A Rule Based Theorem Prover: An Introduction to Proofs in Secondary Education

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Deductive Reasoning in Education

The introduction of automated deduction systems in secondary schools[1] face several bottlenecks:

(A) the absence of the subject in many of the national curricula;

(B) the lack of knowledge (and/or training) by the teachers, Santos and Quaresma [2021];

(C) the dissonance between the (formal) outcomes of the available Automated Theorem Provers (ATP) and the normal practice (informal) of conjecturing and proving.

All forms of reasoning have had, and continue to have, their place in the mathematics curriculum, Hanna [2020].

[1] ISCED 2011, International Standard Classification of Education, UNESCO 2011

Automated Deductive Reasoning in Education

The new Portuguese's curriculum documents, DGE [2018, 2021], also reinforce:

- the importance of promoting and mobilising computational thinking;
- the ability to analyse and define algorithms, allowing a structuring of thinking;
- providing students with more tools to solve problems and prove results.

Is this the time to fight for the introduction of automated theorem provers in education?

(C) $\leadsto$ (A) $\wedge$ (B), the ATP need to be usable for the general public, not only for the ATP experts.

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Since the early attempts, linked to **A**rtificial **I**ntelligence, synthetic provers for geometry, based on inference rules and using forward chaining reasoning can be seen as a more suited for education approach.

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The area of geometry is suitable for the intended goal. From several viewpoints it has some advantages:

- **Logic** a self-contained simple set of inference rules;
- **Educational** a strong visual appealing;
- **Tools** already some very well-known tools available (DGS);
- **GATP** some tools filling the gap, (C).

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The area of geometry is suitable for the intended goal. From several viewpoints it has some advantages:

- **Logic** an (almost) self-contained (almost) simple set of inference rules;
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A Rule Based GATP

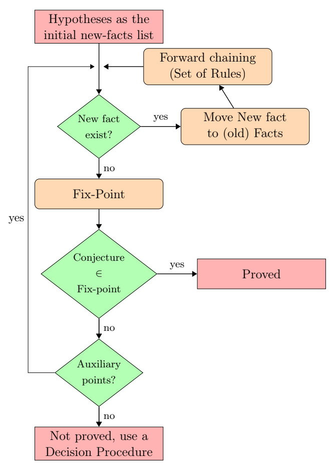
The **deductive database method** (for geometry) is an efficient implementation of a synthetic method, based on a set of inference rules, Chou et al. [2000].

For a given geometric configuration, the program can find all the properties of the geometric construction (the **fix-point**) that can be deduced using a fixed set of geometric rules/axioms.

After finding the fix-point, if the conjecture is among all the deduced facts, **the conjecture is proved**. **A synthetic proof can then be generated with a natural language and visual renderings**.

Deductive Database Method (for Geometry)

The algorithm is simple, a data-based search strategy is used, a list of "new data" is kept and for each new data the system searches the rule set to find and apply the appropriate rule, Chou et al. [2000].



Automated Deductive Reasoning in Secondary Education

In step-wise learning, it is intended that there will be a progressive proficiency in the use of hypothetical-deductive reasoning and mathematical argumentation in primary and secondary school.

Exploring the Portuguese curriculum:

- It is expected that by the 7th year students will be able to elaborate, with some accuracy, small proofs, MEC [2013].
- the most appropriate domain to put that in practice is "Geometry", DGE [2021].

JGEx — A Rule Based Theorem Prover

In a previous work[2] an overview of the automated deduction tools that can be used in education, was presented.

Here we resume that focusing in two implementations of the deductive databases method for geometry: *JGEx* and *OGPCP GDDM*.

JGEx a system which combines an approach for visually dynamic presentation of proofs (VDDP), dynamic geometry software (DGS) and automated theorem prover for geometry (GATP).

OGPCP GDDM an open source implementation of the deductive databases method for geometry as part of the OGPCP[3] library of GATPs.

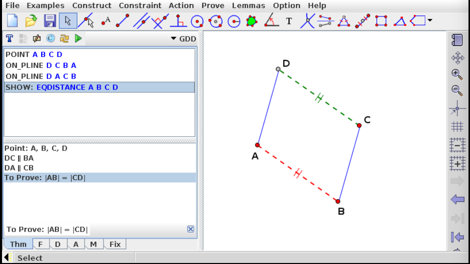
[2] Quaresma and Santos [2022] – P. Quaresma and V. Santos, *Four Geometry Problems to Introduce Automated Deduction in Secondary Schools*, EPTCS 354, DOI: 10.4204/eptcs.354.3

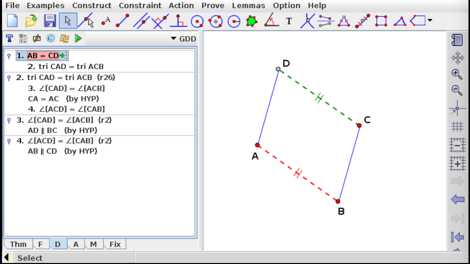
[3] Open Geometry Prover Community Project

A 7^{th} Year Geometry Problem

Theorem (Opposite Sides of a Parallelogram)

If [ABCD] is a parallelogram then the opposite sides are equal, i.e. $\overline{AB} = \overline{CD}$ *and* $\overline{AD} = \overline{BC}$.





