Visual Geometry Proofs in a Learning Context

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International Workshop on Theorem proving components for Educational software, ThEdu'15, July 15, 2015, Washington DC, USA

[1] The first author is partially supported by iCIS project (CENTRO-07-ST24-FEDER-002003) is co-financed by QREN, in the scope of the Mais Centro Program and European Union's FEDER.

Proof and Proving in Mathematics Education

List of the usefulness of proofs and proving in a learning environment (by Gina Hanna, ICMI Study 19 Conference):

- **verification** (concerned with the truth of a statement);
- **explanation** (providing insight into why it is true);
- **systematisation** (the organisation of various results into a deductive system of axioms, major concepts and theorems);
- **discovery** (the discovery or invention of new results);
- **communication** (the transmission of mathematical knowledge);
- **construction of an empirical theory**;
- **exploration of the meaning of a definition** or the consequences of an assumption;
- **incorporation of a well-known fact into a new framework** and thus viewing it from a fresh perspective.

The Best Proof

> *The best proof is one that also helps understand the meaning of the theorem being proved: to see not only that it is true, but also why it is true. Of course such a proof is also more convincing and more likely to lead to further discoveries.*
>
> *Gina Hanna, ICMI Study 19 Conference*

In the classroom, the fundamental question that a proof must address is surely 'why?' (. . .) as an explanation, and in consequence to value most highly those proofs which best help to explain.

Geometry Automated Theorem Provers

There are a couple of DGS/GATP tandems allowing to give a formal answer to a given validation question.

- GCLC - Area Method/Wu's Method/Gröbner basis method.
- Cinderella - randomized theorem proving.
- GeoView - Coq (Area Method).
- GeoGebra 5 - connection with several GATP.

Human-Readable and/or Visual Proofs

An important addition to any learning environment would be a GATP with the capability of **readable formal proofs**, human-readable and/or visual counterparts

The **area method** and the **full-angle method** are two semi-synthetic methods providing human-readable proofs.

A **Coherent Logic** Based Geometry Theorem Prover Capable of Producing Formal and Readable Proofs.

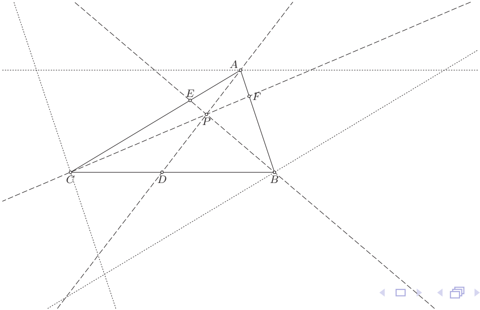
A **long term goal of the Web Geometry Laboratory project**, an adaptive and collaborative blended-learning Web-environment, integrating a dynamic geometry system is **to include GATPs** with the capability of having a **human-readable or even visual** counterpart to the formal **proofs**.

Ceva's Theorem

Example (Ceva's Theorem)

Let $\triangle ABC$ be a triangle and P be an arbitrary point in the plane. Let D be the intersection of AP and BC, E be the intersection of BP and AC, and F the intersection of CP and AB. Then:

$$\frac{\overline{AF}}{\overline{FB}}\frac{\overline{BD}}{\overline{DC}}\frac{\overline{CE}}{\overline{EA}} = 1$$



Ceva's Theorem, Area Method Proof

Proof.

It can be proved that $\frac{\overline{AF}}{\overline{FB}} = \frac{S_{APC}}{S_{BCP}} \ldots$

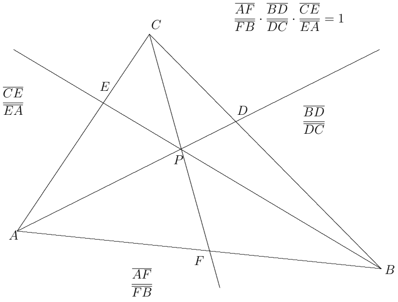
$$\begin{aligned}
\frac{\overline{AF}}{\overline{FB}}\,\frac{\overline{BD}}{\overline{DC}}\,\frac{\overline{CE}}{\overline{EA}} &= \frac{S_{APC}}{S_{BCP}}\,\frac{\overline{BD}}{\overline{DC}}\,\frac{\overline{CE}}{\overline{EA}} && \text{the point } F \text{ is eliminated}\\
&= \frac{S_{APC}}{S_{BCP}}\,\frac{S_{BPA}}{S_{CAP}}\,\frac{\overline{CE}}{\overline{EA}} && \text{the point } D \text{ is eliminated}\\
&= \frac{S_{APC}}{S_{BCP}}\,\frac{S_{BPA}}{S_{CAP}}\,\frac{S_{CPB}}{S_{ABP}} && \text{the point } E \text{ is eliminated}\\
&= 1
\end{aligned}$$

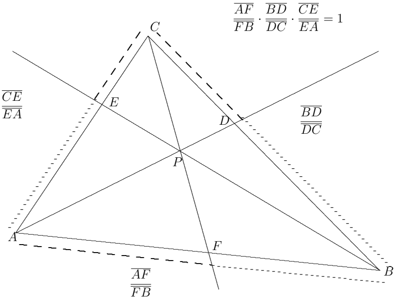
□

The example illustrates how to express a problem using the given geometric quantities and how to prove it, and moreover, how to give a proof that is concise and very easy to understand.

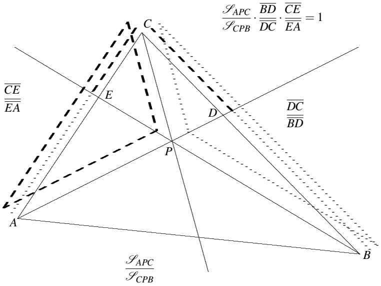
Is it possible to have a visual reading of that proof?

