A Complete Decision Procedure for Linearly Compositional Separation Logic with Data Constraints

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Outline

- Background
- Linearly compositional Separation logic with inductive definitions ($SLID_{LC}$)
- Satisfiability
- Entailment

Background

- **Dynamic data structures**
  - Plenty of them
    - lists,
    - nested lists,
    - trees (binary search trees, red-black trees,…)
  - Widely used in system software
    - operating system kernel
    - drivers
    - …
  - Notoriously difficult to specify, analyze, and verify
    - unbounded size
    - data values over infinite domain
    - pointer arithmetic

Background

- **Separation logic (SL)**
  - Proposed around 2000, by P.O.Hearn, J.C. Reynolds, H. Yang, …
  - Local reasoning about heaps: Separation + Frame rule
  - Syntax

    First order logic + separating operators (separating conjunction and implication)
  - SL with inductive definitions (SLID)

    $$X \neq Z \wedge X \mapsto (next, Y) * lseg(Y, Z)$$

    Inductive definition of *lseg*:

    $lseg(E, F) := E = F \wedge emp,$

    $lseg(E, F) := \exists X.\ E \mapsto (next, X) * lseg(X, F)$

Background

- **SLID solvers & verification tools based on SLID**
  - Smallfoot (UCL, UK),
  - Infer (Facebook, Queen Mary, Univ. of London, UK),
  - VeriFast (KU Leuven, Belgium),
  - SLEEK/HIP (NUS, Sigapore),
  - DRYAD (UIUC, USA),
  - CELIA (LIAFA, France),
  - SPEN (LIAFA, France),
  - SLIDE (Verimag, France)
  - GRASSHopper (NYU, USA)

Background

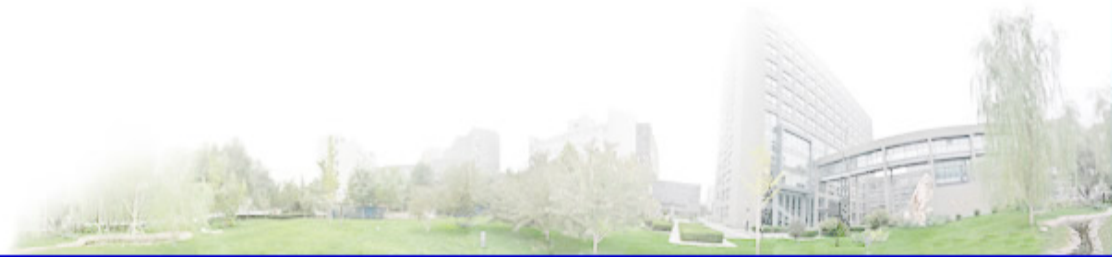
- **Usually incomplete decision procedures, especially when involving data and size constraints, e.g.**
  - VeriFast
  - SLEEK/HIP
  - DRYAD
  - SPEN
- **Why complete decision procedures ?**
  - Theoretically appealing
  - Practical importance, e.g.
    - Specification consistency,
    - Counterexample generation

Background

- **Our contribution**
  - **Linearly compositional** SLID ($SLID_{LC}$)

    lists, doubly linked lists, sorted lists,

    lists with length constraints, lists with tail pointers, …
  - **Complete** Decision procedures for $SLID_{LC}$
    - satisfiability
    - entailment



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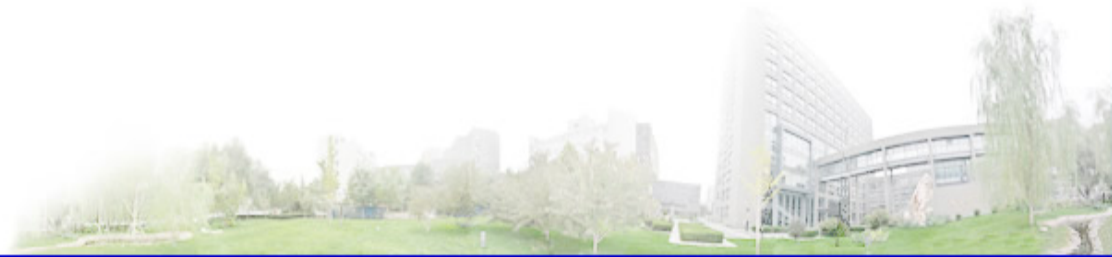


Linearly compositional SLID ($SLID_{LC}$)

- **Inductive predicates $P(E, \alpha; F, \beta; \xi)$**
  - E, α: source parameters,
  - F, β: destination parameters,
  - ξ: static parameters
- **For instance,**
  - *lseg*(E; F): list segments from E to F
  - *slseg*(E, x; F, x'): sorted list segments from E to F, x is the data value in E, x' is the data value in F,
  - *dllseg*(E, P; F, L): doubly linked list segments from E to F
  - *tlseg*(E; F; B): list segments from E to F with tail pointers to B.

