Complexity Optimal Decision Procedure for PDL with Parallel Composition

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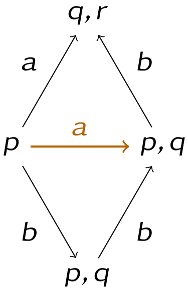
Syntax

$$\alpha, \beta ::= \; a \mid (\alpha \,;\, \beta) \mid (\alpha \cup \beta) \mid \alpha^* \mid \varphi? \qquad \text{(programs)}$$
$$\varphi, \psi ::= \; p \mid \bot \mid \varphi \to \psi \mid \langle \alpha \rangle \varphi \qquad \text{(formulas)}$$

Semantics

$$\langle a \rangle p$$



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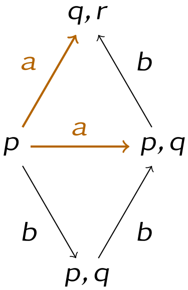
Semantics

$$[a]\, q$$



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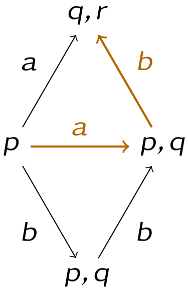
Semantics

$$\langle a \,;\, b \rangle\, r$$



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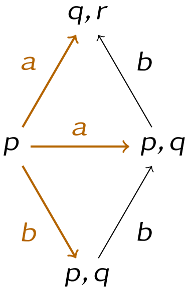

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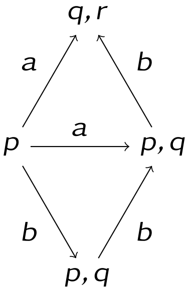
Semantics

$$\langle p? \rangle \top$$



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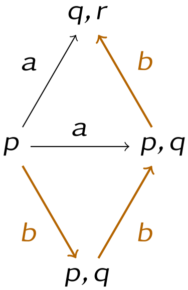
Semantics

$$\langle b^* \rangle r$$



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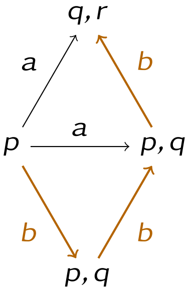
Semantics

$$\langle (p? ; b)^* \rangle r$$

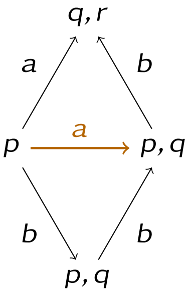


Properties

- Satisfiability problem is EXPTIME-complete.
- Tree-like model property.

Fischer-Ladner closure

$$\langle a \rangle p$$
$$p$$



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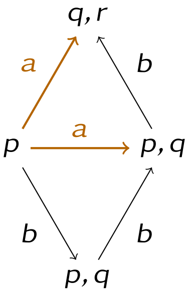
$$[a]\,q$$
$$q$$



Properties

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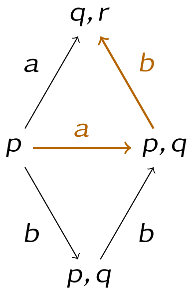
Fischer-Ladner closure

$$\langle a\,;b\rangle r$$
$$\langle a\rangle\langle b\rangle r \quad \langle b\rangle r$$



Properties

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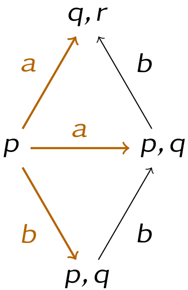
Fischer-Ladner closure

$$[a \cup b]\, q$$

$$[a]\, q \quad [b]\, q \quad q$$



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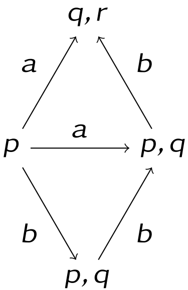
$$\langle p? \rangle \top$$
$$p$$



Properties

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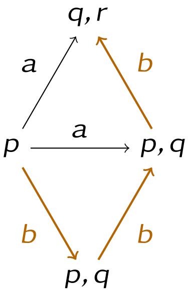
Fischer-Ladner closure

$$\langle b^* \rangle r$$

$$r \quad \langle b \rangle \langle b^* \rangle r \quad \langle b^* \rangle r$$



Properties

- Satisfiability problem is EXPTIME-complete.
- Tree-like model property.

