The challenges schools face with the development of Computer Algebra

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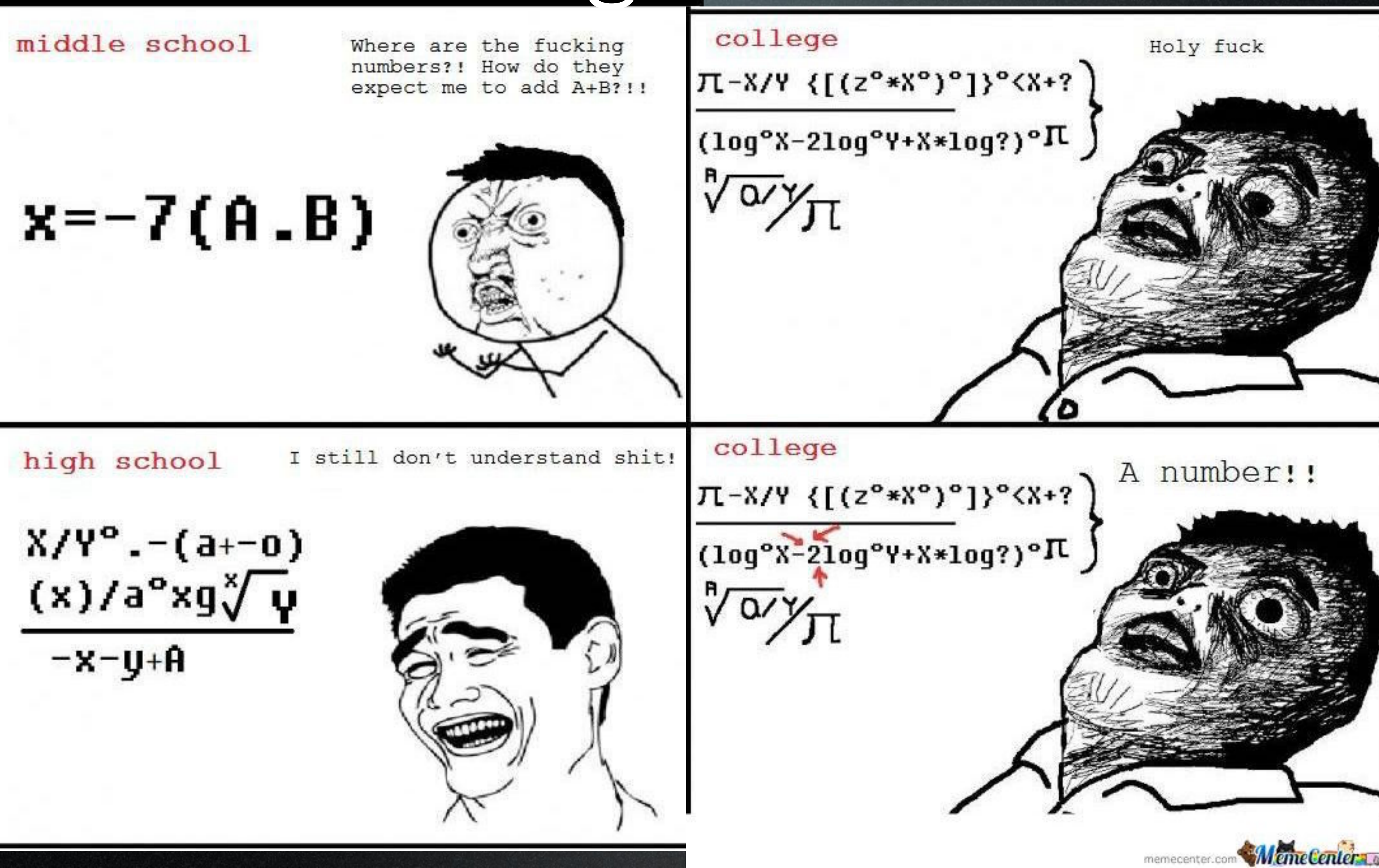
Algebra?

Algebra?

Algebra! Help!

Algebra?



middle school
Where are the fucking numbers?! How do they expect me to add A+B?!!
x=-7(A.B)
high school
I still don't understand shit!
-x-y+A
college
Holy fuck
college
A number!!
memecenter.com
MemeCenter

Algebra?

WHO NEEDS ALGEB

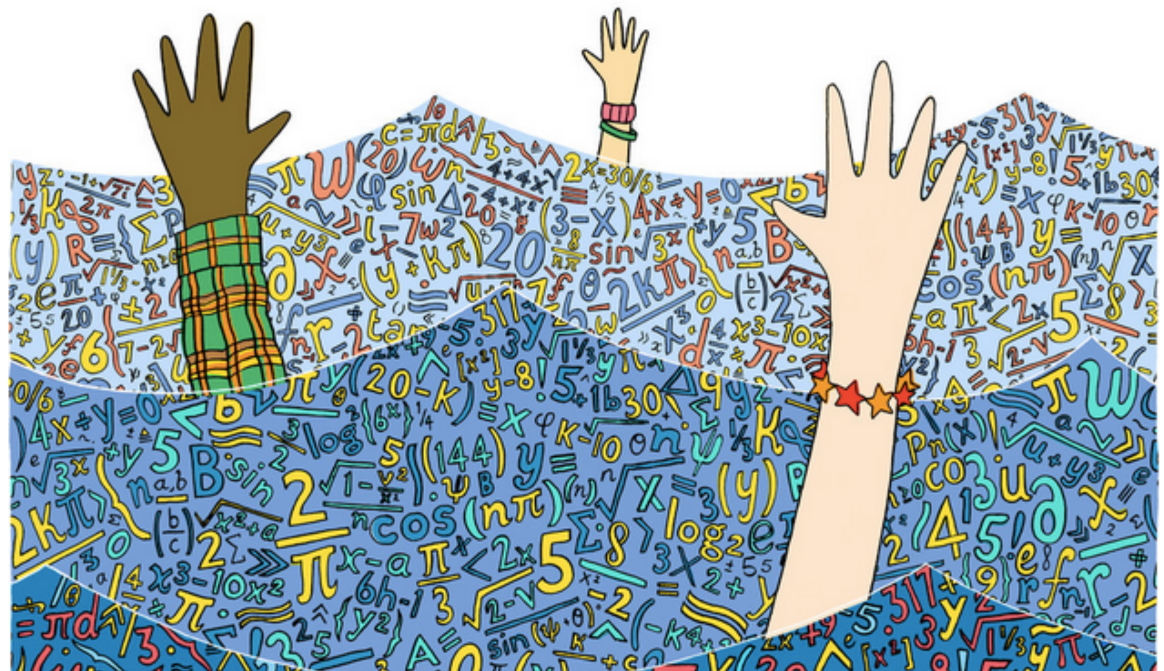
By Colman McCarthy April 20, 1991

America's 12.5 million high school students ought to keep a
superintendent of Dayton, Ohio's, public schools and soon t
system. Smith is an algebra lover who's been pushing for ye
Dayton high school students. If his bizarre idea succeeds, it
country. Currently only one state -- Louisiana -- has compu

SundayReview | OPINION

Is Algebra Necessary?

By ANDREW HACKER JULY 28, 2012



1982

THE DISK WITH THE COLLEGE EDUCATION

HERBERT S. WILF
Department of Mathematics, University of Pennsylvania, Philadelphia PA 19104

The title is somewhat exaggerated, but the calculators-or-no-calculators dilemma that haunts the teaching of elementary school mathematics is heading in the direction of college mathematics, and this article is intended as a distant early-warning signal.

