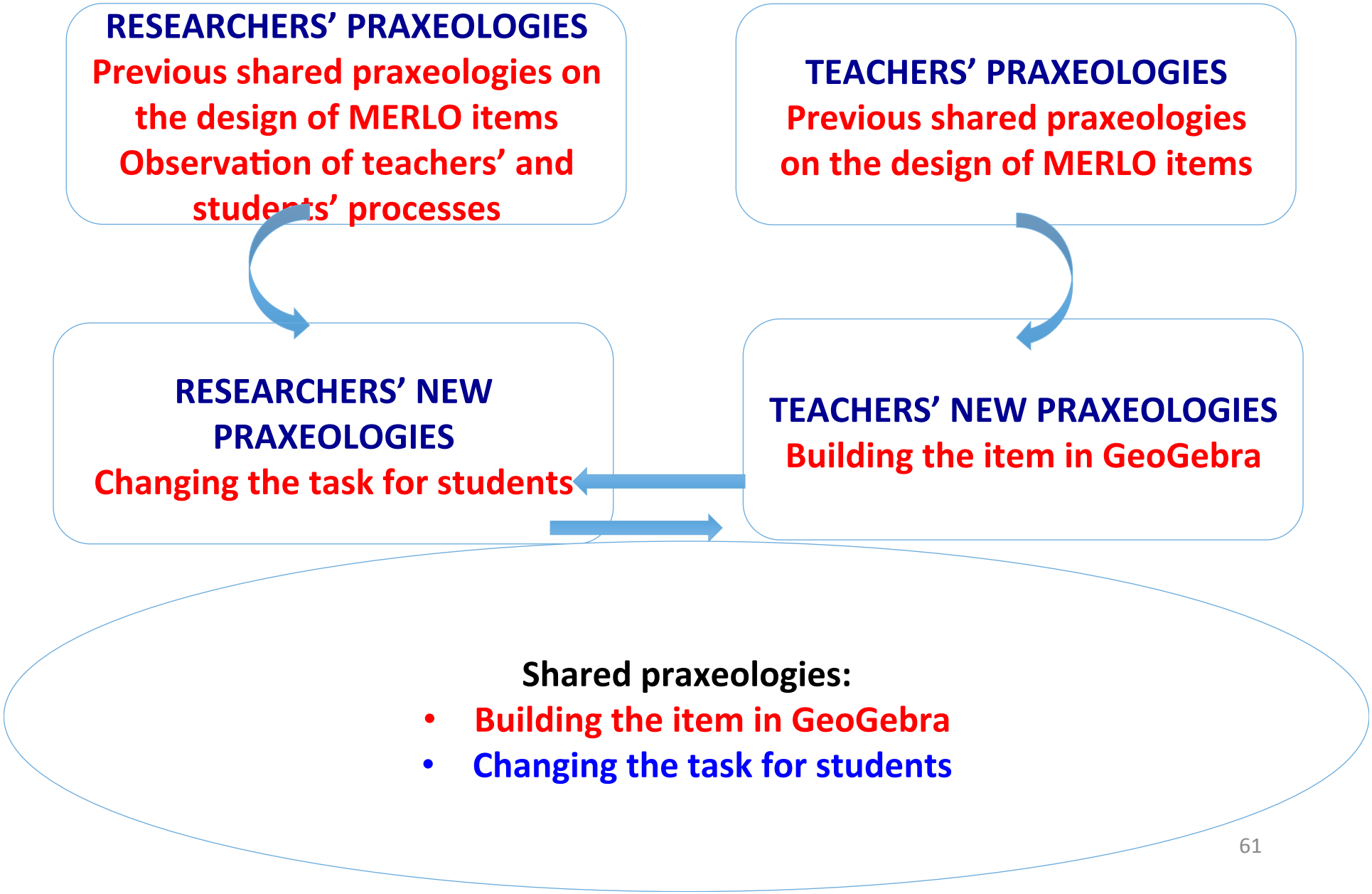
PRAXEOLOGIES COMPONENTS



FIRST DRAFT: STUDENTS' VERSION

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
- Designing the options in a static way
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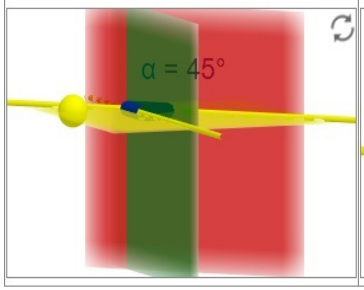
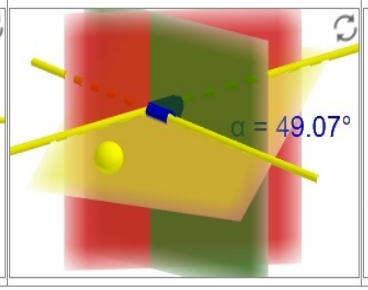
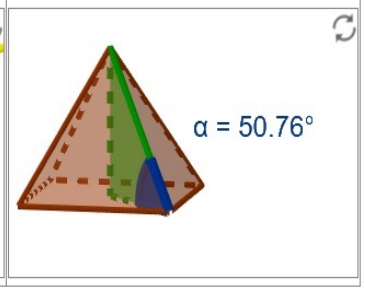
	A	B
1) **mark** the boxes that share the same meaning (two or more) 2) **explain** for EACH box why you chose it or why you excluded it 3) **choose** a title for this activity	The amplitude of a dihedral is defined as the measurement of the angle obtained by cutting the dihedral with a plane perpendicular to the edge of the dihedral.	A measure is defined as a function that assigns a real number to certain subsets of a given set to make quantitative the notion of their extension
C	D	E
		

A. definition of amplitude of a dihedral angle (TS)
B. definition of measure
C. 3D dynamic file with normal planes
D. 3D dynamic file with planes at varying inclinations
E. 3D dynamic file with pyramid