Fischer-Ladner closure

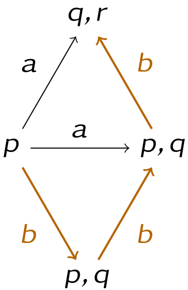
$$\langle (p? \,;\, b)^* \rangle r$$

$$r \quad \langle p? \,;\, b \rangle \langle (p? \,;\, b)^* \rangle r$$

$$\langle p? \rangle \langle b \rangle \langle (p? \,;\, b)^* \rangle r$$

$$p \quad \langle b \rangle \langle (p? \,;\, b)^* \rangle r$$

$$\langle (p? \,;\, b)^* \rangle r$$



Syntax

$$\begin{aligned} \alpha, \beta &::= a \mid (\alpha ; \beta) \mid (\alpha \cup \beta) \mid \alpha^* \mid \varphi? \mid (\alpha \,|\, \beta) && \text{(programs)} \\ \varphi, \psi &::= p \mid \bot \mid \varphi \to \psi \mid \langle \alpha \rangle \varphi && \text{(formulas)} \end{aligned}$$

Semantics

$$\mathcal{L}(\alpha \,|\, \beta) \doteq \mathcal{L}(\alpha) \sqcup \mathcal{L}(\beta)$$

For instance: $\langle a \,|\, b \rangle \varphi \leftrightarrow \langle (a ; b) \cup (b ; a) \rangle \varphi$

Complexity

The satisfiability problem is 2EXPTIME-complete.

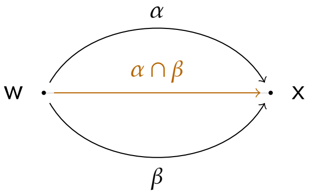
Syntax

$$\alpha, \beta ::= a \mid (\alpha \,;\, \beta) \mid (\alpha \cup \beta) \mid \alpha^* \mid \varphi? \mid (\alpha \cap \beta) \qquad \text{(programs)}$$

$$\varphi, \psi ::= p \mid \bot \mid \varphi \to \psi \mid \langle \alpha \rangle \varphi \qquad \text{(formulas)}$$

Semantics

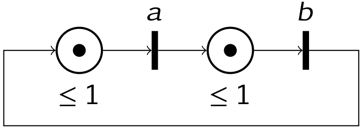
We have: $\langle \alpha \cap \beta \rangle \varphi \to \langle \alpha \rangle \varphi \wedge \langle \beta \rangle \varphi$



Complexity

The satisfiability problem is 2EXPTIME-complete.

Sometimes only parallel programs are executable.



$$\langle a \parallel b \rangle \top \wedge [a] \bot \wedge [b] \bot$$

Syntax

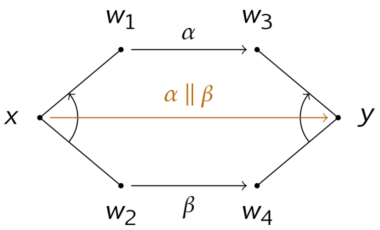
$$\alpha, \beta ::= \ a \mid (\alpha\,;\beta) \mid (\alpha \cup \beta) \mid \alpha^* \mid \varphi? \mid (\alpha \parallel \beta) \qquad \text{(programs)}$$

$$\varphi, \psi ::= \ p \mid \bot \mid \varphi \rightarrow \psi \mid \langle \alpha \rangle \varphi \qquad \text{(formulas)}$$

Semantics

A model is a tuple $\mathcal{M} = (W, R, \triangleleft, V)$ where:

- (W, R, V) is a PDL model,
- $\triangleleft$ is a ternary relation over W.