I have in my home a small personal computer. About 500,000 small personal computers have been sold in this country, of which a healthy fraction are owned by individuals. I use mine primarily for word processing (this article was written on it), for writing programs that do various mathematical jobs related to my teaching or to my research, for playing games, for keeping class rolls, etc.

A new program has recently been made available for my little computer, one whose talents seem worthy of comment here because it knows calculus; in fact, as you read these words, some of

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The author received his Ph.D. degree in 1958 from Columbia University, taught at the University of Illinois, and has been at the University of Pennsylvania since 1962. His principal research interests have been in analysis: numerical, mathematical, and, in the past several years, combinatorial. He is the author of several books and articles, has been a John Simon Guggenheim fellow, has received a Christian and Mary Lindback award for excellence of undergraduate teaching, and is co-editor-in-chief (with D. E. Knuth) of the *Journal of Algorithms*.

1982:
Herbert
Wilf

A new program has recently been made available for my little computer, one whose talents seem worthy of comment here because it knows calculus; in fact, as you read these words, some of your students may be doing their homework with it.

The program is called muMATH; it was written by the Soft Warehouse, and is distributed in the United States by Microsoft Consumer Products of Bellevue, Washington. It costs about $75 and is supplied on a 5-inch floppy disk with an (inadequate) instruction manual.

The program on the disk does numerical calculation to high precision, or symbolic manipulation of expressions. The numerical calculation, which is less important as far as this article is concerned, is in rational arithmetic and is done with 611-digit accuracy. Thus, for example, when the program is loaded, the question

?30!;

yields the instant answer

@265252859812191058636308480000000

The question

?1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7;

elicits

@363/140

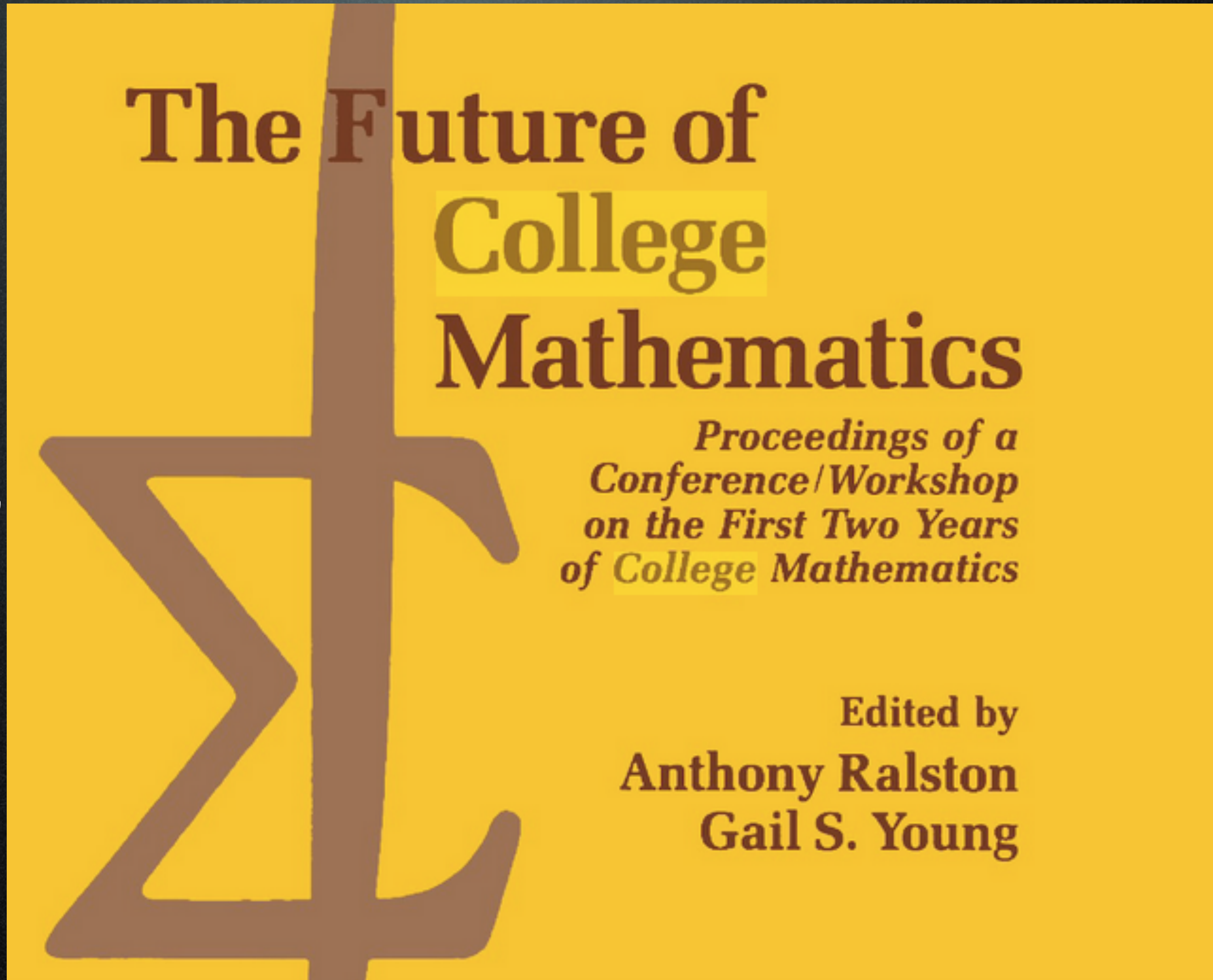
and so forth.

But these are fairly standard calculator-type questions. The first glimmer that a nontrivial intelligence lives on the disk comes with the request for $\sqrt{12}$,

?12↑(1/2);

(the up arrow means "to the power"), whence the response @2*3↑(1/2)

1982:
Herbert
Wilf



meaningful to anyone who has looked into global and local variables in a programming language, and conversely, the former concept strengthens the latter. Another example would be the ideas of a summation sign and a for...next loop, and so it goes.

The prospect of courses emerging in which busywork is much reduced, and in which modern algorithmic ideas are present, both for their own sake and for enhancement of the mathematical concepts, is to me extremely attractive. I look forward to the debates that no doubt will accompany the developmental processes, and to the fruits of those debates which, it seems to me, can only be a significantly improved educational experience for the students and for ourselves.

Reference

1. Herbert S. Wilf, *The disk with the college education*, American Mathematical Monthly, Vol. 89, No. 1, January 1982, pp. 4-8

1982: Herbert Wilf

Barrett: I feel very strongly that mathematics is an experimental science. Thus, I favor letting students experiment with a calculator or computer before they are taught the formal mathematics.

Wilf: I'm hoping that at this conference we can express some of the liberated feeling achieved by symbolic systems and try to find out what we would like to replace some of the techniques by.

On the second part of my paper, the role of algorithms in the curriculum, let me emphasize:

- the importance of understanding the limitations of computers and algorithms, that is computational complexity at a low level
- the need to teach students to think recursively and
- the desirability of expressing algorithms formally enough so that they can be fully understood and analyzed.

Mathematics: An Experimental Science

Herbert S. Wilf

University of Pennsylvania
Philadelphia, PA 19104-6395, USA

1 The mathematician's telescope

Albert Einstein once said "You can confirm a theory with experiment, but no path leads from experiment to theory." But that was before computers. In mathematical research now, there's a very clear path of that kind. It begins with wondering what a particular situation looks like in detail; it continues with some computer experiments to show the structure of that situation for a selection of small values of the parameters of the problem; and then comes the human part: the mathematician gazes at the computer output, attempting to see and to codify some patterns. If this seems fruitful then the final step requires the mathematician to prove that the pattern she thinks she sees is in fact the truth, rather than a shimmering mirage above the desert sands.

