A Tableau System for Quasi-Hybrid Logic

Diana Costa
Manuel A. Martins

CIDMA
Department of Mathematics, University of Aveiro

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The definition of the $\sim$ operator, which will make some definitions clearer.

Definition

Let θ be a formula in NNF and let $\sim$ be a complementation operation such that $\sim\theta = nnf(\neg\theta)$.

Definition

A hybrid structure $\mathcal{H}$ *over L is a tuple* (W, R, N, V)*, where:*

$W \neq \emptyset$ – *domain whose elements are called states or worlds,*

$R \subseteq W \times W$ – *accessibility relation,*

$N : \mathrm{Nom} \to W$ – *hybrid nomination,*

$V : \mathrm{Prop} \to Pow(W)$ – *hybrid valuation.*

Definition

A hybrid bistructure is a tuple (W, R, N, V^{+}, V^{-}) *where* (W, R, N, V^{+}) *and* (W, R, N, V^{-}) *are hybrid structures.*

Definition

For a hybrid bistructure $E = (W, R, N, V^+, V^-)$, a satisfiability relation $\models_d$ called decoupled satisfaction at $w \in W$ for propositional symbols and nominals is defined as follows:

- $E, w \models_d p$ *iff* $w \in V^+(p)$;
- $E, w \models_d i$ *iff* $w = N(i)$;
- $E, w \models_d \neg p$ *iff* $w \in V^-(p)$;
- $E, w \models_d \neg i$ *iff* $w \neq N(i)$.

Definition

A satisfiability relation $\models_s$, called strong satisfaction, is defined as follows:

- $E, w \models_s \top$ *always;*
- $E, w \models_s \bot$ *never;*
- $E, w \models_s \alpha$ *iff* $E, w \models_d \alpha$, $\alpha \in \mathrm{Prop} \cup \mathrm{Nom}$;
- $E, w \models_s \theta_1 \vee \theta_2$ *iff* $[E, w \models_s \theta_1$ *or* $E, w \models_s \theta_2]$ *and* $[E, w \models_s \sim \theta_1 \Rightarrow E, w \models_s \theta_2]$ *and* $[E, w \models_s \sim \theta_2 \Rightarrow E, w \models_s \theta_1]$;
- $E, w \models_s \theta_1 \wedge \theta_2$ *iff* $E, w \models_s \theta_1$ *and* $E, w \models_s \theta_2$;
- $E, w \models_s \Diamond\theta$ *iff* $\exists w'(wRw' \;\&\; E, w' \models_s \theta)$;
- $E, w \models_s \Box\theta$ *iff* $\forall w'(wRw' \Rightarrow E, w' \models_s \theta)$;
- $E, w \models_s @_i\theta$ *iff* $E, w' \models_s \theta$ *where* $w' = N(i)$.

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- $E, w \models_s \theta_1 \wedge \theta_2$ *iff* $E, w \models_s \theta_1$ *and* $E, w \models_s \theta_2$;
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Strong validity is set as follows:

$$E \models_s \theta \text{ iff for all } w \in W, E, w \models_s \theta.$$

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For a hybrid similarity type $L = \langle \mathrm{Prop}, \mathrm{Nom} \rangle$,

- Quasi-hybrid atoms over L:

$\mathrm{QHAt}(L) = \{@_i p,\ @_i \Diamond j \mid i, j \in \mathrm{Nom},\ p \in \mathrm{Prop}\}$;

- Quasi-hybrid literals over L:

$\mathrm{QHLit}(L) = \{@_i p,\ @_i \neg p,\ @_i \Diamond j,\ @_i \Box \neg j \mid i, j \in \mathrm{Nom},\ p \in \mathrm{Prop}\}$;

