Technology of Deduction for "Systems that Explain Themselves"

A new generation of educational software for engineering mathematics

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ThEdu'17 at CADE 26
6. Aug. 2017

Outline

1. Misunderstandings ???
2. TP's benefits for educational math software
   - TP's knowledge is self-contained
   - each step is justified
   - . . . + computation: Lucas-Interpretation
   - TPs are open source
3. Promises of future software based on TP
   - Foster abstraction by operation
   - Cover all phases of problem solving
   - Support independent learning
   - Connect application — theory
4. Watch the demo at xx:xx !!!

Misunderstandings ???

- Proof assistants (TPs) are self-explanatory NO !

  Rather: use TP technology to *build new* SW !
- Computers can only compute (finite objects) NO !

  Rather: TPs proof Kepler Conjecture, verify SW, etc
- Human thought can*not* be mechanised YES/NO !

  Rather: *every*thing can be mechanised
  as soon as it is mathematised !
- "Systems that Explain Themselves" replace teachers NO !

  Rather: free teachers for non-mechanical aspects !

deductive technologies $\Longrightarrow$? $\Longleftarrow$ **learning mathematics**

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TP is self-contained

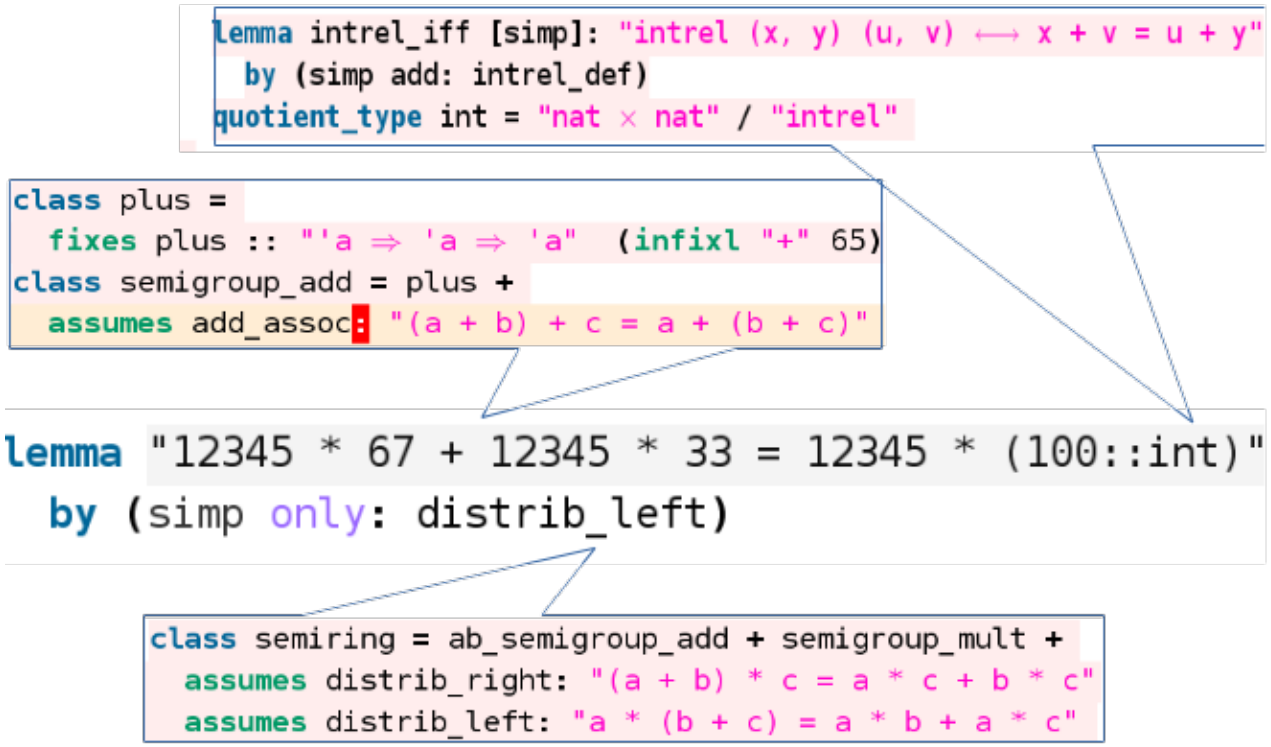

Figure : Knowledge underlying $12345 * 67 \ldots$ in TP Isabelle

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211 `Specification`:
212 `Solution`:

2121 forces of springs

2122 $[F_{c1} = c_1 x_1,\ F_{c2} = c_2(x_2 - x_1),\ F_{c3} = c_1 x_2]$

2123 forces of dampers

2124 $[F_{d1} = d\dot{x}_1,\ F_{d2} = d\dot{x}_2]$

2125 mass times acceleration equals sum of all forces

2126 $[m\ddot{x}_1 = -F_{c1} + F_{c2} - F_{d1},\ \ m\ddot{x}_2 = -F_{c2} - F_{c3} - F_{d2} + F_2]$