Linearly compositional SLID ($SLID_{LC}$)

- **Syntax**

$\Pi ::= E = F \mid E \neq F \mid \Pi \wedge \Pi$ (Pure formula)

$\Delta ::= true \mid x \circ c \mid x \circ y + c \mid \Delta \wedge \Delta$ (Data formula)

$\Sigma ::= emp \mid E \mapsto \rho \mid P(E, \alpha; F, \beta; \xi) \mid \Sigma * \Sigma$ (Spatial formula)

$\rho ::= (f, X) \mid (d, x) \mid \rho, \rho$

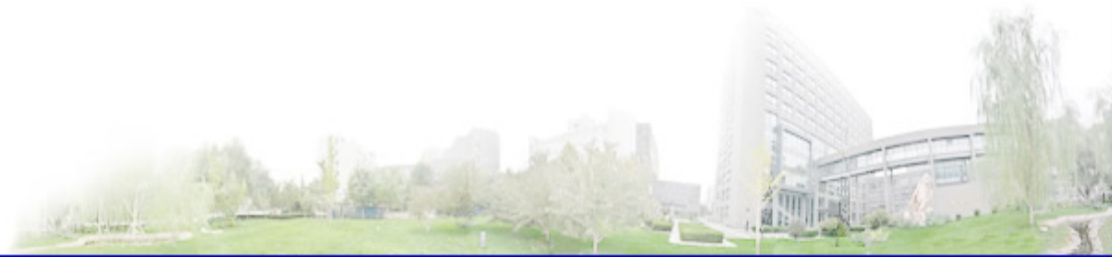
where $\circ \in \{=, \leq, \geq\}$, $c \in \mathbb{Z}$, $f \in \mathcal{F}$, $d \in \mathcal{D}$ and $P(E, \alpha; F, \beta; \xi)$ is a linearly compositional inductive predicate.

Linearly compositional SLID ($SLID_{LC}$)

- **An inductive predicate $P(E, \alpha; F, \beta; \xi)$ is linearly compositional if it is defined by the following two rules**
  - base rule: $P(E, \alpha; F, \beta; \xi) ::= E = F \wedge \alpha = \beta \wedge emp$,
  - inductive rule:
    $P(E, \alpha; F, \beta; \xi) ::= \exists X \exists x.\ \Delta \wedge E \mid\!\rightarrow \rho * P(Y, \gamma; F, \beta; \xi)$ s.t.
    - none of the variables from F, β occurs in $\Delta \wedge E \mid\!\rightarrow \rho$ (which guarantees the composition lemma $P(E_1,\alpha_1; E_2,\alpha_2; \xi) * P(E_2,\alpha_2; E_3,\alpha_3; \xi) \Rightarrow P(E_1,\alpha_1; E_3,\alpha_3; \xi)$)
    - each conjunct of Δ is of the form $\alpha_i \circ \xi_j + c$ or $\alpha_i \circ \gamma_i + c$
    - *for each* α_i, if α_i is a data variable, then either α_i occurs in ρ, or $\alpha_i = \gamma_i + c$ is a conjunct of Δ
    - ...

Linearly compositional SLID ($SLID_{LC}$)

- **Semantics**
  - A state (s, h),
    - s a store: assignment of values to variables
    - h a heap:
      a partial function of finite domain from
      $L \times (F \cup D)$ to $L \cup Z$
  - The semantics is defined by the relation $(s, h) \models \varphi$
    - $(s, h) \models \varphi_1 * \varphi_2$ if h is partitioned into two domain-disjoint heaps h_1 and h_2 s.t. $(s, h_1) \models \varphi_1$ and $(s, h_2) \models \varphi_2$
    - The semantics of $P(E, \alpha; F, \beta; \xi)$ is defined as the least fixed point.



Linearly compositional SLID ($SLID_{LC}$)

- **Examples**
  - Sorted list segments,

    $slseg(E, x; F, x') ::= E = F \wedge x{=}x' \wedge emp,$

    $slseg(E, x; F, x') ::= \exists X, x''.\ x \leq x'' \wedge E|{\rightarrow}((next, X),(data, x))^* slseg(X, x''; F, x')$
  - Doubly linked list segments,

    $dllseg(E, P; F, L) ::= E = F \wedge P = L \wedge emp,$

    $dllseg(E, P; F, L) ::= \exists X.\ E\ |{\rightarrow}((next, X),(prev, P))^* dllseg(X, E; F, L)$
  - Doubly linked list segments with length function,

    $ldllseg(E, P, x; F, L, x') ::= E = F \wedge P = L \wedge x{=}x' \wedge emp,$

    $ldllseg(E, P, x; F, \mathrm{L}, x') ::= \exists X, x''.\ x{=}x''{+}1 \wedge E|{\rightarrow}((next, X),(prev,P)) * ldllseg(X, E, x''; F, L, x')$
  - ...