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Definition

Let $L = \langle \mathrm{Prop}, \mathrm{Nom} \rangle$ *be a hybrid similarity type, and consider a hybrid bistructure over* L, $E = (W, R, N, V^{+}, V^{-})$. *The* ***elementary paraconsistent diagram*** *of* E*, denoted by* $Pdiag(E)$*, is the set of quasi-hybrid literals over* $L(W)$ *that hold in* $E(W)$*, i.e.,*

$$Pdiag(E) = \{\alpha \in \mathrm{QHLit}(L(W)) \mid E(W) \models_s \alpha\}$$

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$$Pdiag(E) = \{\alpha \in \mathrm{QHLit}(L(W)) \mid E(W) \models_s \alpha\}$$

Given L, W and N being the identity, the paraconsistent diagram of a bistructure is unique. Therefore, in the sequel, a bistructure $E = (W, R, N, V^+, V^-)$ will be represented by its (finite) paraconsistent diagram $Pdiag(E)$.

A Tableau for Quasi-Hybrid Logic

This new tableau system is a fusion between the tableau system for Quasi-classical logic and the tableau system for Hybrid logic.

We will consider a database Δ of hybrid formulas that express real situations where inconsistencies may appear at some states, and we will check if a query φ is a consequence of the database, *i.e.*, we will want to check if every bistructure that strongly validates all formulas in Δ also validates φ weakly.

We will restrict our attention to formulas which are satisfaction statements.

Definition

We define weak satisfaction, $\models_{\mathrm{w}}$, as strong satisfaction ($\models_s$), except for the case of disjunction, which we will consider as a classsical disjunction:

$$E, w \models_{\mathrm{w}} \theta_1 \vee \theta_2 \textit{ iff } E, w \models_{\mathrm{w}} \theta_1 \textit{ or } E, w \models_{\mathrm{w}} \theta_2$$

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Note that for any θ, $E, w \models_s \theta$ implies $E, w \models_{\mathrm{w}} \theta$. And that, by contraposition, $\not\models_{\mathrm{w}} \subseteq \not\models_s$.

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Similarly to the definition of strong validity, we define *weak validity* as follows: $E \models_{\mathrm{w}} \theta$ iff for all $w \in W$, $E, w \models_{\mathrm{w}} \theta$.

Quasi-Hybrid Consequence Relation

Definition (Quasi-Hybrid Consequence Relation)

Let Δ be a set of satisfaction statements called database, and φ be a satisfaction statement, called query. We say that φ is a consequence of Δ in quasi-hybrid logic if and only if, for all bistructures E which are quasi-hybrid models of Δ, φ is weakly valid.
Formally,

$$\Delta \models_{\mathrm{QH}} \varphi \textit{ iff } \forall E\,(E \models_s \Delta \Rightarrow E \models_{\mathrm{w}} \varphi)$$

Definition

Given a hybrid similarity type $L = \langle \mathrm{Prop}, \mathrm{Nom} \rangle$, we denote the set of satisfaction statements of L as $L_{@}$.
We duplicate the set of satisfaction statements by considering starred copies. The extended set is denoted by $L_{@}^{*}$ and is defined as: $L_{@}^{*} = L_{@} \cup \{\varphi^{*} \mid \varphi \in L_{@}\}$.

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We duplicate the set of satisfaction statements by considering starred copies. The extended set is denoted by $L^*_{@}$ and is defined as: $L^*_{@} = L_{@} \cup \{\varphi^* \mid \varphi \in L_{@}\}$.

Definition

We extend both weak and strong satisfaction relations to starred formulas as follows:

$E, w \models_s \varphi^*$ *iff* $E, w \not\models_s \varphi$

$E, w \models_{\mathrm{w}} \varphi^*$ *iff* $E, w \not\models_{\mathrm{w}} \varphi$

Weak and strong validity of starred formulas are defined in the natural way.