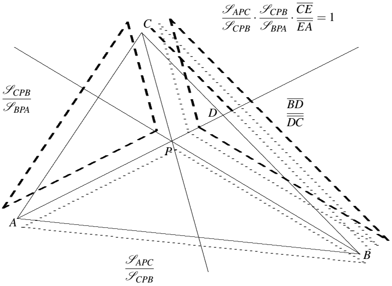
Ceva's Theorem, Visual Proof I



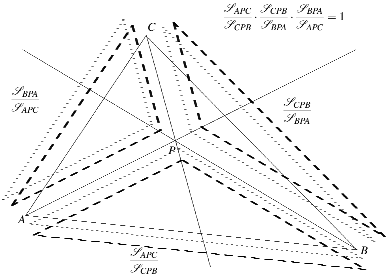


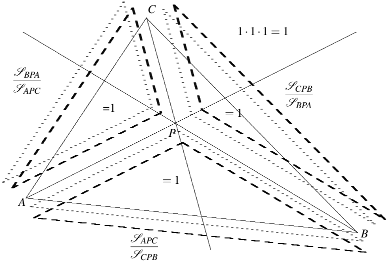
Ceva's Theorem, Visual Proof II





Ceva's Theorem, Visual Proof III





Visual Proofs in JGEX

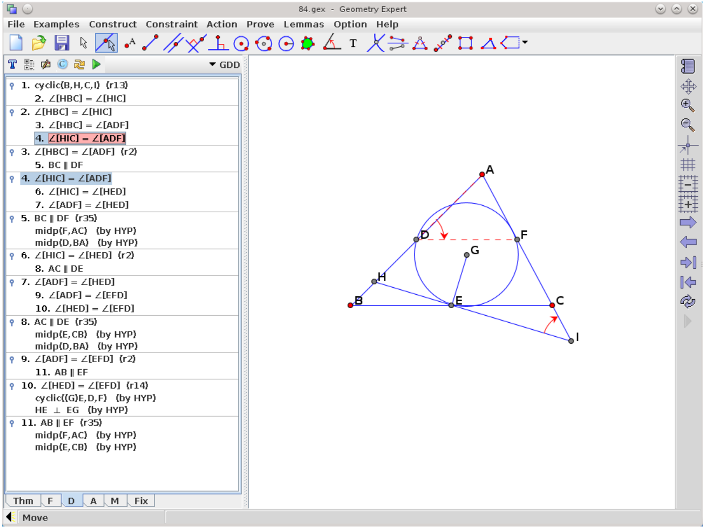
Using the **JGEX** system we can build a given construction, state a conjecture about it and then, using one of the built-in GATPs, prove it. Using the **full-angle method** based GATP we can produce examples where the formal proof has a visual counterpart.

A big set of examples.

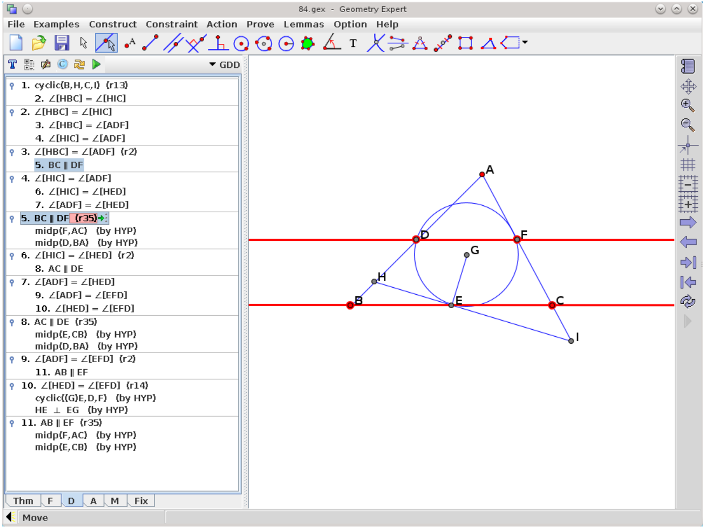
When '**clicking**' on a given step of the formal proof, a visual **animation of the step** is given on the construction.

At first the related relations between objects of the construction are shown '**blinking**', then the became fixed but **using colours to** clearly showing the corresponding relations being established in the formal proof.

JGEX—Example 84 – I



JGEX—Example 84 – II



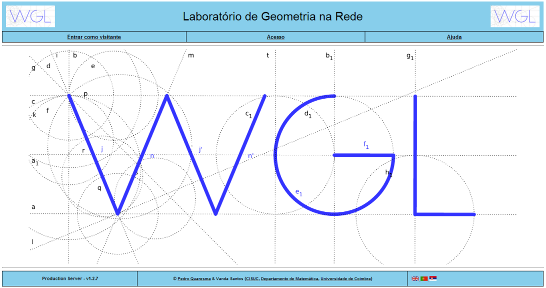
Future Work

Hybrid Language for Geometry, a pair of controlled languages (natural and visual) with common semantics.
By considering figures as sentences in a visual language sharing semantics with the natural language of geometric statement, we can get interaction between parts of text and corresponding figures, connecting formal proofs to natural languages description, to visual descriptions.

Open Geo Prover a open-source library of GATPs.

Web Geometry Laboratory is already a collaborative and adaptive blended learning platform being used in Portugal and Serbia.
The integration of GATPs into the *Web Geometry Laboratory* will allow students to explore the connection between the visual content and its formal specification, consolidating the geometric knowledge of the students.

Thank you all for your attention



IJCAR 2016, International Joint Conference on Automated Reasoning, 27 June - 2 July, 2016, Coimbra, Portugal.