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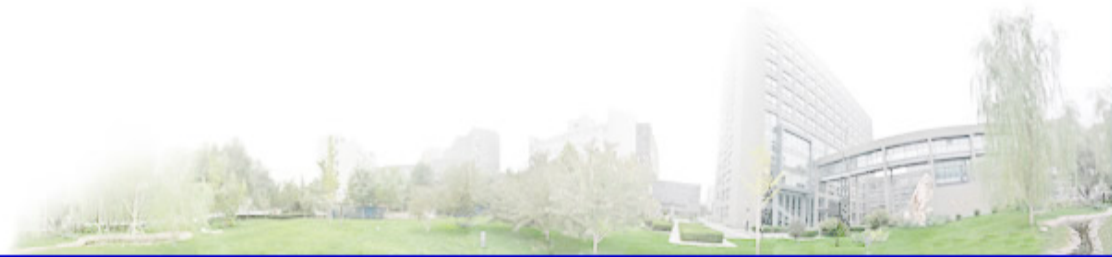
- Entailment



Satisfiability

- **Thm. Satisfiability of $SLID_{LC}$ is in NP.**
  - Reducuction to satisfiability of quantifier free linear arithmetic constraint (discharged by some SMT solver e.g. Z3)
  - E.g. $\varphi = E_1 = E_4 \wedge x_1 > x_2 + 1 \wedge ldllseg(E_1, E_3, x_1; E_2, E_4, x_2)$
    - The formula φ is unsatisfiable
    - $Abs(\varphi) = E_1 = E_4 \wedge x_1 > x_2 + 1 \wedge$
      $(\neg[E_1, 1] \wedge \neg[E_4, 1] \wedge E_1 = E_2 \wedge E_3 = E_4 \wedge x_1 = x_2 \wedge k_1 = 0) \vee$
      $([E_1, 1] \wedge [E_4, 1] \wedge E_1 = E_4 \wedge k_1 = 1 \wedge x_1 = x_2 + k_1) \vee$
      $([E_1, 1] \wedge [E_4, 1] \wedge E_1 \neq E_4 \wedge k_1 \geq 2 \wedge x_1 = x_2 + k_1)$
      where $[E_1, 1]$, $[E_4, 1]$ are the newly introduced Boolean variables
      The formula $Abs(\varphi)$ is unsatisfiable.

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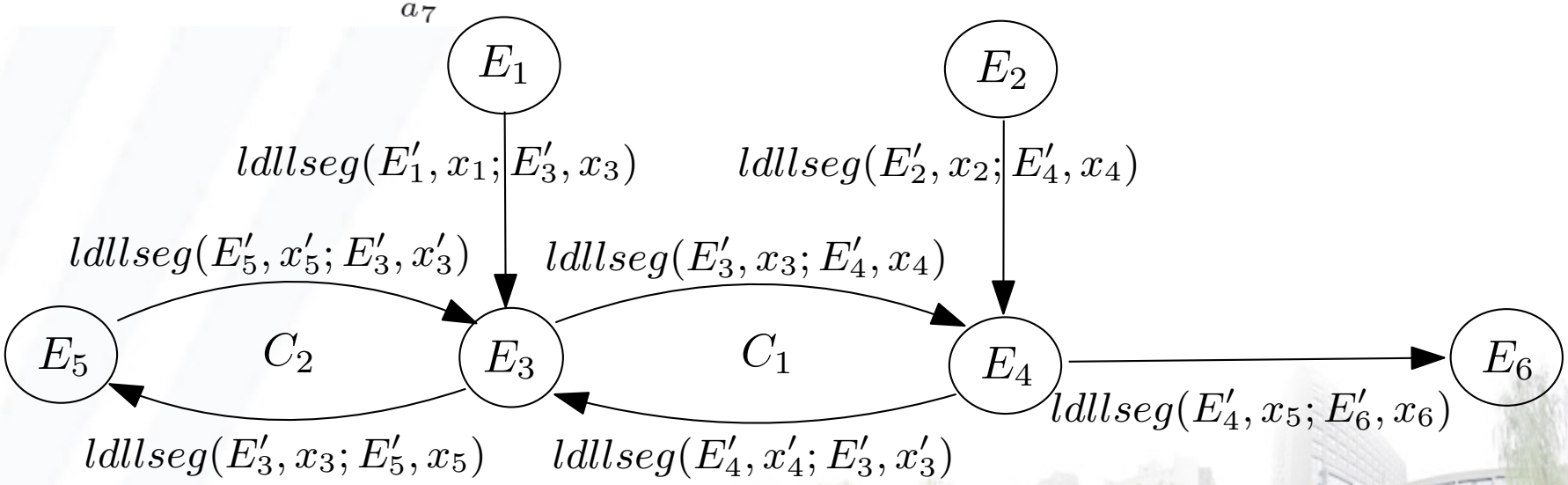
Entailment

- **Thm. Entailment of $\mathbf{SLID}_{\mathbf{LC}}$ is in Π_3^P**
  - The problem $\varphi \models \psi$ s.t. $\mathrm{Vars}(\psi) \subseteq \mathrm{Vars}(\varphi)$, and $\mathit{Flds}(\varphi) = \mathit{Flds}(\psi)$.
    E.g. $\mathit{ldllseg}(E, P, x; F, L, x') \models \mathit{dllseg}(E, P; F, L)$
  - Idea: Extend the idea of graph homomorphisms (Cook et al. CONCUR 2011)
  - Proof sketch
    - $\varphi \Rightarrow G_\varphi$ (resp. $\psi \Rightarrow G_\psi$):
      $E \sim F$ iff $\mathit{Abs}(\varphi) \models E = F$ (resp. $\mathit{Abs}(\psi) \models E = F$)
    - Allocating plans $\mathcal{AP}$ of G_φ :
      $G_{\mathcal{AP}}$: DAG-like graphs obtained by simplifying G_φ
    - For each allocating plan $\mathcal{AP}$,
      check the existence of a homomorphism from G_ψ to $G_{\mathcal{AP}}$

