Four Geometry Problems to Introduce Automated Deduction in Secondary Schools

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Introducing Automated Deduction in Secondary Schools

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- the absence of the subject of rigorous mathematical demonstrations in the curricula;
- the lack of knowledge by the teachers about the subject;
- the difficulty of tackling the task by automatic means.

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- meaningfull but simple to manipulate by students and teachers;
- reasonably easy to be dealt by automatic means;
- with a natural language and geometric renderings.

Problem 1

Theorem

Show that for any given convex quadrilateral, [ABCD], that [EFGH], where each of the points is the mid point of a segment in [ABCD], is a parallelogram.

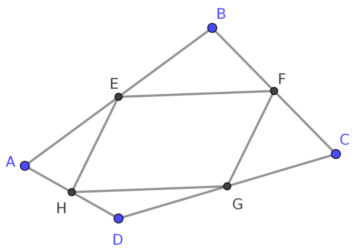

Figure: *Problem 1*

Problem 1 — Rigorous Proof

Proof.
Consider any convex quadrilateral $[ABCD]$, as shown in figure 1 where E, F, G and H are the midpoints of $[AB]$, $[BC]$, $[CD]$ and $[DA]$ respectively.
(...)
Given that, the angles $\angle BAC$ and $\angle BEF$ are congruent and as these angles have a common side, then $[EF] \| [AC]$.
(...)
Like $[EF] \| [GH]$ and $[EH] \| [FG]$ then the vertices of the midpoints of any quadrilateral convex are vertices of a parallelogram. □

Problem 1 — Formal Proofs

GeoGebra visual dynamic and numeric check, algebraic formal proof — without a proof script.

JGEX Geometry deductive database method; full-angle method; Gröbner bases method; Wu's method (less then 1 second) — formal proofs with visual helps.

GCLC area method, 0.001 seconds; Wu's method, 0.051 seconds; Gröbner bases method 0.132 seconds — with formal proof scripts.

Vampire Geometry deductive database axioms, resolution method 15.382 seconds — formal proof script.

This is a **good example**, at least from the point of view of GATP.

GeoGebra JGex GCLC Vampire

Problem 2

Theorem

Consider the convex quadrilateral [ABCD], like the one represented in the figure 2, Assuming that $|BD| > |BC|$ *and that* $\alpha = \angle BAC > \angle ABC = \beta$*, show that* $|BD| > |AC|$.

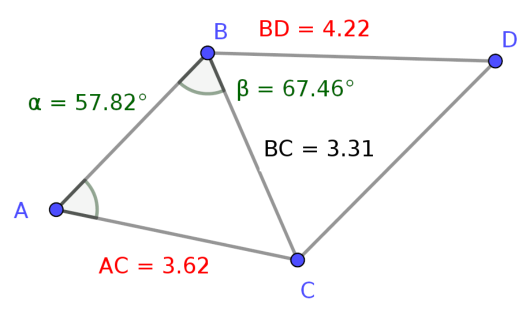

Figure: *Problem 2*

Problem 2 — Rigorous Proof

Proof.

Given that in any triangle, the angle with the greatest amplitude is opposed to the side with the longest length, we have that in the triangle ΔABC, $|BC| > |AC|$.

Given that $|BD| > |BC|$, by hypothesis and, $|BC| > |AC|$, just proved above, due to the transitive property of the $>$ order relation we have $|BD| > |AC|$. □

Problem 2 — Formal Proof

The different formal proving approaches are not possible given that the current axiomatic systems/methods do not deal with inequality (some new approaches are being proposed).

GeoGebra visual (dynamic) check.

JGEX visual (dynamic) check.

From the view point of the student this is "one more geometric conjecture" but. . . , from the view point of a formal proofs we are not anymore in a "simple" geometric proof, but one involving a field (real numbers).

GeoGebra

New Approaches / Old Approaches Made New

Proof systems sound, not necessarily complete, tailored to the needs of the secondary school.

- Deductive databases approaches.
- Prolog rule systems.
- Maude Axiomatic theories.

Readable Proofs

- Synthetic proofs with proof scripts.
- Connection between the synthetic proof and the construction.

Tutorial Systems

Deductive Database Tools

Axiom systems "à la carte"

Geometry Deductive Database Method

```
fof(ruleD1,axiom,( ! [A,B,C] :
    (coll(A,B,C) => coll(A,C,B)) )).
fof(ruleD2,axiom,( ! [A,B,C] :
    (coll(A,B,C) => coll(B,A,C)) )).
fof(ruleD3,axiom,( ! [A,B,C,D] :
    ((A!=B & coll(A,B,C) & coll(A,B,D)) => coll(C,D,A)) )).
fof(ruleD4,axiom,( ! [A,B,C,D] :
    (para(A,B,C,D) => para(A,B,D,C)) )).
fof(ruleD5,axiom,( ! [A,B,C,D] :
    (para(A,B,C,D) => para(C,D,A,B)) )).
(...)
```

Apart the rules concerning the properties of the geometric objects in consideration, any kind of high-level lemmas can also be added.

Forward-chaining is usually used, in a synthetic geometric proof.

Maude equational (and rewriting) logic system

Implementing Tarski's axiom system, as described by Art Quaife in Maude.

```
*** System Tarski over G3cp
fmod FORMULA is
pr QID . *** Maude's Qualified Identifiers ('a,'b, etc).
sorts Prop Formula Point Segments. *** Atomic propositions, Formulas, Points, Segments
subsort Qid < Prop < Formula .
subsort Qid < Point .
*** Tarski geometry primitive relations
op p : —> Point [ctor] .
op pl : —> Point [ctor] .
op pll : —> Point [ctor] .
op _*_ : Point Point —> Segment [ctor comm] .
op betweenness : Point Point Point —> Prop [ctor] .
op equidistance : Segment Segment —> Prop [ctor comm] .
op innerPasch : Point Point Point Point Point —> Point .
endfm
mod Tarski is
*** Tarski' Geometry (Art Quaife (1989), JAR 5, 97——118.
*** A7 Inner Pasch
rl [ip1] : C , betweenness(U,V,W), betweenness(Y,X,W) |—— betweenness(V,innerPasch(U,V,W,X,Y),Y) , C' =>
      proved .
rl [ip2] : C , betweenness(U,V,W), betweenness(Y,X,W) |—— betweenness(X,innerPasch(U,V,W,X,Y),U) , C' =>
      proved .
```

Again, any kind of high-level lemmas can also be added.

Tutorial Systems

A new tutorial system with automated deduction in the background.

The **QED-tutrix** tutorial system builds the *Hypothesis, Properties, Definitions, Intermediate results and Conclusion graph* (HPDIC-graph).

The HPDIC-graph contains all possible proofs for a given problem, using a given set of axioms (using *Prolog* rule-based logical query mechanism).

Conclusions & and Future Research

- \+ many GATP available.
- \+ integration in larger systems DGS, Tutorial systems, etc.
- \- gap between the teachers knowledge and practice and formal proofs.
- \- proof rendering (natural and visual languages).

I would like to say, "Automated Deduction is making is way into Secondary Schools" . . . but this is, for now, a bit too optimistic.

- \+ "à la carte" axiom systems;
- \+ integration between DGS and GATP.

Obrigado