2005: Princeton Companion

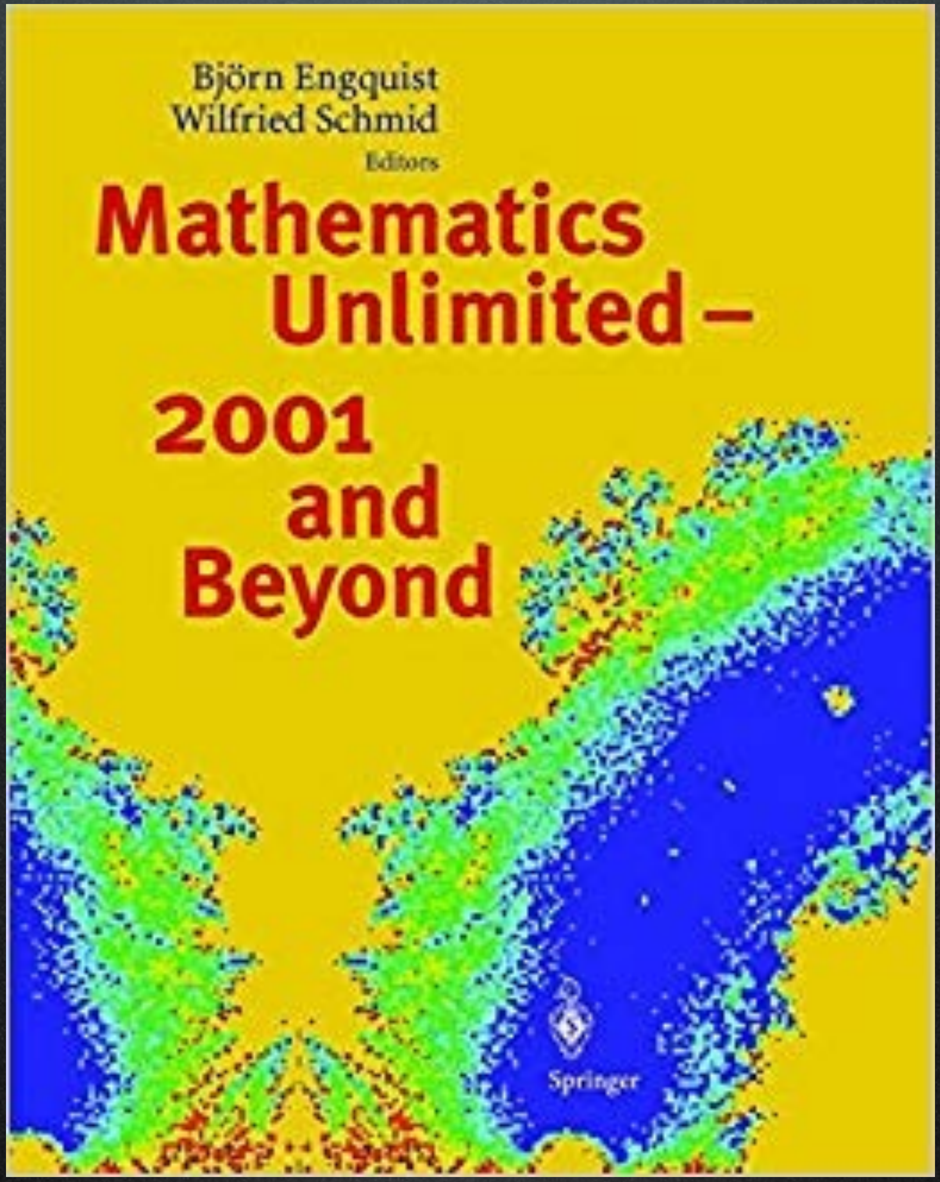
2 Some of the tools in the toolbox

2.1 The CAS

The mathematician who enjoys using computers will find an enormous number of programs and packages available, beginning with the two major Computer Algebra Systems (CAS), Maple and Mathematica. Either of these programs will provide so much assistance to a working mathematician that they must be regarded as essential pieces of one's professional armamentarium. They are extremely user-friendly and capable.

Typically one uses a CAS in interactive mode, meaning that you type in a one line command and the program responds with its output, then you type in another line, etc. This *modus operandi* will suffice for many purposes but for best results one should learn the programming languages that are embedded in these packages. With a little knowledge of programming, one can ask the computer to look at larger and larger cases until something nice happens, then take the result and use another package to learn something else, and so forth. Many are the times when I have written little programs in Mathematica or Maple and then gone away for the weekend leaving the computer running and searching for interesting phenomena.

2000: mathematics unlimited



How Should We Prepare the Students of Science and Technology for a Life in the Computer Age?

HANS PETTER LANGTANGEN · ASLAK TVEITO

1. Models in Science and Technology

1.1 Computer Modeling

Science is modeling; we derive models of Nature for the purpose of understanding how everything works. Technology is the application of scientific models for developing devices capable of increasing the quality of life in general.

Certainly, "model" is a very general term, and its content vary throughout the branches of science. In the natural sciences, the mathematical models have played an important role during the last centuries, and there is no doubt that

2. The Educational System

2.1 Paper and Pencil; Ready for the Museums?

The future importance of computers in science and engineering is obvious to most of us. We would therefore expect that the entire educational system had changed accordingly. This has, however, not been the case so far. Browsing through virtually any recent textbook in physics, geophysics, petroleum engineering etc., the picture is the same; over-simplified models are analyzed with paper and pencil methods throughout. Where are all the books that reflect the importance of computers in these applied fields? Even in applied mathematics, which by nature is close to the computer revolution, the curricula have changed very little for the past 30 years. It seems obvious that a complete revision of the basic education in both science and engineering is necessary to meet the demands of modern candidates and their employers.

Much of the current focus on algebraically challenging, lengthy, error-prone paper and pencil work can be significantly reduced. In fact, we seriously doubt

Natural Sines

	0' 0.0°	6' 0.1°	12' 0.2°	18' 0.3°	24' 0.4°	30' 0.5°	36' 0.6°	42' 0.7°	48' 0.8°	54' 0.9°	Mean Differences 1'	2'	3'	4'	5'
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0.0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	0.0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	0.0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	0.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	0.1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	0.1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	0.1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	0.1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10°	0.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
11	0.1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	0.2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	0.2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	0.2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	0.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	0.2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	0.2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	0.3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	0.3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20°	0.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	0.3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	0.3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	0.3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	0.4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	0.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	0.4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	0.4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	0.4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	0.4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30°	0.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	0.5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	0.5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	0.5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	0.5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	0.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
36	0.5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	0.6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	0.6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	0.6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40°	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	0.6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	0.6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	0.6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	0.6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