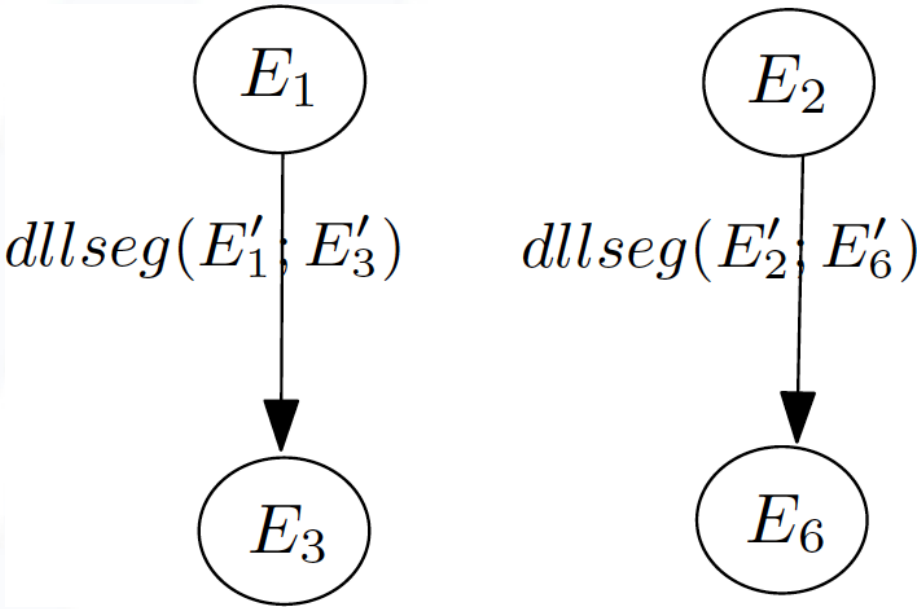
Graph representation G_φ

$$\varphi = \underbrace{ldllseg(E_1, E_1', x_1; E_3, E_3', x_3)}_{a_1} * \underbrace{ldllseg(E_2, E_2', x_2; E_4, E_4', x_4)}_{a_2} *$$
$$\underbrace{ldllseg(E_3, E_3', x_3; E_4, E_4', x_4)}_{a_3} * \underbrace{ldllseg(E_4, E_4', x_4'; E_3, E_3', x_3')}_{a_4} *$$
$$\underbrace{ldllseg(E_3, E_3', x_3; E_5, E_5', x_5)}_{a_5} * \underbrace{ldllseg(E_5, E_5', x_5'; E_3, E_3', x_3')}_{a_6} *$$
$$\underbrace{ldllseg(E_4, E_4', x_5; E_6, E_6', x_6)}_{a_7}.$$