Strong rules (S-rules)

- For connectives and operators:

$$\frac{@_i(\alpha \vee \beta)}{(@_i(\sim \alpha))^* \mid @_i\beta}\ (\vee_1) \qquad \frac{@_i(\alpha \vee \beta)}{(@_i(\sim \beta))^* \mid @_i\alpha}\ (\vee_2)$$

$$\frac{@_i(\alpha \vee \beta)}{@_i\alpha \mid @_i\beta}\ (\vee_3)$$

$$\frac{@_i(\alpha \wedge \beta)}{@_i\alpha, @_i\beta}\ (\wedge) \qquad \frac{@_i@_j\alpha}{@_j\alpha}\ (@) \qquad \frac{@_i\Box\alpha, @_i\Diamond t}{@_t\alpha}\ (\Box)$$

$$\frac{@_i\Diamond\alpha}{@_i\Diamond t, @_t\alpha}\ (\Diamond)(\text{i})$$

- For nominals:

$$\frac{}{@_i i}\ (Ref)(\text{ii}) \qquad \frac{@_a c, @_a\varphi}{@_c\varphi}\ (Nom_1)(\text{iii}) \qquad \frac{@_a c, @_a\Diamond b}{@_c\Diamond b}\ (Nom_2)$$

Weak rules (W-rules)

- For connectives and operators:

$$\frac{(@_i(\alpha \vee \beta))^*}{(@_i\alpha)^*, (@_i\beta)^*}\ (\vee^*) \qquad \frac{(@_i(\alpha \wedge \beta))^*}{(@_i\alpha)^* | (@_i\beta)^*}\ (\wedge^*)$$

$$\frac{(@_i @_j \alpha)^*}{(@_j\alpha)^*}\ (@^*) \qquad \frac{(@_i \Box \alpha)^*}{@_i \Diamond t, (@_t\alpha)^*}\ (\Box^*)(\text{iv})$$

$$\frac{(@_i \Diamond \alpha)^*, @_i \Diamond t}{(@_t\alpha)^*}\ (\Diamond^*) \qquad \frac{(@_i \Box \neg t)^*}{@_i \Diamond t}\ (\Box^*_{\neg i})$$

- For nominals:

$$\frac{@_a c, (@_a\varphi)^*}{(@_c\varphi)^*}\ (Nom^*_1)(\text{iii}) \qquad \frac{@_a c, (@_a \Box b)^*}{(@_c \Box b)^*}\ (Nom^*_2)$$

(i) t a new nominal, α not a nominal
(ii) i in the branch.
(iii) $\varphi \in \mathrm{Prop} \cup \mathrm{Nom}$
(iv) t a new nominal, α not of the form $\neg j$, for j a nominal.

Theorem (Soundness)

The tableau rules are sound in the following sense:

- *for any r-rule* $\frac{\Lambda}{\Sigma}$*, any bistructure* M*, and any state* $w \in W$*,* $M, w \models_r \Lambda$ *implies* $M, w \models_r \Sigma$*.*
- *for any r-rule* $\frac{\Lambda}{\Sigma \mid \Gamma}$*, any bistructure* M*, and any state* $w \in W$*,* $M, w \models_r \Lambda$ *implies* $M, w \models_r \Sigma$ *or* $M, w \models_r \Gamma$*,*

for Λ*;* Σ *and* Γ *lists of formulas in* $L^*_{@}$*,* $r \in \{s, \mathrm{w}\}$*.*

Properties of the Tableau System

Definition

We say that a formula $\chi \in L^*_@$ is a strong occurrence/s-occurs if it is the result of applying a strong rule. Analogously we say that χ is a weak occurrence/w-occurs if it is the result of applying a weak rule. A formula occurs if it s-occurs or w-occurs.

Definition

The notion of a subformula is defined by the following conditions:
– ϕ is a subformula of ϕ;
– if $\psi \wedge \theta$ or $\psi \vee \theta$ is a subformula of ϕ, then so are ψ and θ;
– if $@_a\psi$, $\Diamond\psi$ or $\Box\psi$ is a subformula of ϕ, then so is ψ.

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The tableau system T_{QH} satisfies the following quasi-subformula property:

Theorem (Quasi-subformula property)

If a formula $@_a\varphi$ s-occurs in a tableau where φ is not a nominal and φ is not of the form $\Diamond b$, then φ is a subformula of a root formula. If a formula $(@_a\varphi)^*$ w-occurs in a tableau, then φ is a subformula of the premise in the applied rule.