[AB] denotes the line segment with endpoints A and B, $\overline{AB}$ denotes the length measure of [AB], $[A_1A_2\ldots A_n]$ denotes the polygon with vertices $A_1, A_2, \ldots A_n$, $A\hat{B}C$ denotes the measure of the amplitude of the angle ABC.

JGEx — Deductive Database Method for Geometry

- +++ visually dynamic presentation of proofs;
- ++ DGS and GATP integrated;
- + Rule-based synthetic method (among others);
- – unknown (at least in Portugal);
- – interface in English (only);
- – – unusual geometric construction language;
- – – unusual set of rules;
- – – – **Not currently maintained by the original authors**.

In GitHub (since 2016) as an open source project, and (maybe) being resumed by Zoltán Kovács, and also by Pedro Quaresma.

OGPCP GDDM, Deductive Database Method for Geometry

The *OGPCP GDDM* is an open source implementation of the deductive database method geometry, that uses an in-memory relational database to enjoy the efficient information management and query mechanism provided by the SQL engine.

- ++ implemented as a library, so usable within any software system;
- ++ integrated in the OGPCP;
- + Rule-based synthetic method;
- ± raw GATP, no GUI interface provided;
- ∓ modular, it is expected that it can accept different set of rules;
- ∓ **still being developed**.

A Set of Rules for the Secondary Education

A different set of rules, the *LF set of rules*, (*QED-Tutrix*, Font [2021], Font et al. [2018], Gagnon et al. [2017]), more adapted to the 7th year in the Portuguese curricula, DGE [2018, 2021], MEC [2013].

- R1 (definition of parallelogram) If $[ABCD]$ is a parallelogram then AB is parallel to CD and BC is parallel to AD.
- R2 (definition of alternate interior angles);
- R3 If two lines are parallel, the alternate interior angles determined by a transversal are equal;
- R4 (ASA criterion of equality of triangles) Two triangles are equal if two corresponding angles and the side in between are equal;
- R5 Given two equal triangles $[ABC]$ and $[DEF]$, the sides and the corresponding angles are equal $\overline{AB} = \overline{DE}$, $\overline{BC} = \overline{EF}$, $\overline{CA} = \overline{FD}$, and for the angles $B\widehat{A}C = E\widehat{D}F$, $C\widehat{B}A = F\widehat{E}D$ and $A\widehat{C}B = D\widehat{F}E$;
- R6 (definition of rectangle) If $[ABCD]$ is a rectangle then the interior angles are all right angles;
- R7 If ABC is a right angle then the lines AB and BC are perpendicular;
- R8 Two lines perpendicular to a third line are parallel to each other;
- R9 In a quadrilateral $[ABCD]$, if the lines AB and CD are parallel and the lines AD and BC are also parallel, then $[ABCD]$ is a parallelogram;
- R10 (SAS criterion of equality of triangles): Two triangles are equal if two corresponding sides and the angle in between are equal.

Problem 1's Proof

This, still manual, proof, is expected to be produced by a rule-based theorem prover, implementing the *LF set of rules*, and having a natural language rendering.

Proof.

[*ABCD*] is a parallelogram, by rule **R1** (parallelogram definition), the lines *AB* and *CD* are parallel and the lines *AD* and *BC* are also parallel.

By rule **R2** (definition of alternate interior angles) the angles *BAC* and *DCA* are alternate interior angles determined by the lines *AB* and *CD* cut by a transversal *AC*.

By rule **R3**, since the lines *AB* and *CD* are parallel, the angles *BAC* and *DCA* are equal.

By rule **R2** (alternate interior angles definition) the angles *ACB* and *CAD* are alternate interior angles determined by the lines *AD* and *BC* cut by the transversal *AC*.

By rule **R3**, since the lines *AD* and *BC* are parallel, the angles *ACB* and *CAD* are equal.

Since *BAC* and *DCA* are equal angles, *ACB* and *CAD* are equal angles and, $\overline{AC} = \overline{CA}$, by rule **R4** (ASA criterion of equality), triangles [*ABC*] and [*CDA*] are equal.

Finally, using rule **R5**, we have $\overline{AB} = \overline{CD}$ and $\overline{BC} = \overline{DA}$. □

Conclusions and Future Work

The goal: A natural language set of rules; a rule-based GATP; a natural language and visual renderings of the proof.

- An open source implementation of the deductive database method for geometry; ✓
- The formalisation of the natural language set of rules;
- An interface that can "wrap" the GATP and provide natural language and visual renderings of the proofs;
- Case-study(ies) with pre-service teachers.

Obrigado

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