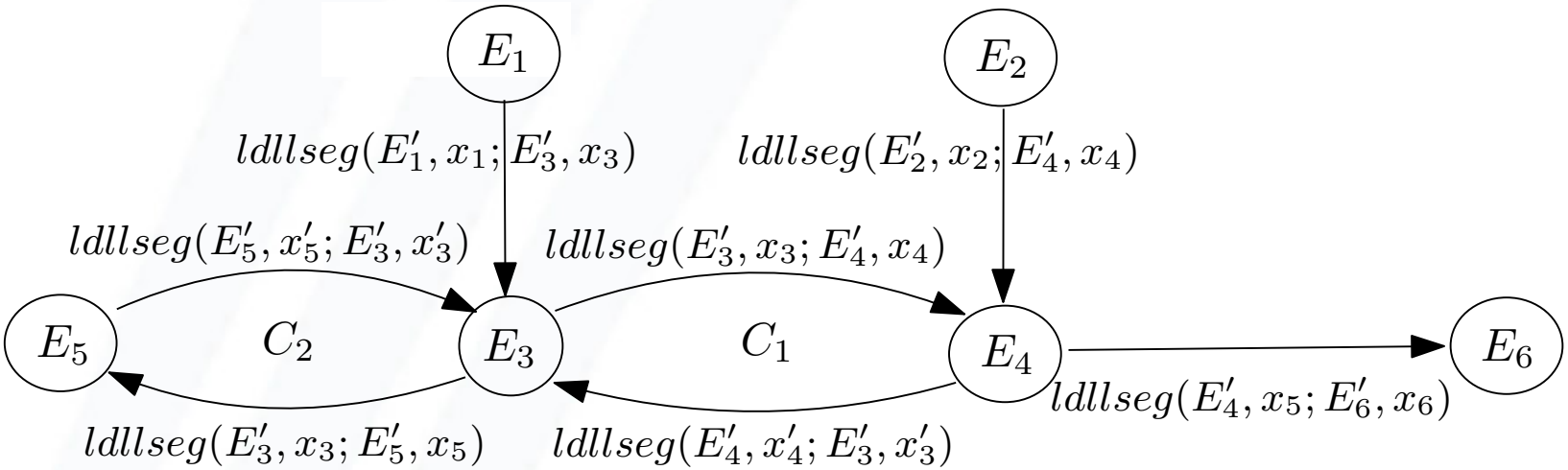


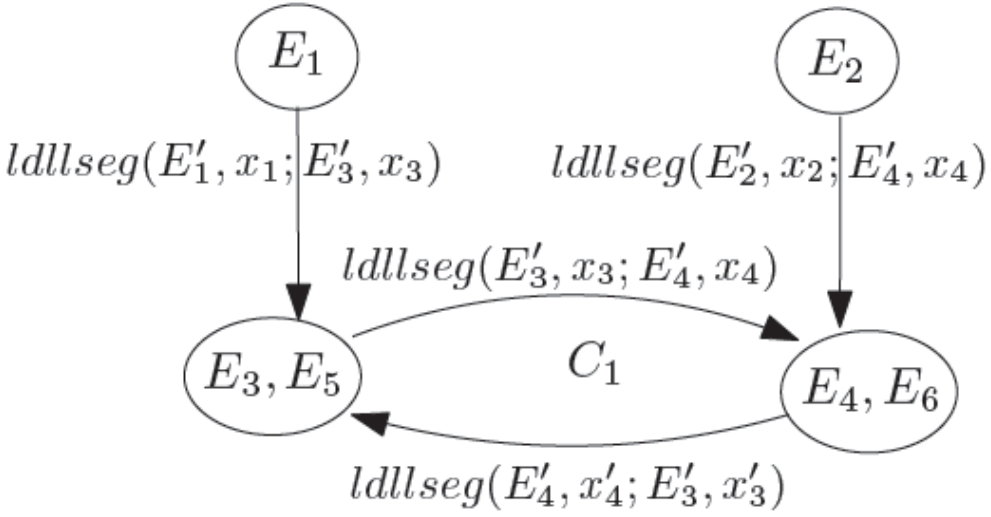
Graph representation G_ψ

- $\psi = dllseg(E_1, E_1'; E_3, E_3') * dllseg(E_2, E_2'; E_6, E_6')$



Allocating plans of G_φ





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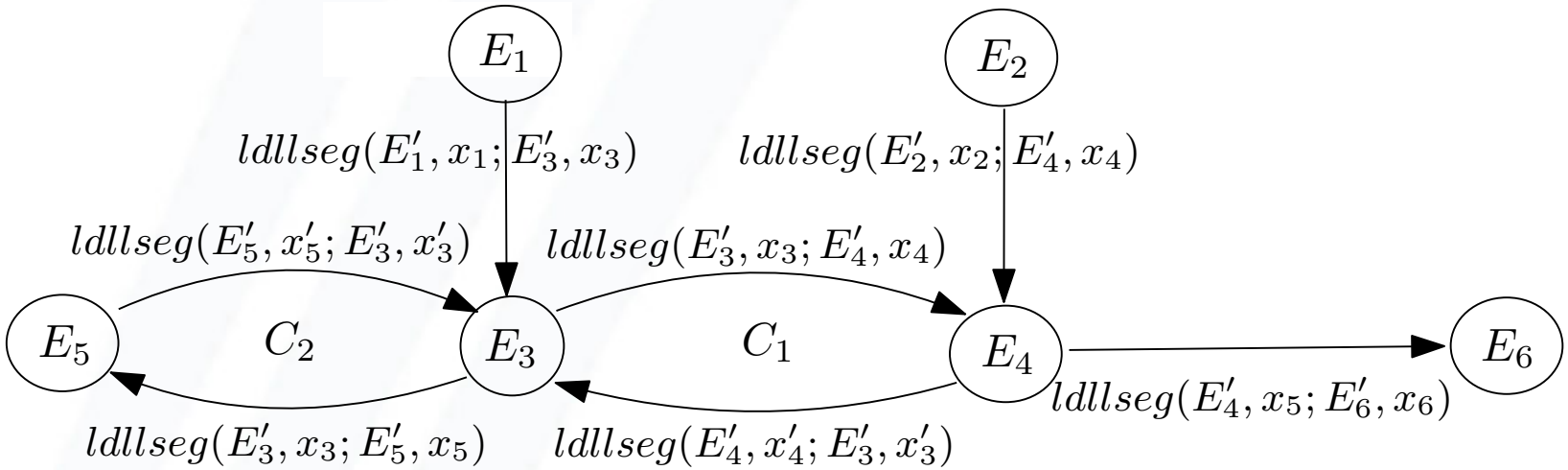