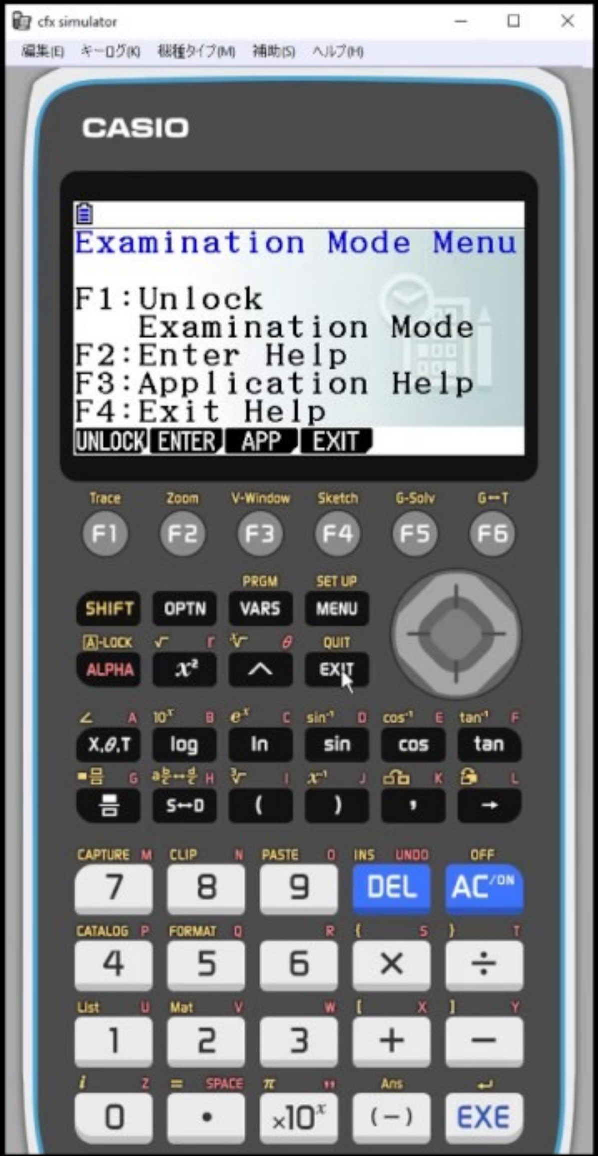
2.2 Do You S… e Sine Function?

The authors re… they had to learn com-
puting sines a… vas there, actually we
all had our ow… yet adapted to the new
technology. Ba… es when we ignore the
existence of M… software, which solves
virtually any ex… a. Sometimes it appears
that many teach… as a threat towards their
profession. Th… ling; proper application
of software wo… alculus, by enabling the
students to lea… he difficult issues rather
than wasting th…

4. Computational Mathematics from Day One?

Forty years ago the Noble Prize Laureate, Richard P. Feynman at Caltech, initiated a major revision of the introduction to physics. One of his objectives was to maintain the enthusiasm students have as they enter the university. We believe that the situation is somewhat similar today; students entering universities today expect to find a modern environment fully utilizing the power of computers, which they know from their own experience and from what they read in newspapers etc. Instead, the situation today is that many students do not seriously use a computer beyond text processing. Modern concepts like parallel computing are only presented for a small group of students at advanced levels in science and technology. We should work more in the spirit of Feynmann; start day one with computing, also in parallel (see section 5.3), and let the students experience how powerful mathematical models are to predict the behavior of Nature and technical devices.

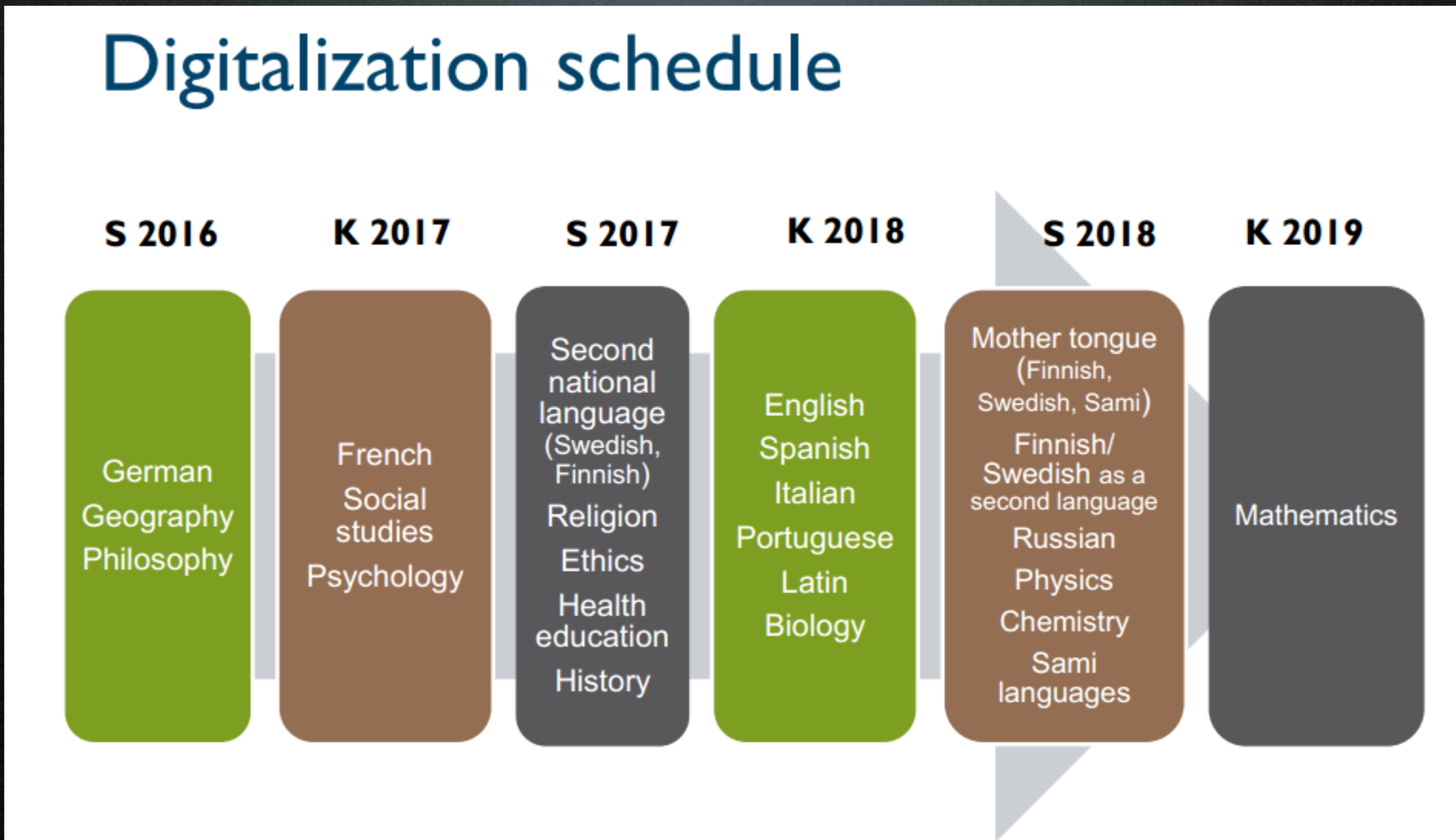
Exam mode



This mode restricts access to Memory mode, Program mode, eActivity, vector commands, Program commands(W(output command), : (multi-statement command), l(carriage return)), E-CON3 mode, data transfer, add-in applications, add-in languages, storage memory access, user name editing, so these modes and functions are not available during exams.