FIRST DRAFT: STUDENTS' VERSION

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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	A X	B
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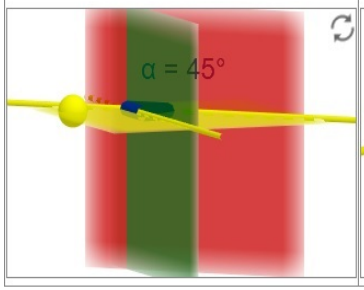	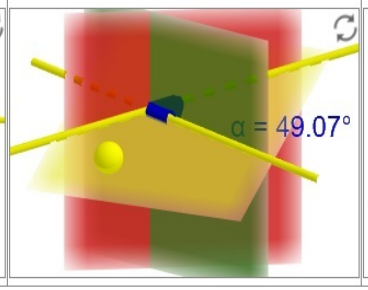	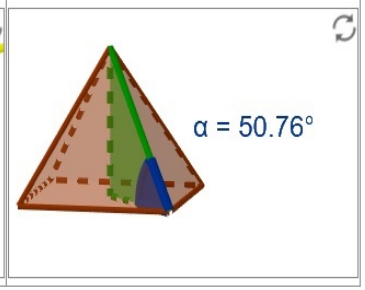

A. definition of amplitude of a dihedral angle (TS)
B. definition of measure
C. 3D dynamic file with normal planes
D. 3D dynamic file with planes at varying inclinations
E. 3D dynamic file with pyramid

FIRST DRAFT: STUDENTS' VERSION

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
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	A X	B X ?
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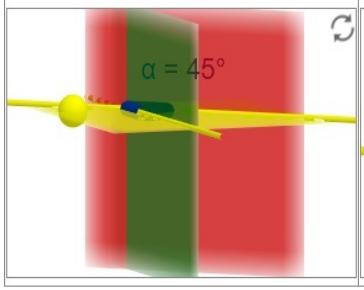
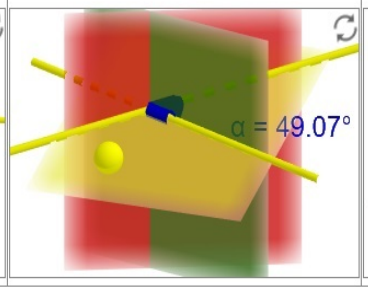
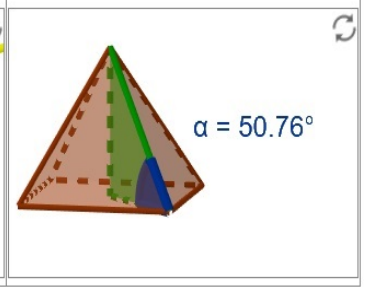
		

A. definition of amplitude of a dihedral angle (TS)
B. definition of measure
C. 3D dynamic file with normal planes
D. 3D dynamic file with planes at varying inclinations
E. 3D dynamic file with pyramid

DISCUSSION BETWEEN TWO TEACHERS

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
- Designing the options in a static way
- **Building the item in GG**
- Re-designing the item

GERMANA: then the item is not "dihedral", it is "measure in the plane", a measure in the plane and a measure in space

GIULIA: it's "measure"

GERMANA: it is "measure" so it is no longer "dihedral"

GIULIA: yes yes I understand what you are saying

GERMANA: if I'm working in 3D, we're here concentrate on the dihedral concept ... insert a box in which we have an example in 2d in which we have the distance from a point to a line and again the concept of perpendicular …

GIULIA: there is the need to formulate the definition of measure so that it is univocal

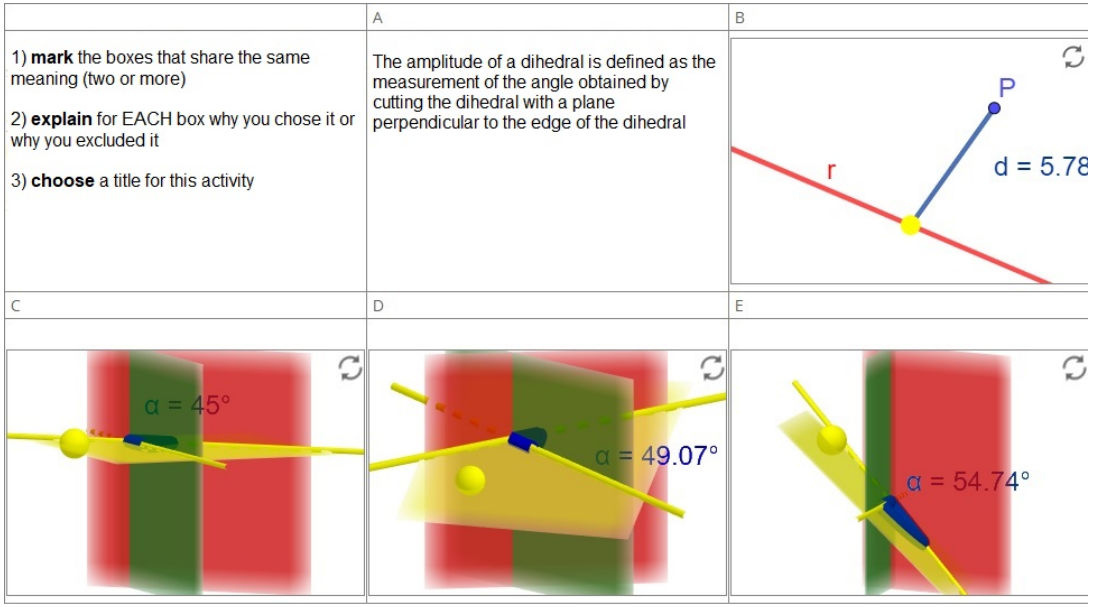
GERMANA: yes, but if we are working on the dihedral my doubt is: if I insert this element can it confuse?

GIULIA: it could, however, in my experience if you can reconnect the 3D concepts to those in 2D with which they are familiar, they can extend to the third dimension more easily.