Definition

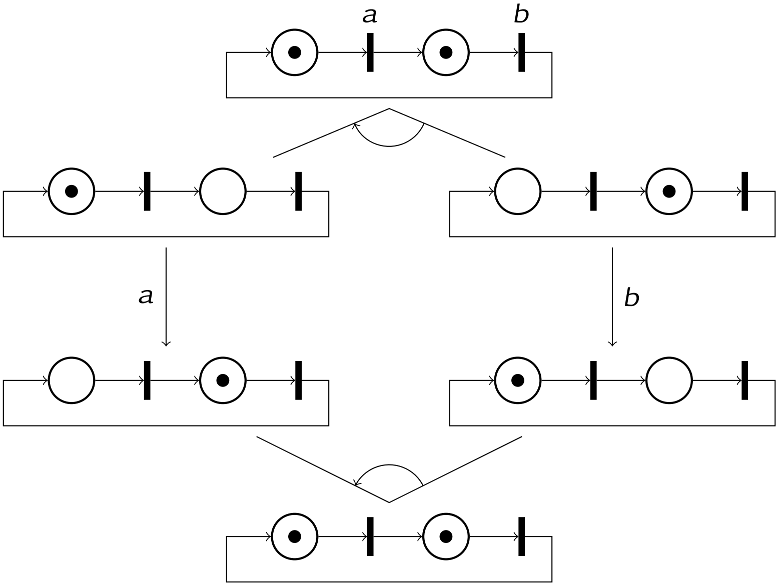
A model is $\lhd$-deterministic iff there is at most one way to merge any pair of states:

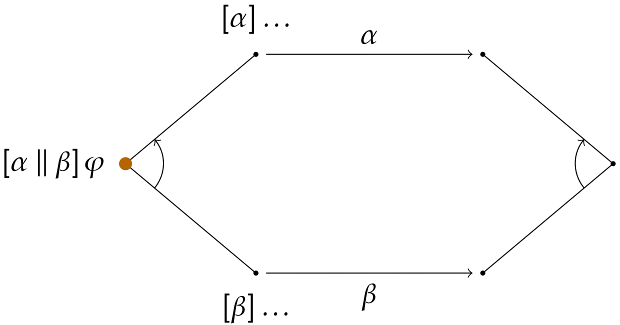
$$\text{if } w_1 \lhd (x, y) \text{ and } w_2 \lhd (x, y) \text{ then } w_1 = w_2$$

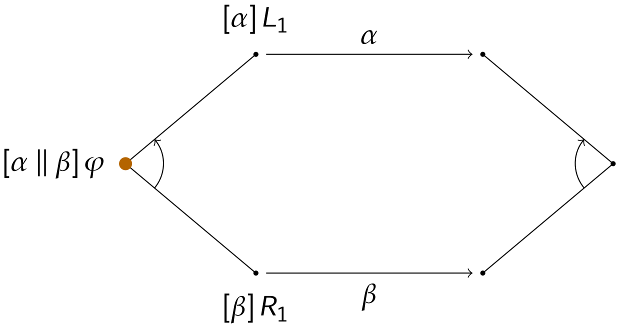
Rationale

- There is a partial binary operator $\bullet$ such that $w \lhd (x, y) \Leftrightarrow w = x \bullet y$.
- Usual constraint in modal logics with a binary modality (arrow logics, ambient logics, separation logic...).