we have a problem

Finland: Ylioppilastutkinto



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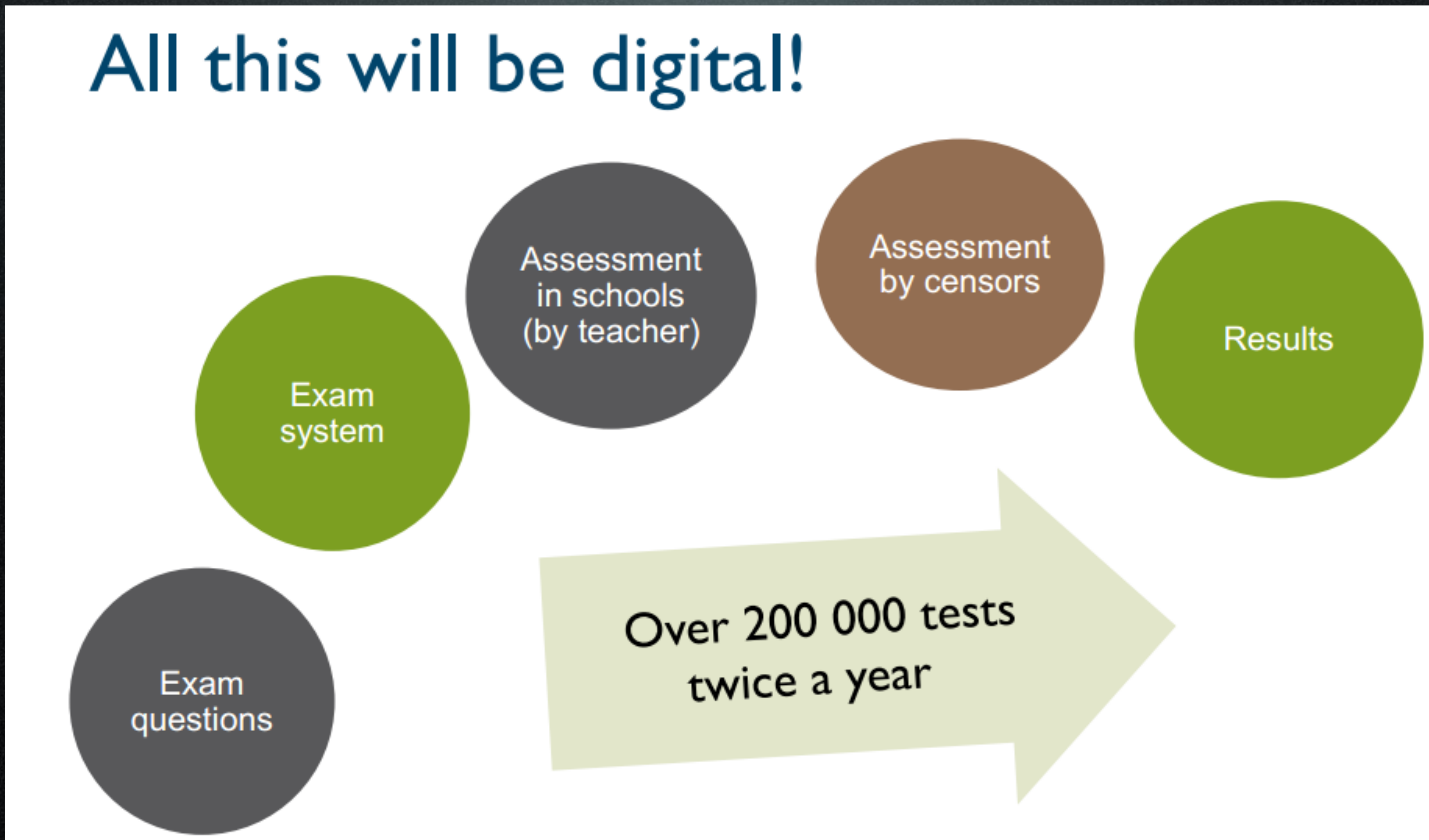

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Finland: Ylioppilastutkinto

Finland: Ylioppilastutkinto

Casio ClassPad Manager	symbolic computation (CAS)	commercial, available in the Matriculation Examination and Abitti	https://edu.casio.com/products/classroom/cp2m/
Dia	vector graphics	free, open source	http://dia-installer.de/
Geogebra	symbolic computation (CAS)	free, open source	http://www.geogebra.org/cms/en/
GIMP	graphics editor	free, open source	http://gimp.org/
InkScape	vector graphics	free, open source	http://inkscape.org
LibreOffice	word processing, spreadsheet, presentations, vector graphics	free, open source	https://www.libreoffice.org/
LoggerPro	analysis of measurement data	commercial, available in the Matriculation Examination but not in Abitti	http://www.vernier.com/products/software/lp/

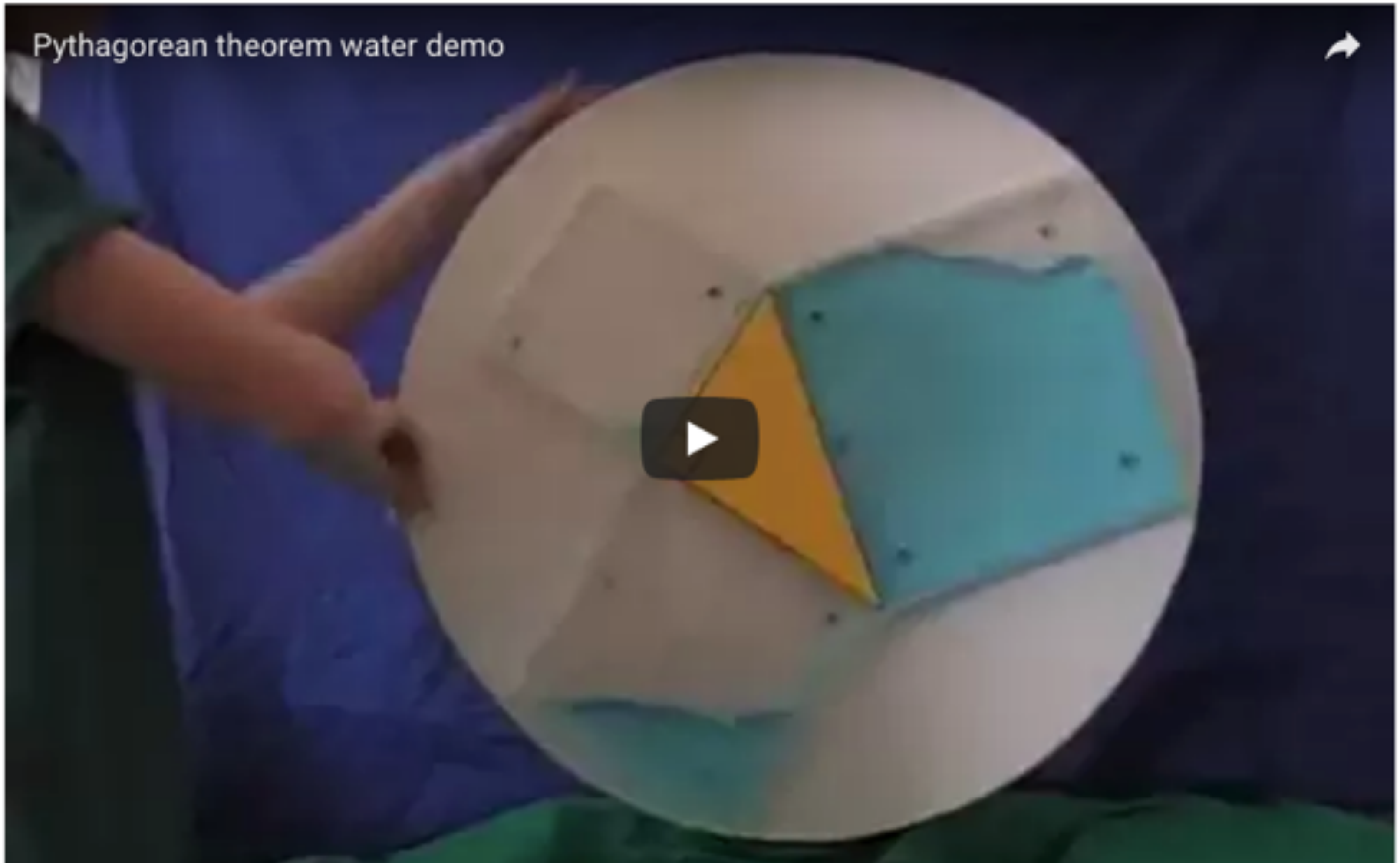
Finland: Ylioppilastutkinto

MarvinSketch	structural formulas in chemistry	commercial, free for students and teachers, available in the Matriculation Examination and Abitti	https://www.chemaxon.com/products/marvin/marvinsketch/
Pinta	graphics editor	free, open source	http://www.pinta-project.com/
Texas Instruments TI-Nspire CAS	symbolic computation (CAS)	commercial, available in the Matriculation Examination and Abitti	https://education.ti.com/fi/products/computer-software/ti-nspire-cx-cas-student-sw
wxMaxima	symbolic computation (CAS)	free, open source	http://andrejv.github.io/wxmaxima/
MAOL digital tables	table application for mathematics, physics and chemistry	commercial, available in the Matriculation Examination, restricted content in Abitti	https://oppimisenpalvelut.otava.fi/tuotteet-ja-palvelut/lukio/maol-taulukot/