SECOND DRAFT: STUDENTS' VERSION

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
- Designing the options in a static way
- **Building the item in GG**
- Re-designing the item

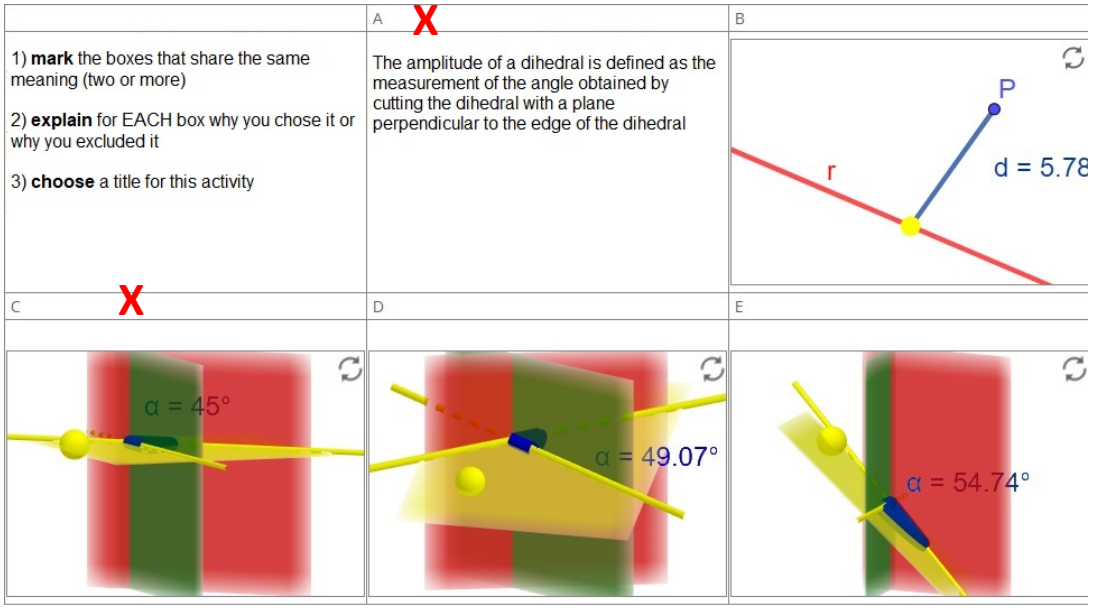


A. definition of amplitude of a dihedral angle (TS)
B. measure of the distance from a point to a line
C. 3D dynamic file with normal planes
D. 3D dynamic file with planes at varying inclinations
E. 3D dynamic file with planes at fixed inclination (not normal)

SECOND DRAFT: STUDENTS' VERSION

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
- Designing the options in a static way
- **Building the item in GG**
- Re-designing the item



A. definition of amplitude of a dihedral angle (TS)
B. 2D dynamic file with segment connecting a point to a line at varying inclinations
C. 3D dynamic file with normal planes
D. 3D dynamic file with planes at varying inclinations
E. 3D dynamic file with planes at fixed inclination (not normal)

SPLITTING THE ITEM INTO 2 ITEMS

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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Dihedral angle

Measure

DISCUSSING...

https://youtu.be/2p9hAo1qSik?t=37s

GIULIA: Because initially we had thought to do only two dynamic figures three-dimensional one with the two planes sectioned with a plane with variable inclination and the second with two planes sectioned by a plane... let's say a bundle of parallel perpendicular planes, but then we thought of adding the third possibility: of the planes with fixed inclination, therefore a bundle of planes all parallel, but not perpendicular and therefore increase the complexity.

If I cut with a bundle of planes with variable inclination I get so many amplitudes - all different - and therefore I can not correctly attribute a measure to that; if I cut with planes that are all parallel but not orthogonal to the edge I get only one measure but I have to let them understand that it is not good; and then the third element is dihedral, a bundle of planes all parallel, but finally orthogonal to the corner and to see that this amplitude is that of the definition.

RESULT OF THE SPLITTING: 1st ITEM

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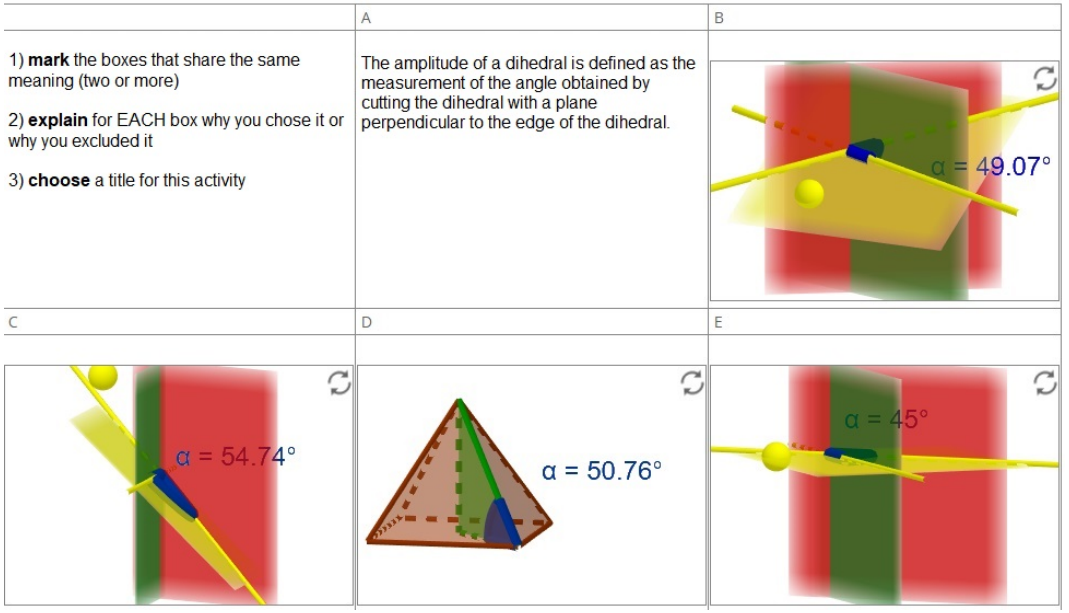
Dihedral angle



A. definition of amplitude of a dihedral angle (TS)
B. 3D dynamic file with planes at varying inclinations
C. 3D dynamic file with planes at fixed inclination (not normal)
D. 3D dynamic file with pyramid
E. 3D dynamic file with normal planes