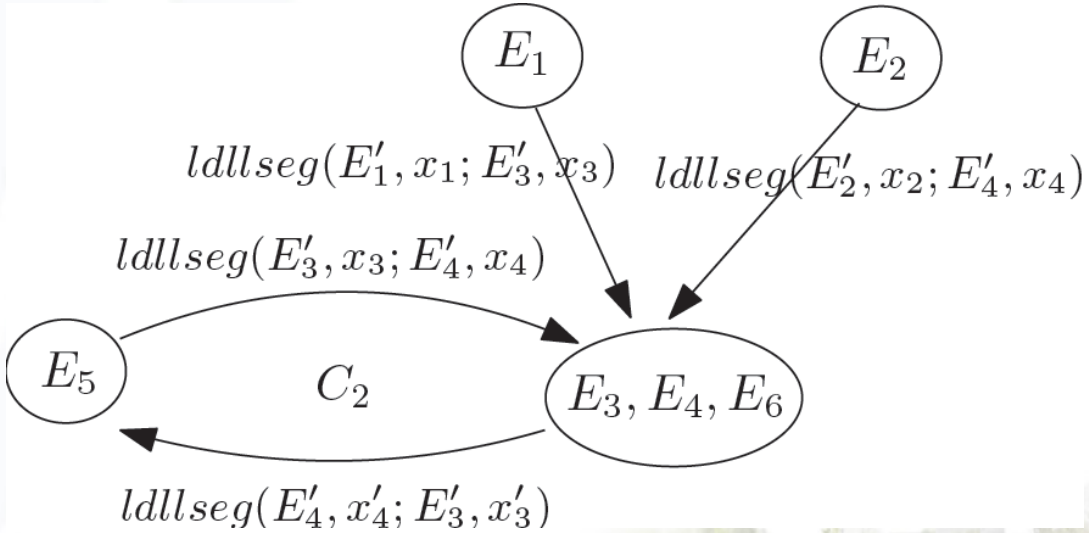

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