Definition

Let Θ be a branch of a tableau and let Nom^{Θ} be the set of nominals occurring in the formulas of Θ. Define a binary relation $\sim_{\Theta}$ on Nom^{Θ} by $a \sim_{\Theta} b$ if and only if the formula $@_a b$ occurs on Θ.

Definition

Let b and a be nominals occurring on a branch Θ of a tableau in T_{QH}. The nominal a is said to be included in the nominal b with respect to Θ if the following holds:

- *for any subformula φ of a root formula, if the $@_a\varphi$ s-occurs on Θ, then $@_b\varphi$ also s-occurs on Θ; and*
- *if $(@_a\varphi)^*$ w-occurs on Θ, then $(@_b\varphi)^*$ also w-occurs on Θ.*

If a is included in b with respect to Θ, and the first occurrence of b on Θ is before the first occurrence of a, then we write $a \subseteq_{\Theta} b$.

Tableau Construction

Definition (Tableau construction)

Given a database Δ *of satisfaction statements and a query* $@_a\varphi$ *of* QH*, one wants to verify if* $@_a\varphi$ *is a consequence of* Δ*. In order to do so, we define by induction a sequence* $\tau_0, \tau_1, \tau_2, \cdots$ *of finite tableaux in* T_{QH}*, each of which is embedded in its successor.*
Let τ_0 *be the finite tableau constituted by the formulas in* Δ *and* $(@_a\varphi)^*$*.* τ_{n+1} *is obtained from* τ_n *if it is possible to apply an arbitrary rule to* τ_n *with the following three restrictions:*

- *If a formula to be added to a branch by applying a rule already occurs on the branch, then the addition of the formula is simply omitted.*

Definition (continuation)

- *After the application of a destructive rule to a formula occurrence φ on a branch, it is recorded that the rule was applied to φ with respect to the branch and the rule will not again be applied to φ with respect to the branch or any extension of it.*
- *The existential rules $(\Diamond, \Box^*)$ are not applied to a formula occurrence $@_a\Diamond\varphi$ or $(@_a\Box\varphi)^*$ on a branch Θ if there exists a nominal b such that $a \subseteq_\Theta b$.*

A branch is *closed* iff there is a formula ψ for which ψ and ψ^* are in that branch. A QH tableau is *closed* iff every branch is closed. A branch is *open* if it is not closed and there are no more rules to apply. A tableau is *open* if it has an open branch.

A terminal tableau is a tableau where the rules have been exhaustively used *i.e.*, there are no more rules applicable to the tableau obeying the restrictions of the tableau construction.

Definition

Let U be the subset of Nom^{Θ} containing any nominal a having the property that there is no nominal b such that $a \subseteq_{\Theta} b$. Let $\approx$ be the restriction of $\sim_{\Theta}$ to U.

Note that U contains all nominals present in the root formulas since they are the first formulas of the branch Θ. Θ is closed under the rules (Ref) and (Nom1), so both $\sim_{\Theta}$ and $\approx$ are equivalence relations.

Given a nominal a in U, we let $[a]_{\approx}$ denote the equivalence class of a with respect to $\approx$ and we let $U/\approx$ denote the set of equivalence classes.

Definition

Let R be the binary relation on U defined by aRc if and only if there exists a nominal $c' \approx c$ such that one of the following two conditions is satisfied:

1. *The formula $@_a\Diamond c'$ occurs on Θ.*
2. *There exists a nominal d in Nom^{Θ} such that the formula $@_a\Diamond d$ occurs on Θ and $d \subseteq_{\Theta} c'$.*

We let $\bar{R}$ be the binary relation on $U/\approx$ defined by $[a]_{\approx}\bar{R}[c]_{\approx}$ if and only if aRc.

Definition

Let $\bar{N} : U \to U/\approx$ be defined as $\bar{N}(a) = [a]_{\approx}$.