RESULT OF THE SPLITTING: 1st ITEM

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
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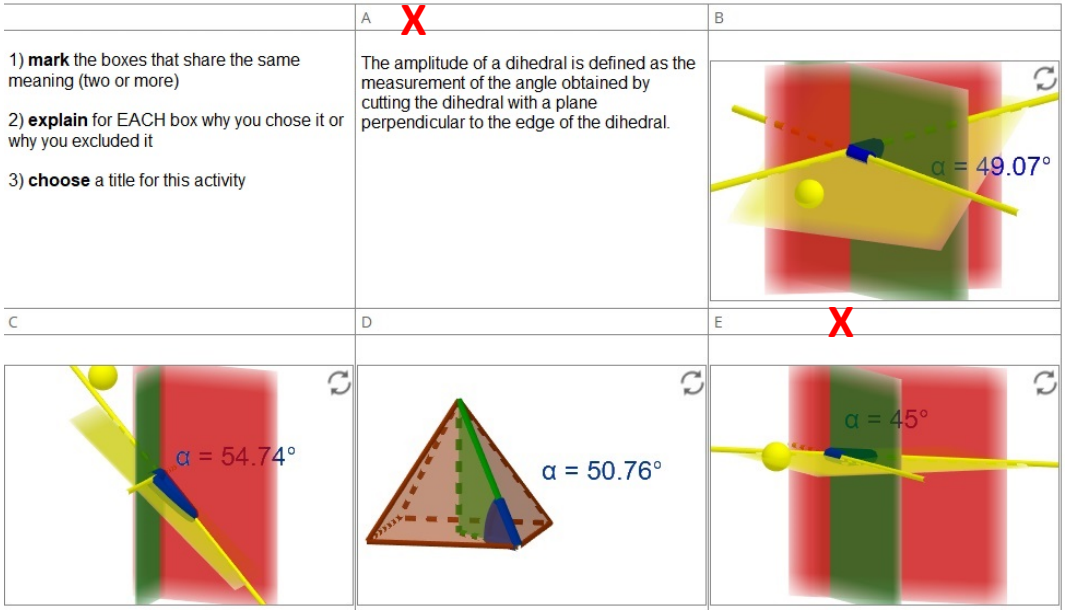
- Re-designing the item

Dihedral angle



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B. 3D dynamic file with planes at varying inclinations
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E. 3D dynamic file with normal planes

RESULT OF THE SPLITTING: 2nd ITEM

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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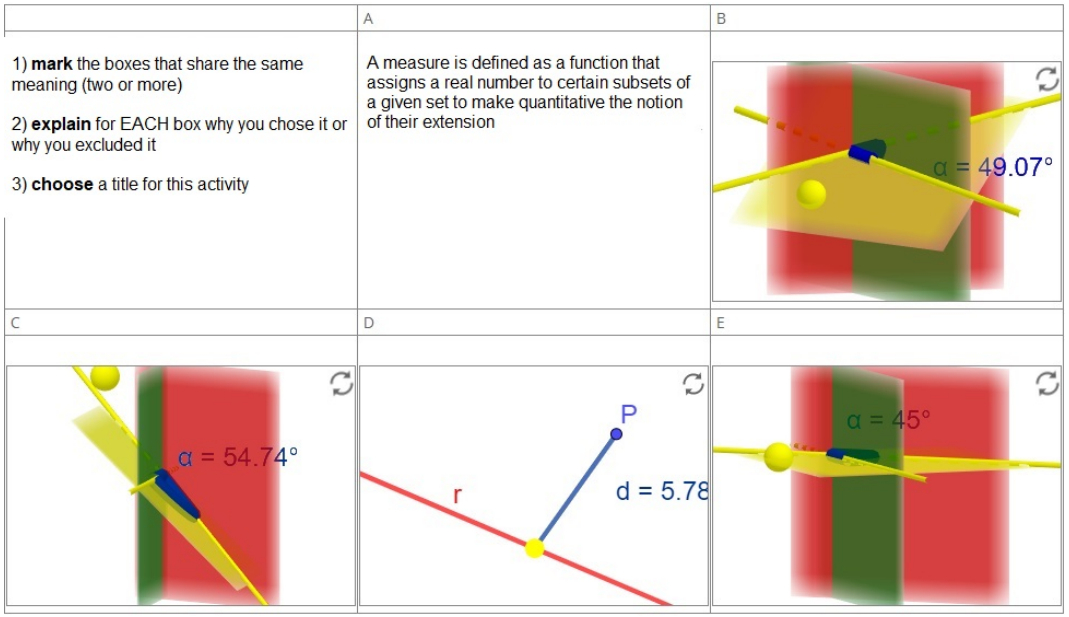
- Designing the options in a static way
- **Building the item in GG**
- Re-designing the item

Measure



A. definition of measure (TS)
B. 3D dynamic file with planes at varying inclinations
C. 3D dynamic file with planes at fixed inclination (not normal)
D. 2D dynamic file with segment connecting a point to a line at varying inclinations
E. 3D dynamic file with normal planes

RESULT OF THE SPLITTING: 2nd ITEM

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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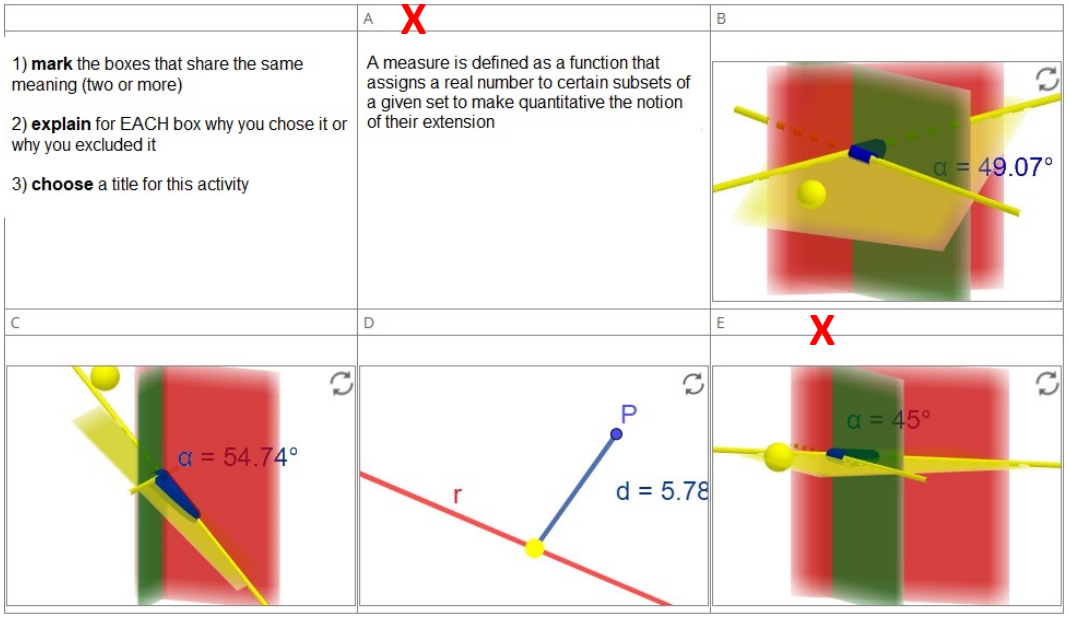
Measure



