A sub-Riemannian optimal control problem giving rise to quasi-geodesics on Stiefel manifolds

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Stiefel manifolds, as well as Grassmann manifolds, play an important role in engineering applications where the underlying models have a linear subspace structure. This is the case of a wide range of computer vision applications, such as dynamic textures, human activity modelling and recognition, video based face recognition and pattern recognition.

Each point in a Grassmann manifold is identified with a k-dimensional subspace of an n-dimensional Euclidean space, while a point in the Stiefel manifold corresponds to a particular choice of a frame that generates that subspace. Since both are Riemannian manifolds, differential geometry provides elegant and computationally efficient methods to analyse and propose solutions with considerable performance improvements when compared with those obtained with more traditional methods. Many of the iterative methods on Euclidean spaces can be directly modified for manifolds just replacing everything by their Riemannian counterparts. Geodesic curves play a crucial role in these processes, but their use is effective from an applied viewpoint only if explicit formulas are available. Unfortunately, this is not always the case for Stiefel manifolds.

Quasi-geodesics are simple curves with constant geodesic curvature that have been used successfully to solve interpolation problems on Stiefel manifolds. This is due to the fact that under mild conditions there are explicit formulas to compute the quasi-geodesic that joins 2 given points. These interesting curves have a surprising geometry, and we will show that they are projections of sub-Riemannian geodesics on certain Lie groups that act transitively and effectively on those manifolds. To reach this conclusion we use results from optimal control theory on Lie groups.

References

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