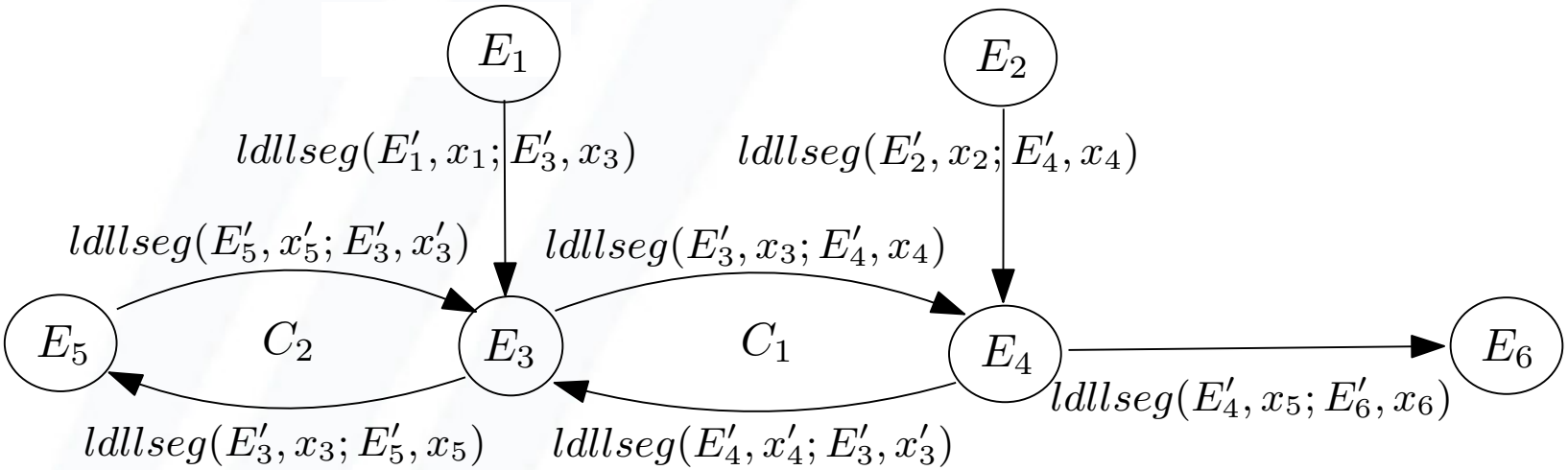
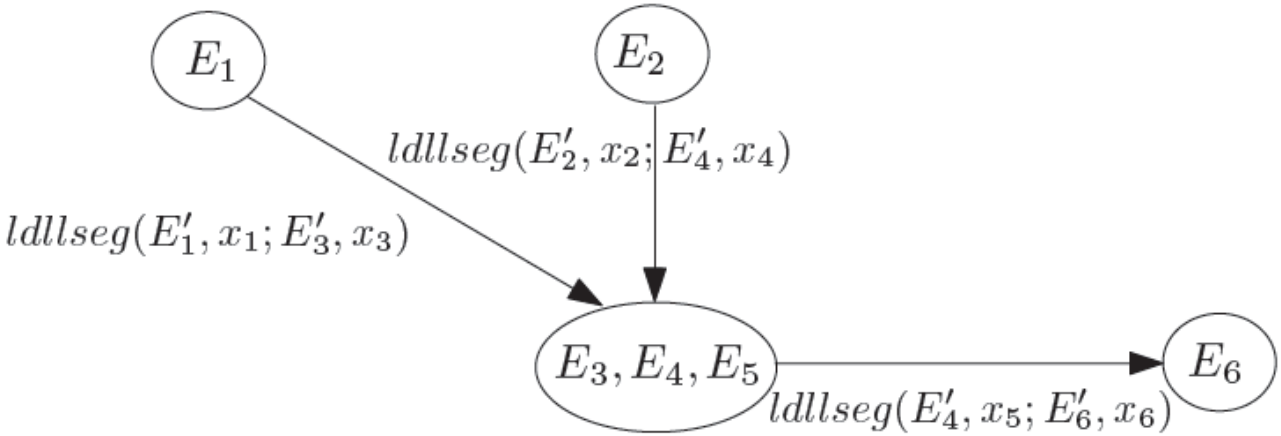
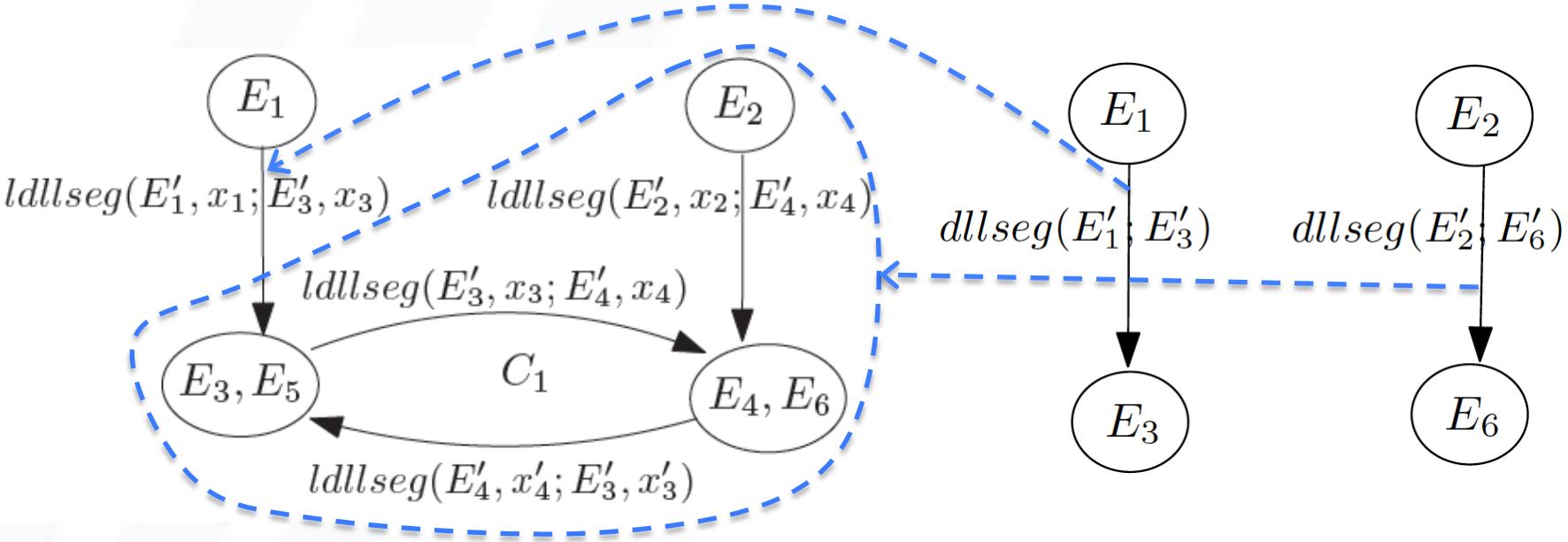