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THE CHOICE OF A NEW DEFINITION OF MEASURE

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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- **Re-designing the item**

ITEM MEASURE: NEW VERSION

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

- Choosing the concept in the institutional reference
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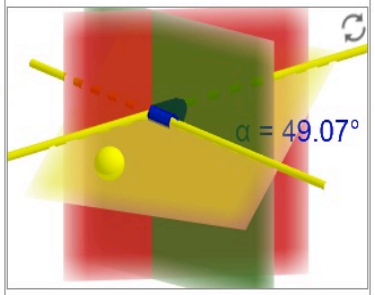
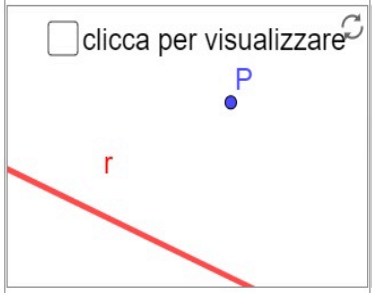
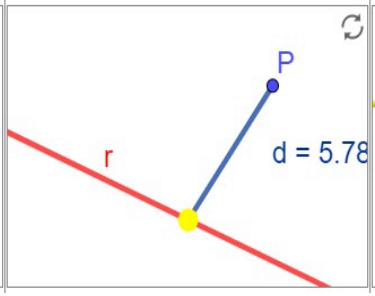
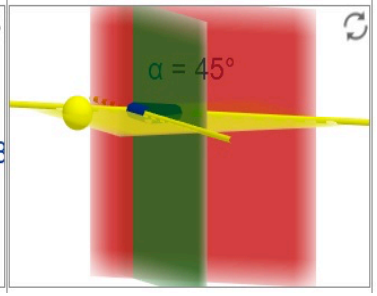
- Building the item in GG
- **Re-designing the item**

Measure

	A	B
1) **segna** le caselle che condividono lo stesso significato (due o più) 2) **spiega** per CIASCUNA casella perchè l'hai scelta o perchè l'hai esclusa 3) **scegli** un titolo per questa attività	Si definisce misura una funzione che assegna un numero reale a taluni sottoinsiemi di un dato insieme per rendere quantitativa la nozione della loro estensione.	
C	**D**	**E**
		

A. A new definition of measure (TS)
B. 3D dynamic file with planes at varying inclinations
C. 2D dynamic file with a point and a line (distance)
D. 2D dynamic file with segment connecting a point to a line at varying inclinations
E. 3D dynamic file with normal planes

ITEM MEASURE: NEW VERSION

DESIGN OF MERLO ITEMS IN GEOMETRY WITH GEOGEBRA:

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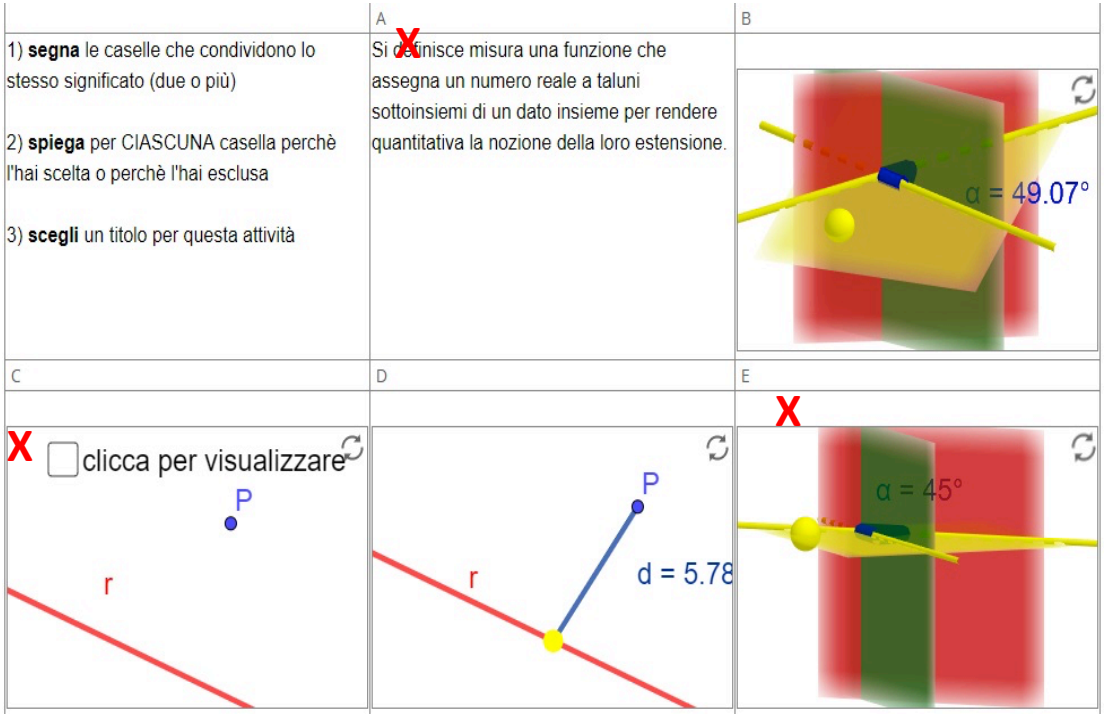