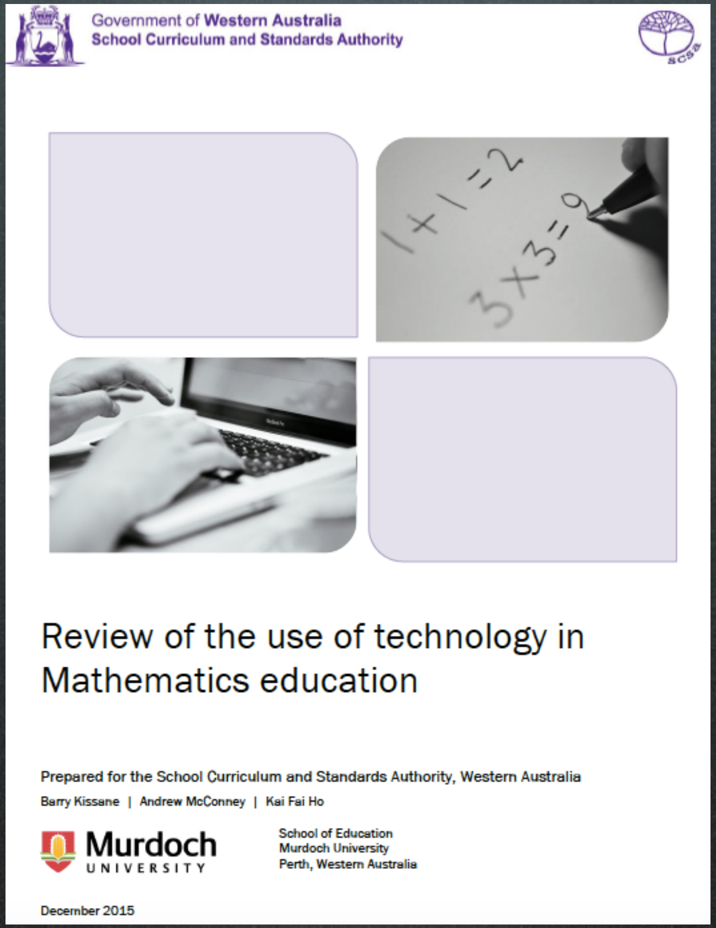
Testes digitais DIGABI: https://digabi.fi/

(a) Explain what the mathematical aspects of the Pythagorean phrase are from YouTube video.

(b) Explain why this empirical experiment does not form a mathematically valid certificate for the Pythagorean phrase.



2015 Report (Australia)



Kissane, B., McConney, A., & Ho, K.F. (2015, December). Review of the use of technology in Mathematics education and the related use of CAS calculators in external examinations and in post school tertiary education settings. Perth, WA: School Curriculum and Standards Authority.

2015 Report (Australia)

- Empirical research summaries have consistently suggested that the use of graphics calculators and CAS calculators by secondary school students can result in improvements in conceptual understanding in mathematics, although the improvements are modest and depend on the extent to which teachers and students make effective classroom use of them. Definitive large-scale studies on the effectiveness of sound use of CAS in secondary schools are not yet available.

2015 Report (Australia)

- In practice, CAS calculators have often been used to replace traditional procedures more than they have been used to enhance students' conceptual understanding.

2015 Report (Australia)

- Research and careful analysis have highlighted some of the challenges of effective use of CAS in particular, requiring careful consideration of the nature of algebra and calculus especially in both CAS and non-CAS environments, and developing suitable expertise by both students and teachers to integrate the tools appropriately.

2015 Report (Australia)

- Since it has become a common practice for students to be assessed both with and without access to technology when CAS is used, the use of CAS has been recognised as creating special difficulties for the assessment of student learning, especially in timed examinations.

2015 Report (Australia)

- In the case of the five Western Australian universities, neither CAS calculators nor graphics calculators are systematically used for instruction in first year mathematics classes, and technology use in assessment is mostly confined to scientific calculators.

2015 Report (Australia)

- This situation is well-known to many local teachers, who often interpret it as an argument against the use of these technologies in school mathematics.

2015 Report (Australia)

- Mathematics teachers in local universities are generally unfamiliar with the use of CAS calculators or graphics calculators as learning tools, do not use them for teaching purposes and regard them only as computational devices.

2015 Report (Australia)

- Both the existing Mathematics syllabuses (concluding with Year 12 in 2015) and the new syllabuses (starting with Year 11 in 2015) explicitly recognise roles for technology. In each case, however, there is very little specific advice and guidance offered to readers to clarify in any detail how that technology might be used for teaching, for learning or for assessment.

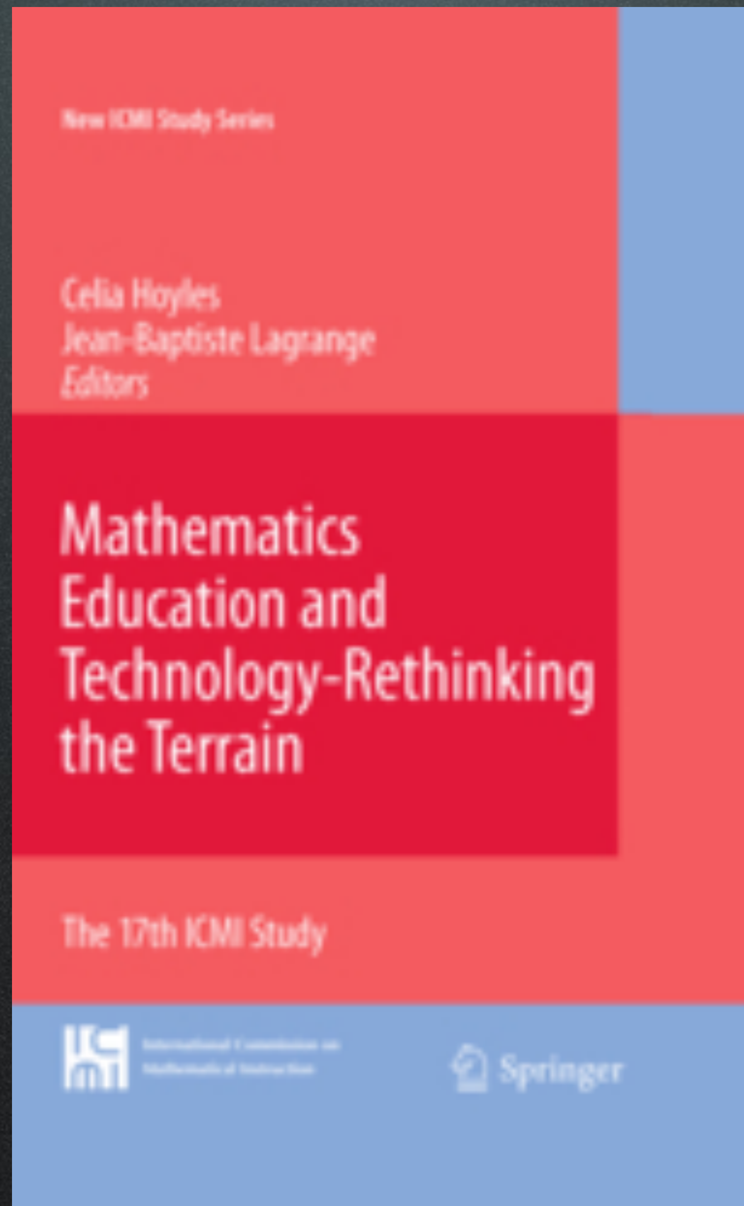
2015 Report (Australia)

- In particular, there seems to be no substantial advice offered regarding the use of the computer algebra capabilities that distinguish CAS calculators from their predecessors, graphics calculators.

