

A Coupled Wave-Diffusion Model for Enhanced Drug Delivery

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ABSTRACT

Enhanced and controlled delivery of molecules (e.g. drug, proteins, DNA) into cells with the aid of electric fields is a hot topic in molecular medicine. In this context, the role of the electric field is two-fold: one, to permeabilize the cellular membrane allowing the introduction of the molecules into the cell, a process known as electroporation; two, to advance and control the migration of the charged molecules into the cell. This last technique, known as electrophoresis, is particularly crucial when the cells have a short resealing time or when large molecules need to be loaded into cells.

The mathematical modeling of these biological and physical processes involves two main equations, a parabolic convection-diffusion equation that describes the evolution of the concentration of molecules, in and outside the cell, and Maxwell's equations for the electromagnetic waves. Ignoring the magnetic field, the Maxwell system can be reduced to a wave equation for the electric field.

In this work we study a finite difference method for the wave-diffusion coupled problem that is based on piecewise linear finite element approximations. Second order error estimates with respect to a discrete H^1 -norm are established provided that, for each time level, the solution of the coupled problem is in $H^3(\Omega) \times H^3(\Omega)$, where Ω is the spatial domain.