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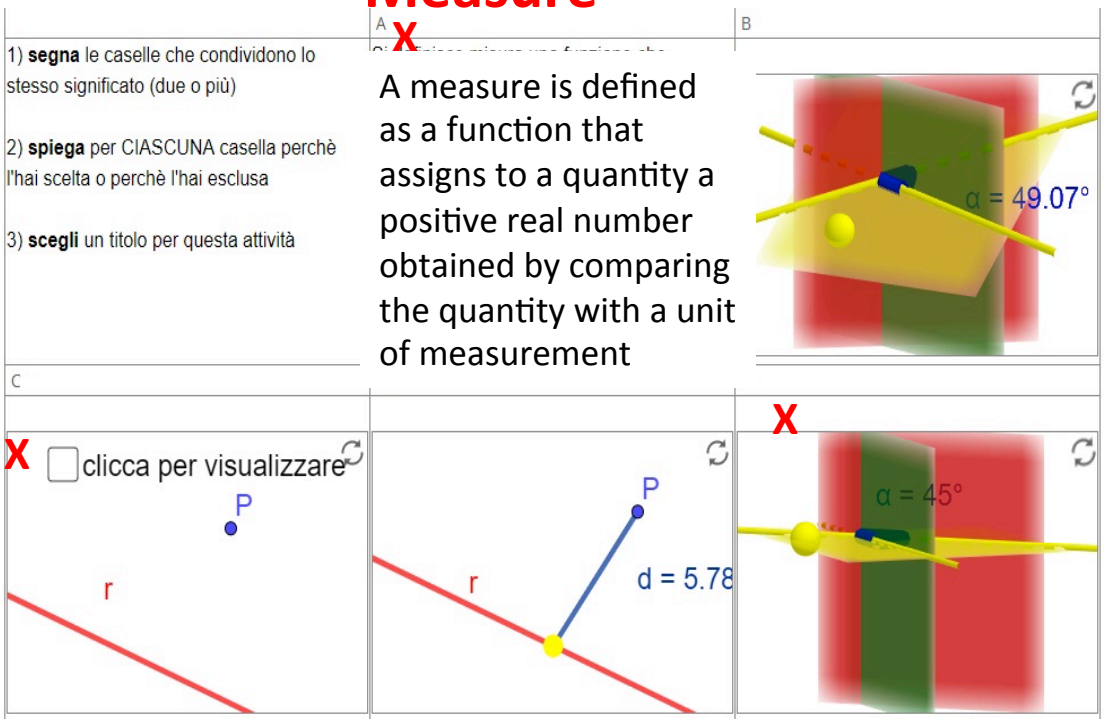

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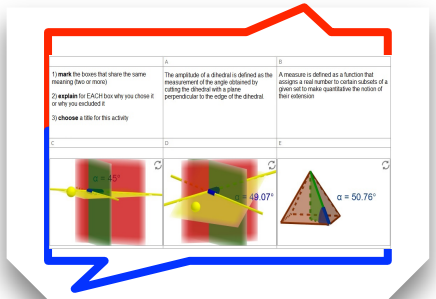
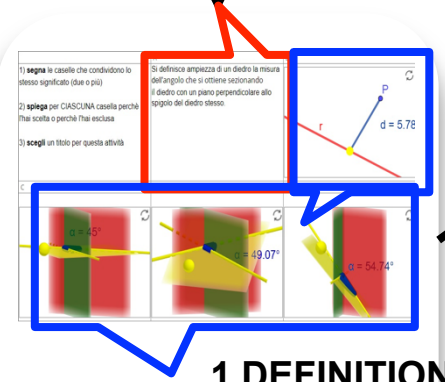
PROCESS OF DESIGN