2015 Report (Australia)

- Study of recent examination papers in mathematics reveals that there are typically few questions that require students to use CAS calculator capabilities for efficient solutions (especially for lower level courses) and that there are also questions for which use of a CAS calculator would be inadvisable or even inappropriate.

ICMI Study 17 (2010)



ICMI Study 17 (2010)

8.3.2 *CAS and Problem Spotting*

To avoid these pitfalls and problems, it has been suggested that students use algebra as long as possible when doing complicated calculations, hoping that the algebra postpones rounding errors as long as possible (Sträßer 2001a). For many of the problems related to computer-based arithmetic, this works fine. In Computer Algebra Systems (CAS) the problems normally start as soon as calculations go beyond the simple development of a formula. Spotting problem situations like reducing $(x-1)^2/(x^2-1)$ to $(x-1)/(x+1)$ will be correctly handled by most modern CAS programs (recalling that the reduction is invalid for $x = 1$). Without greater understanding about the inner mechanisms of automatic algebraic calculations, the problem of this type of manipulation of equations and formulae serves as a warning to check the algebraic reductions provided by a particular CAS program.

Problems using CAS in a school setting

From the analysis of students' experience while working with computer algebra the following list of obstacles can be drawn up:

1. The difference between the algebraic representations provided by the CAS and those students expect and conceive as 'simple'.
2. The difference between numerical and algebraic calculations and the implicit way the CAS deals with this difference.
3. The limitations of the CAS and the difficulty in providing algebraic strategies to help the CAS to overcome these limitations.
4. The inability to decide when and how computer algebra can be useful.
5. The flexible conception of variables and parameters that using a CAS requires.

PAUL DRIJVERS (2000)
STUDENTS ENCOUNTERING OBSTACLES USING A CAS

Problems using CAS in a school setting

The need to make sense of the CAS outputs, and the ability to coordinate these with existing theoretical notions and paper-and-pencil techniques, was fundamental to the students' theoretical and paper-and-pencil-technical progress.

The co-emergence of machine techniques,
paper-and-pencil techniques, and theoretical reflection:
A study of CAS use in secondary school algebra
Carolyn Kieran, Paul Drijvers (2006)

Problems using CAS in a school setting

However, the students did not manage all of this on their own, even with the aid of the CAS. The teacher was essential to the process. The students clearly experienced difficulties at times, but they were not abandoned in these situations.

Without the teacher orchestrating the theoretical and technical development of the task situation, and asking key questions at the right moment, the advances made by the students would likely have been less dramatic.

The co-emergence of machine techniques,
paper-and-pencil techniques, and theoretical reflection:
A study of CAS use in secondary school algebra
Carolyn Kieran, Paul Drijvers (2006)

Problems using CAS in a school setting

Graphic calculators have allowed the introduction of families of functional objects, but the mathematical work involved tends to be situated at a graphical level. CAS allows the connection of such graphical work with symbolic work, so that regularities graphically observed can be proved in a symbolic way.

MICHÈLE ARTIGUE (2002)
LEARNING MATHEMATICS IN A CAS ENVIRONMENT:
THE GENESIS OF A REFLECTION ABOUT INSTRUMENTATION
AND THE DIALECTICS BETWEEN TECHNICAL AND
CONCEPTUALWORK

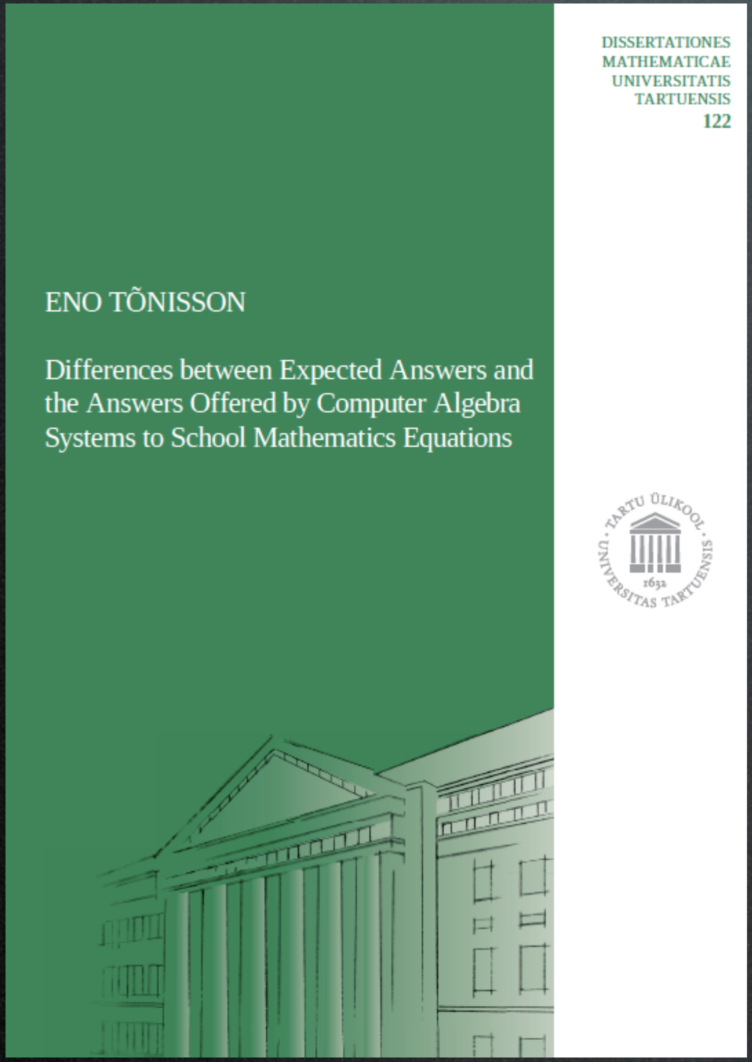
Problems using CAS in a school setting

...research carried out up to now, allows us to better understand:

- the difficulties of the effective integration of CAS into mathematics teaching;
- some of the possible reasons for the success of some of our experiments and the failure of others;

MICHÈLE ARTIGUE (2002)
LEARNING MATHEMATICS IN A CAS ENVIRONMENT:
THE GENESIS OF A REFLECTION ABOUT INSTRUMENTATION
AND THE DIALECTICS BETWEEN TECHNICAL AND
CONCEPTUALWORK

ENO TÕNISSON (2017)
Differences between Expected Answers and the Answers Offered by Computer Algebra Systems to School Mathematics Equations



University of Tartu
Tartu, Estonia

ENO TÕNISSON (2017)
Differences between Expected Answers and the Answers Offered by Computer Algebra Systems to School Mathematics Equations

- while many answers offered by CAS (CAS answers) do not differ from the answers expected in a school context (school answers), there are some exceptions.