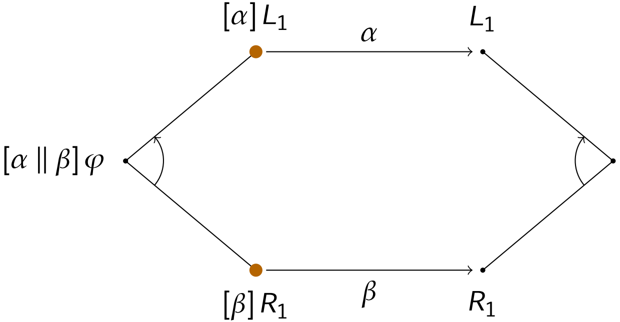
The Petri net example

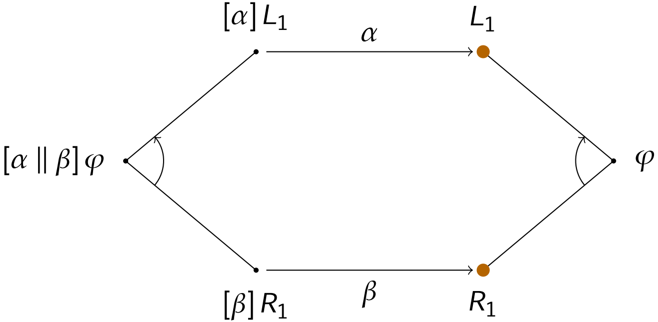


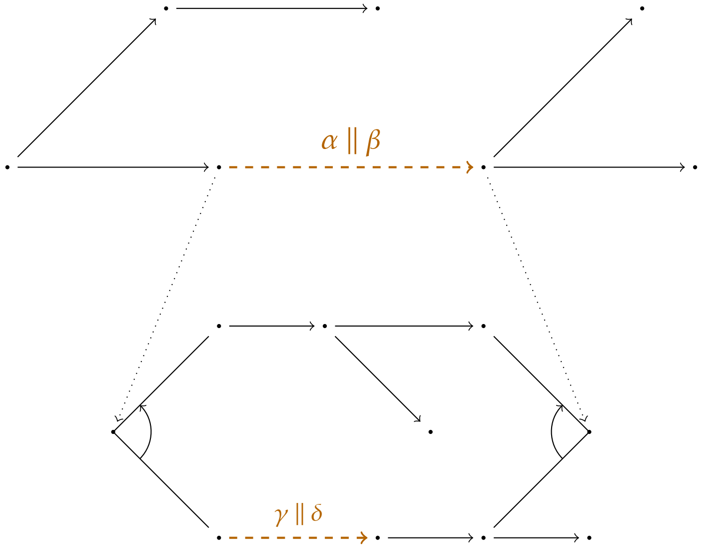

$[\alpha]\ldots$
α
$[\alpha \parallel \beta]\varphi$
$[\beta]\ldots$
β



$[\alpha]\,L_1$
α
$[\alpha \parallel \beta]\,\varphi$
$[\beta]\,R_1$
β


$[\alpha]\, L_1$
α
L_1
$[\alpha \parallel \beta]\, \varphi$
$[\beta]\, R_1$
β
R_1


$[\alpha]\, L_1$
L_1
α
$[\alpha \parallel \beta]\, \varphi$
φ
$[\beta]\, R_1$
β
R_1


$\alpha \parallel \beta$
$\gamma \parallel \delta$

Hintikka sets

Maximal consistent subsets of the Fischer-Ladner closure.

Plugs

Triples $(\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3)$ of Hintikka sets s.t. $L_i \in \mathcal{H}_2$ and $R_i \in \mathcal{H}_3$ for some $i \in \{1, 2\}$.

Sockets

Sets of zero, one or two plugs.

States of the pseudo-models

Pairs $(\mathcal{H}, \mathcal{S})$ where $\mathcal{H}$ is a Hintikka set and $\mathcal{S}$ is a socket.

$$(\mathcal{H}_1, \mathcal{S}_1) \lhd ((\mathcal{H}_2, \mathcal{S}_2), (\mathcal{H}_3, \mathcal{S}_3)) \text{ iff } (\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3) \in \mathcal{S}_2 = \mathcal{S}_3$$

Theoretical result

The addition of deterministic separating parallel compositions to PDL does not increase the complexity of the satisfiability problem.

Future works

- Design an optimal and implementable decision procedure.
- Add commutativity and associativity.