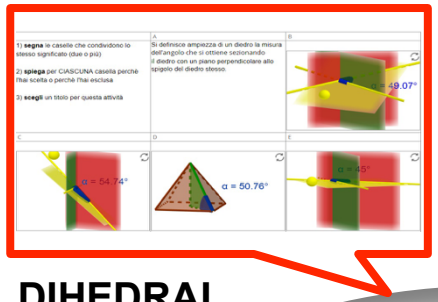
FIRST DRAFT
1 ITEM



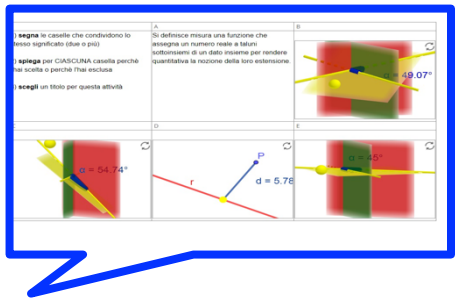
2 DEFINITIONS
3 DIAGRAMS



1 DEFINITION
4 DYNAMIC DIAGRAMS



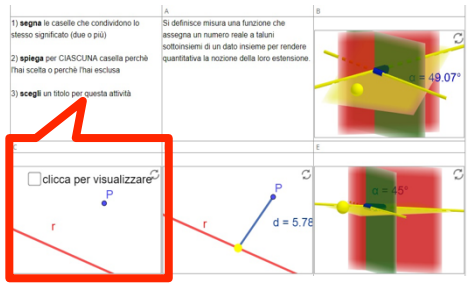
DIHEDRAL ANGLE



MEASURE

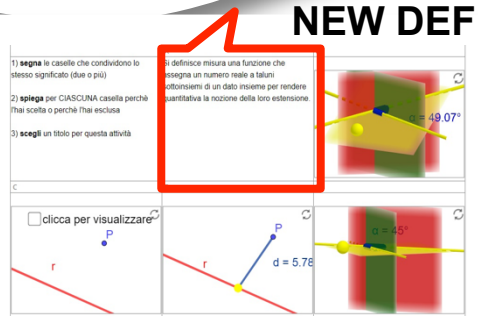
FINAL VERSION:
2 ITEMS

- **1 definition**
- **4 dynamic diagrams**

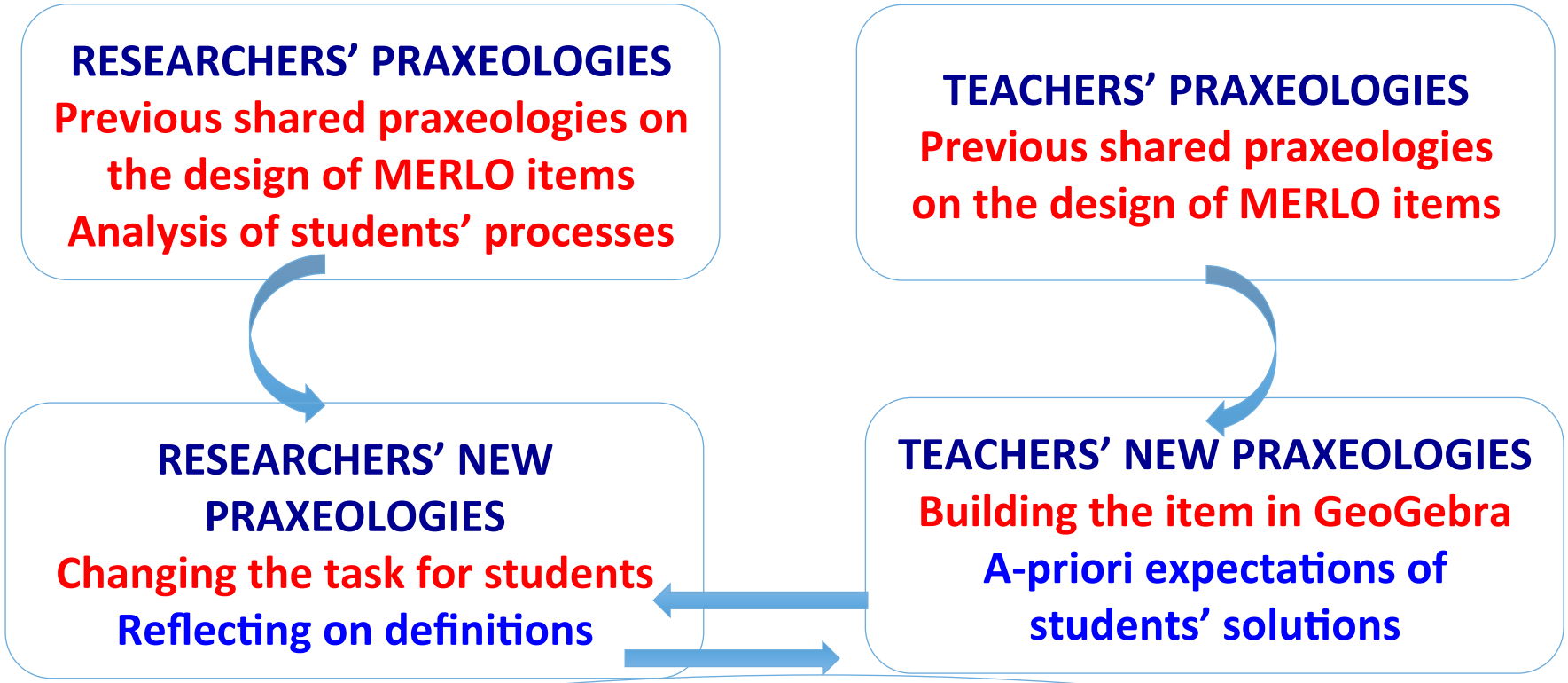


LINKING 2D & 3D MEASURES

NEW DEF



PRAXEOLOGIES COMPONENTS



Shared praxeologies:

- **Building the item in GeoGebra (TASK, TECHNIQUE)**
- **Changing the task for students (TECHNIQUE, JUSTIFICATION)**
- **Reflecting on definitions (JUSTIFICATION, THEORY - MAT)**
- **A-priori expectations of students' solutions (THEORY– MAT EDU)**
- **Analysis of students' processes (THEORY – MAT EDU)**

STUDENTS' SOLUTIONS

THE STORY MUST GO ON …

- **Teaching experiments in classroom environments**

GIULIA BINI presentation on Thursday at 9.00

RESULT OF THE SPLITTING: 2nd ITEM NEW-NEW-NEW VERSION

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- Re-designing the item
- **Re-re-designing the item**

IN TERMS OF LEARNING OF THE TEACHERS WORKING TOGETHER AND WITH RESEARCHERS…

MATHEMATICS

- Reflecting on definitions: dihedral angle, measure

USE OF GEOGEBRA

- Building the item in GeoGebra (Google site, GeoGebratube, HTLM interface)

DIDACTICS

- Task for students
- Dynamic MERLO item

RESEARCH

- A-priori expectations of students' solutions
- Analysis of students' processes (and also of teachers)

IN TERMS OF LEARNING OF THE TEACHERS WORKING TOGETHER AND WITH RESEARCHERS…

MATHEMATICS

- **Reflecting on definitions: dihedral angle**

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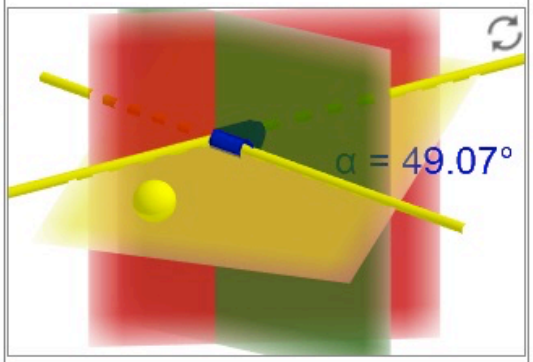
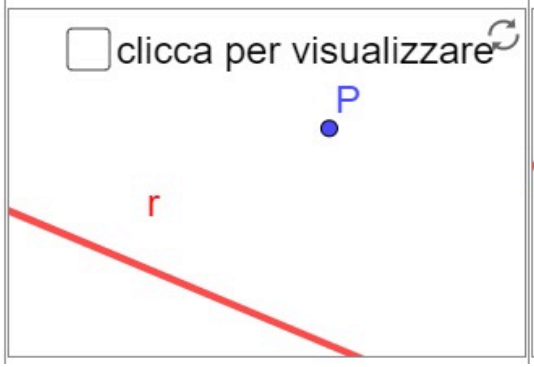
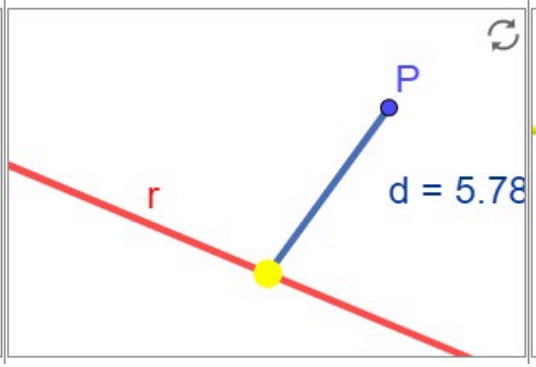
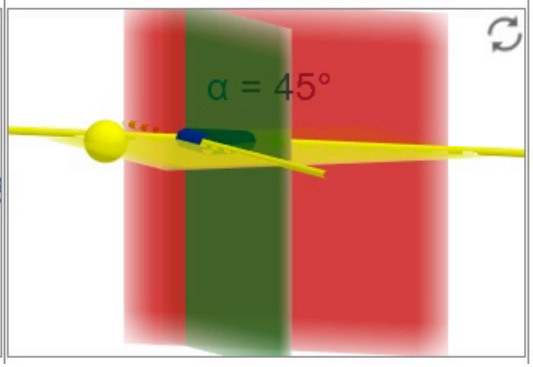
1) **segna** le caselle che condividono lo stesso significato (due o più) 2) **spiega** per CIASCUNA casella perchè l'hai scelta o perchè l'hai esclusa 3) **scegli** un titolo per questa attività	Si definisce misura una funzione che assegna un numero reale a taluni sottoinsiemi di un dato insieme per rendere quantitativa la nozione della loro estensione.	
C	D	E
		