ENO TÕNISSON (2017)
Differences between Expected Answers and the Answers Offered by Computer Algebra Systems to School Mathematics Equations

$Equation$: $\sqrt{2x+1} = \sqrt{x}$
$Answer_{Axiom}$, $Answer_{Maxima(to\_poly\_solver)}$, $Answer_{Sage(to\_poly\_solve)}$,
$Answer_{WolframAlpha(SolveEquation)}$: -1

(Wiris and WolframAlpha give no solutions in the real domain.)

Number -1 is, of course, a real number, but substitution to the equation gives $\sqrt{-1}$.

What happens when we substitute the solution for the equation in CAS?

$Equation$: $\sqrt{2\cdot(-1)+1} = \sqrt{(-1)}$
$Answer_{Axiom}$: $\sqrt{-1} = \sqrt{-1}$
$Answer_{Maxima}$, $Answer_{Sage}$: $i = i$
$Answer_{WolframAlpha}$: $True$

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$Equation$: $\sin x = \cos x$

$Answer_{Textbook}$: $\frac{\pi}{4} + n\pi, n \in \mathbb{Z}$

$Answer_{WolframAlpha(Equation)}$: $2(\pi n - tan^{-1}(1 - \sqrt{2})), n \in \mathbb{Z}$ and $2(\pi n - 2tan^{-1}(1 + \sqrt{2})), n \in \mathbb{Z}$

$Answer_{Maxima(to\_poly\_solver)}$: $\frac{4\pi\%z1 + \pi}{4}, n \in \mathbb{Z}$

It is explainable that $\arctan(1 - \sqrt{2})$ is equal to $-\frac{\pi}{8}$ and $\arctan(1 + \sqrt{2})$ to $\frac{3\pi}{8}$.

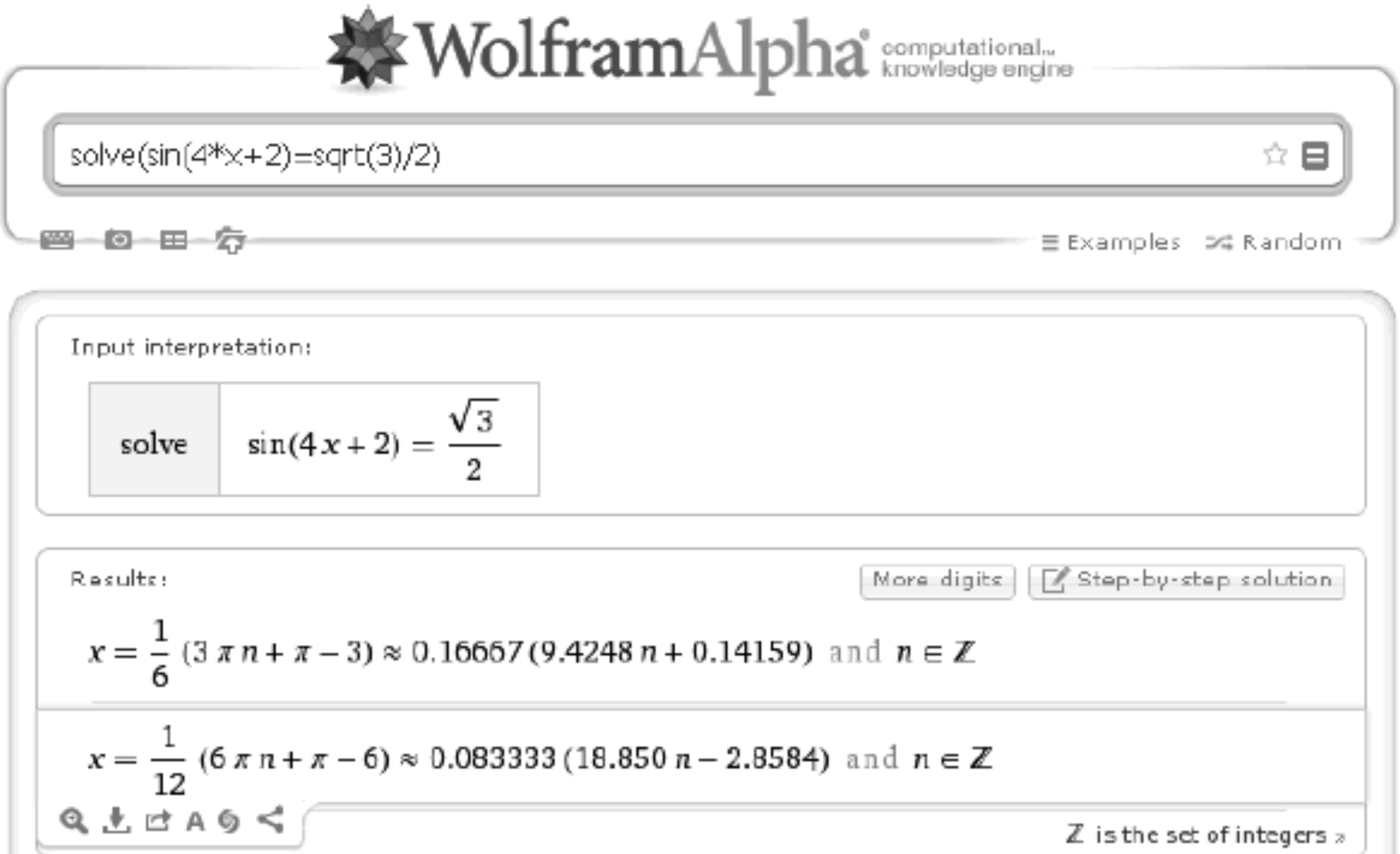

Figure 22. $\sin(4x+2)=\frac{\sqrt{3}}{2}$ (WolframAlpha)

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$$\text{solve}(\tan(x)^3=\tan(x)) \rightarrow \left\{\{x=0\},\{x=\pi\},\left\{x=\frac{\pi}{4}\right\},\left\{x=\frac{3\cdot\pi}{4}\right\},\left\{x=\frac{5\cdot\pi}{4}\right\},\left\{x=-\frac{\pi}{4}\right\}\right\}$$

Figure 23. $\tan^3 x = \tan x$ (Wiris)

```
(%i6) %solve(cos(x-%pi/6)=0.5,x);
```

$$(\%o6)\ \%union\left([x=2\pi\ \%z6-\frac{\pi}{6}],[x=2\pi\ \%z8+\frac{\pi}{2}]\right)$$

Figure 24. $\cos\left(x-\frac{\pi}{6}\right)=0.5$ (Maxima)

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[redacted] It should be noted that a new version of a CAS can behave differently from a previous version in the case of a particular equation. [redacted]

CAS and visions

Main Visions are:

- to make a CAS very intuitive to use. Teachers and students should not waste time in getting lost by considering technical details.
- to develop and to find new possibilities for teaching and learning mathematics which are based on the power of a CAS.

What is Happening with CAS in Classrooms?
Example Austria
Josef Böhm, ACDCA, Austria (2007)

CAS and visions

Main Visions are:

- to integrate the various technologies which are important for math education: CAS and Dynamic Geometry, Spreadsheet, Statistics Package, Probes, Text processing, Programming, CAD, Portability into web formats.

What is Happening with CAS in Classrooms?
Example Austria
Josef Böhm, ACDCA, Austria (2007)

CAS and visions

Main Visions are:

- to find some agreement about basic knowledge, basic skills, basic concepts. Which amount is indispensable without CAS?

Future needs

- Teacher/Faculty preparation
- (powerful and more realistic) Mathematics models with CAS
- Which basic algebra
- Integration of numerical, graphic and symbolic
- Which new pitfalls

to understand really the practical connections of Algebra and CAS in school in the 21st century

The challenges schools face with the development of Computer Algebra

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