Homomorphism from G_{ψ} to $G_{\mathcal{AP}}$



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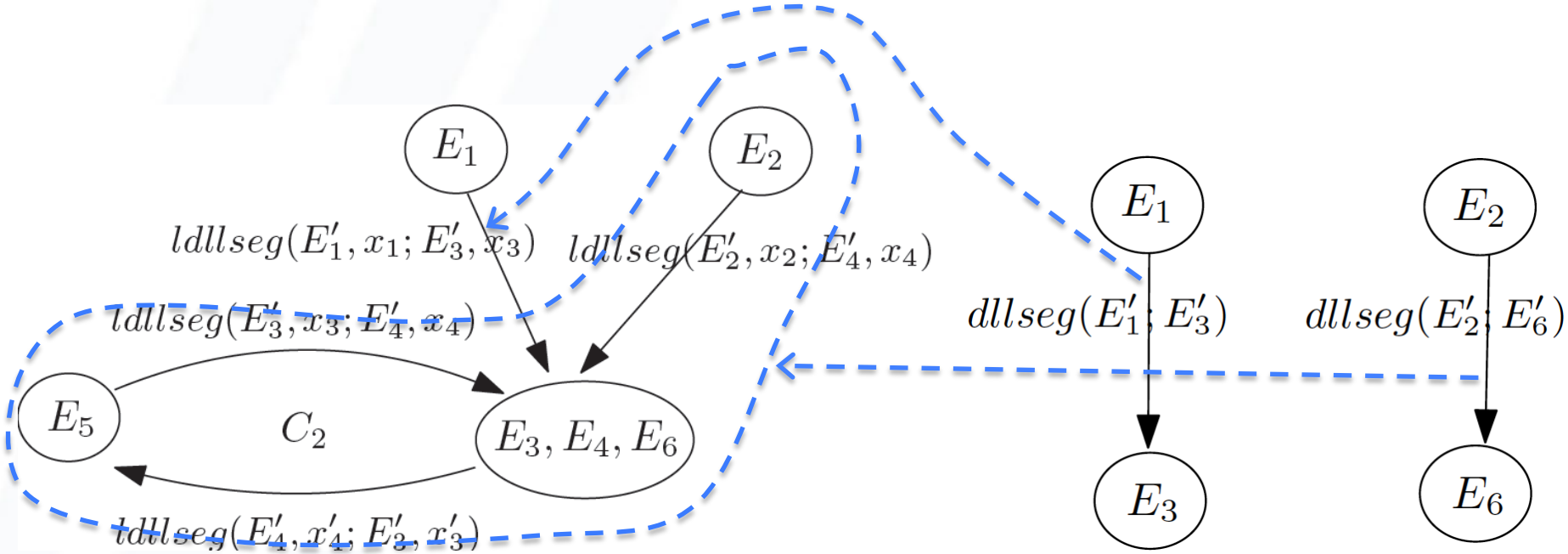

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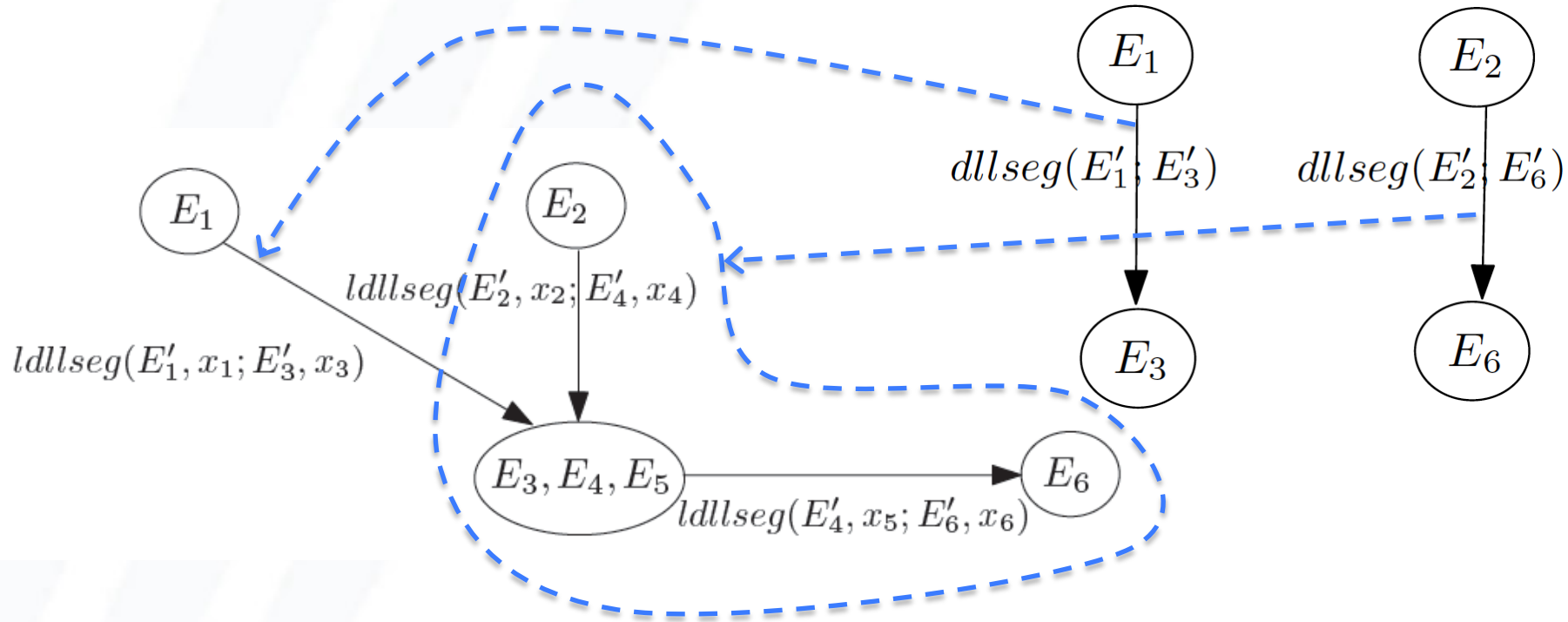

Conclusion and future work

- **To summarize**
  - Linearly compositional separation logic with inductive definitions ($SLID_{LC}$)
  - Complete decision procedures for the satisfiability and entailment problem of $SLID_{LC}$
- **Ongoing and future work**
  - We are implementing the decision procedures (Based on SPEN)
  - Extend to set, multiset constraints
  - Extend to tree structures (e.g. binary search trees, red-black trees, …)