A Nondegenerate version of the Maximum Principle for State-Constrained Nonholonomic Systems[1]

S. O. Lopes
CFIS, DMA, Universidade do Minho, Guimaraes, Portugal
sofialopes@math.uminho.pt

M. M. A. Ferreira and M.d R. de Pinho and F.A.C.C. Fontes
Systec-ISR Porto, Faculdade de Engenharia, Universidade do Porto, 4200-465 Porto, Portugal
mmf,mrpinho,faf@fe.up.pt

In optimal control problems with state constraints, the candidates to minimizer obtained from the necessary conditions of optimality can coincide with the whole set of points that satisfy the constraints, making the optimality conditions useless. For problems satisfying additional Constraint Qualifications, it is possible to derive stronger, nondegenerate forms of the Maximum Principle. Such nondegenerate forms can be found in the literature, see for example [5] and references within. Two common constraint qualification are of the types:

(CQd1) (e.g. [6])$\exists\delta > 0$, $\exists\hat{u}$ such that for t near 0

$$h_x(t, x_0) \cdot f(t, x_0, \hat{u}) < 0$$

(CQd2) ([7]) $\exists\delta > 0$, $\exists\hat{u}$ such that for t near 0

$$h_x(t, x_0) \cdot (f(t, x_0, \hat{u}) - f(t, x_0, \bar{u}(t))) < -\delta$$

While **(CQd1)** mainly states that there exists a control that pushes the trajectory away from the boundary at the initial point, **(CQd2)** covers the case when there exists a control that pushes the trajectory away from the boundary faster than the optimal control again at the initial time. Each of them have its merits, as it was discussed in [4]. A more recent version combining these two constraint qualification was introduced in [2], where **(CQd1)** or **(CQd2)** have to be satisfied for the times at which $\bar{x}$ has an outward pointing velocity.

Another constraint qualification of interest is the integral type condition: **(CQI)** $\exists\delta > 0$, $\exists\hat{u}$ such that for all t near 0

$$\int_0^t h_x(\tau, x_0) \cdot (f(\tau, x_0, \hat{u}(\tau)) - f(\tau, x_0, \bar{u}(\tau)))d\tau \leq -\delta t.$$

In this constraint qualification, the inward pointing condition has to be satisfied for some, not all, instants of a neighbourhood of the initial time. The work [3] provides a nondegenerate Maximum Principle that holds under **(CQI)** and also gives an example in which **(CQd1)** and **(CQd2)** are not satisfied, but **(CQI)** is.

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More recently, the authors of [1] have studied non-holonomic systems, namely the Brocket integrator, for which **(CQd1)**, **(CQd2)** also do not hold, but a new constraint qualification using Lie brackets guarantees the existence of a neighbouring feasible trajectory [6] and thereby ensuring nondegeneracy.

Here, we consider a well-known nonholonomic system, a car-like system, and show that a variant of **(CQI)** holds while all the other aforementioned constraint qualifications do not.

References

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