IN TERMS OF LEARNING OF THE TEACHERS WORKING TOGETHER AND WITH RESEARCHERS…

MATHEMATICS

- **Reflecting on definitions: measure**

A measure is defined as a function that assigns a real number to certain subsets of a given set to make quantitative the notion of their extension

A measure is defined as a function that assigns to a quantity a positive real number obtained by comparing the quantity with a unit of measurement

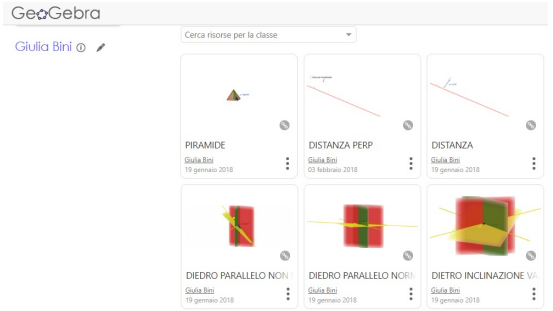
Measure or invariant?

IN TERMS OF LEARNING OF THE TEACHERS WORKING TOGETHER AND WITH RESEARCHERS…

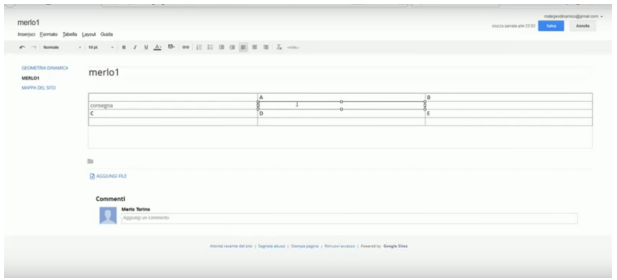
USE OF GEOGEBRA

- **Building the item in GeoGebra (Google site, GeoGebratube, HTLM interface)**

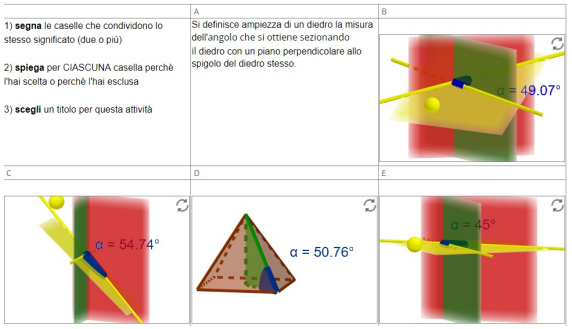
1 - Upload single dynamic objects to GeoGebratube



2 – Create the MERLO frame in a Google site



3 - Embed the dynamic objects using the HTLM interface



IN TERMS OF LEARNING OF THE TEACHERS WORKING TOGETHER AND WITH RESEARCHERS…

DIDACTICS

- **Task for students**

1. Mark the statements (at least two) that share the same mathematical meaning.
2. Write the reasons that guided your choice.

- **Dynamic MERLO item**

1. Mark the statements (at least two) that share the same mathematical meaning.
2. For each statements, explain the reasons for having chosen it or not.
3. Give a title to the item.

In which ways using dynamic representations in the design of MERLO items, to support students' solutions?

IN TERMS OF LEARNING OF THE TEACHERS WORKING TOGETHER AND WITH RESEARCHERS…

RESEARCH

- **A-priori expectations of students' solutions**

  In terms of mathematical learning, but not only: also discussing, arguing, reflecting on meaning and representations.

- **Analysis of students' processes (and also of teachers)**

  1. Teachers learn to observe students' processes.
  2. Teachers learn from students' approaches and re-design the item.
  3. Teachers learn to do research.

GIULIA BINI presentation on Thursday at 9.00

CONTENTS

TEACHERS' LEARNING

OPEN PROBLEM FOR RESEARCHERS:

HOW TO OBSERVE/MEASURE/ANALYSE TEACHERS' LEARNING?

TASKS
INTERVIEWS
QUESTONNAIRES
OBSERVATIONS IN THEIR CLASSROOMS
…

BOUNDARY OBJECT

RESEARCH QUESTIONS

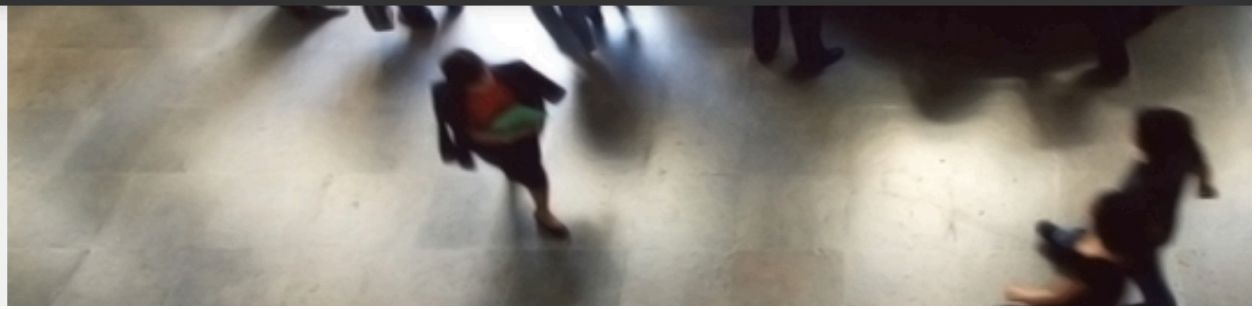
1. CAN MERLO ITEM BE CONSIDERED A **BOUNDARY OBJECT** ACROSS THE COMMUNITIES OF RESEARCHERS AND THAT OF TEACHERS? WITH WHICH STRUCTURE?

2. HOW CAN WE DESCRIBE THE EVOLUTION OVER TIME OF THE MERLO BOUNDARY OBJECT, THROUGH **BOUNDARY ENCOUNTERS**?

3. WHICH **INTERACTIONS,** WHICH **PRAXEOLOGIES** SUPPORT THIS EVOLUTION?

UC.PT

CADGME2018



Thank you