Definition

Let V^{+} be the function that to each ordinary propositional symbol assigns the set of elements of U where that propositional variable occurs, i.e., $a \in V^{+}(p)$ iff $@_a p$ occurs on Θ. Analogously, let V^{-} be the function that to each ordinary propositional symbol assigns the set of elements of U where the negation of that propositional variable occurs, i.e., $a \in V^{-}(p)$ iff $@_a \neg p$ occurs on Θ.
We let $V^{+}_{\approx}$ be defined by $V^{+}_{\approx}(p) = \{[a]_{\approx} \mid a \in V^{+}(p)\}$. We define $V^{-}_{\approx}$ analogously: $V^{-}_{\approx}(p) = \{[a]_{\approx} \mid a \in V^{-}(p)\}$.

Given a branch Θ, let $M^{\Theta} = \left(U/\approx, \bar{R}, \bar{N}, V^{+}_{\approx}, V^{-}_{\approx}\right)$. We will omit the reference to the branch in M^{Θ} if it is clear from the context.

Theorem (Model Existence)

Assume that the branch Θ is open. For any satisfaction statement $@_a\varphi$ which contains only nominals from U, the following conditions hold:

(i) *If $@_a\varphi$ s-occurs on Θ, then $M, [a]_{\approx} \models_s \varphi$*

(ii) *If $@_a\varphi$ w-occurs on Θ, then $M, [a]_{\approx} \models_{\mathrm{w}} \varphi$*

(iii) *If $(@_a\varphi)^*$ s-occurs on Θ, then $M, [a]_{\approx} \not\models_s \varphi$.*

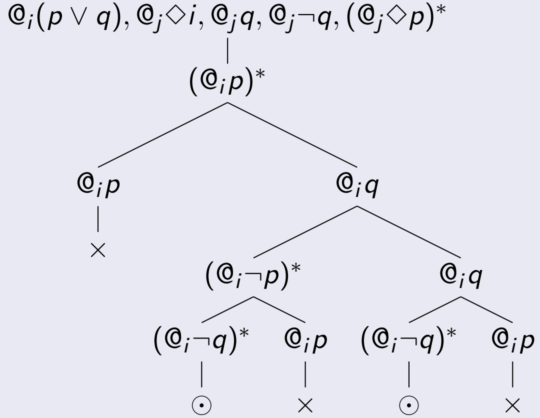
(iv) *If $(@_a\varphi)^*$ w-occurs on Θ, then $M, [a]_{\approx} \not\models_{\mathrm{w}} \varphi$.*

Decision Procedure

Given a database Δ and a query $@_a\varphi$ whose consequence from Δ we want to decide, let τ_n be a terminal tableau generated by the tableau construction algorithm. If the tableau is closed, then $@_a\varphi$ is a consequence of Δ. Analogously, if the tableau is open, then $@_a\varphi$ is not a consequence of Δ.

Example

Let $\Delta = \{@_i(p \vee q), @_j\Diamond i, @_j q, @_j\neg q\}$ *be a database and consider a query* $\varphi = @_j\Diamond p$.



The tableau is open, thus φ *is not a consequence of* Δ.

Example

Let $\Delta = \{@_t(p \wedge q \wedge r), @_i\Box\neg p, @_i\Diamond t\}$ *be a database and consider a query* $\varphi = (@_t(p \wedge \neg p))$.

$$@_t(p \wedge q \wedge r),\ @_i\Box\neg p,\ @_i\Diamond t,\ (@_t(p \wedge \neg p))^*$$

$$|$$

$$@_t p, @_t q, @_t r$$

$$|$$

$$@_t \neg p$$

$$(@_t p)^* \qquad (@_t \neg p)^*$$

$$| \qquad\qquad |$$

$$\times \qquad\qquad \times$$

Note that the database has an inconsistency and the query itself is inconsistent. However, from the tableau procedure we verify that, since it is closed, φ *is a consequence of* Δ.

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