2127 `Substitute` $[F_{c1}, F_{c2}, F_{c3}, F_{d1}, F_{d2}]$

2128 $[m\ddot{x}_1 = -c_1 x_1 + c_2(c_2 - x_1) - d\dot{x}_1,\ \ m\ddot{x}_2 = -c_2(c_2 - x_1) - c_1 x_2 - d\dot{x}_2 + F_2]$

2129 `Rewrite_Set` normalise

212a $[m\ddot{x}_1 + d\dot{x}_1 + c_1 x_1 - c_2(x_2 - x_1) = 0,\ \ m\ddot{x}_2 + d\dot{x}_2 + c_2(x_2 - x_1) + c_1 x_1 = F_2]$

212b switch to vector representation

212c $$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} d & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{pmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_1 + c_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ F \end{pmatrix}$$

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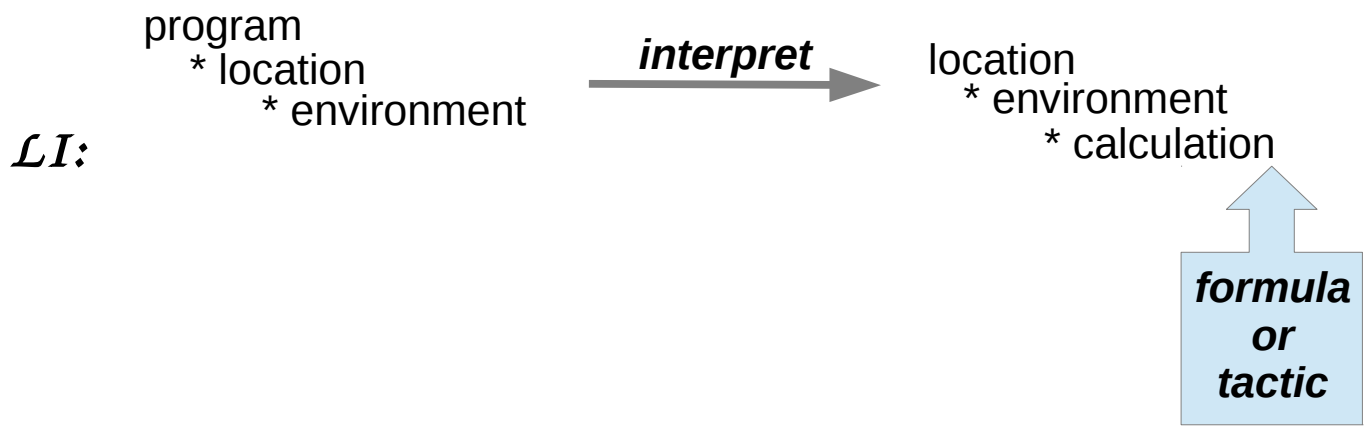

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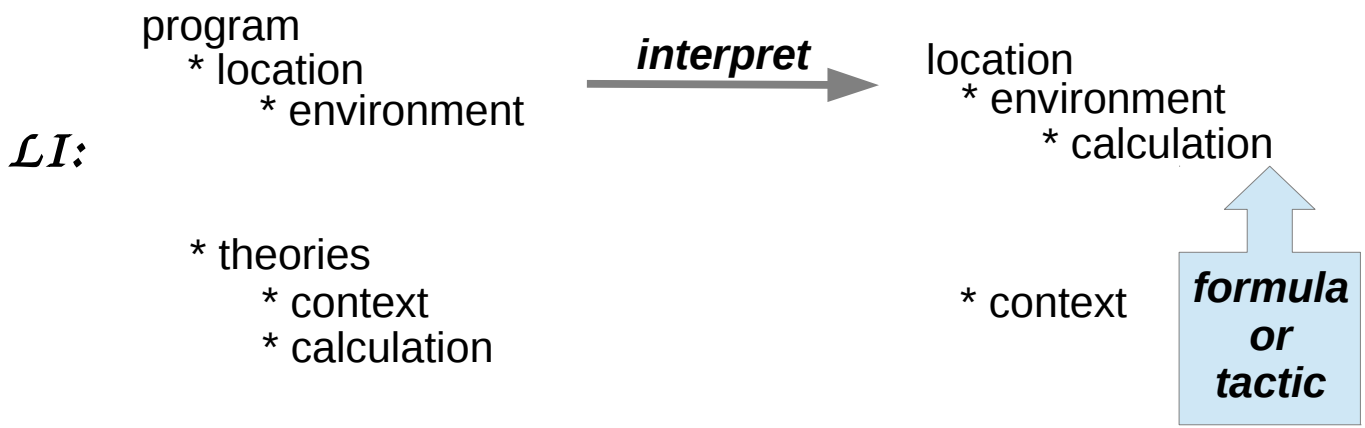

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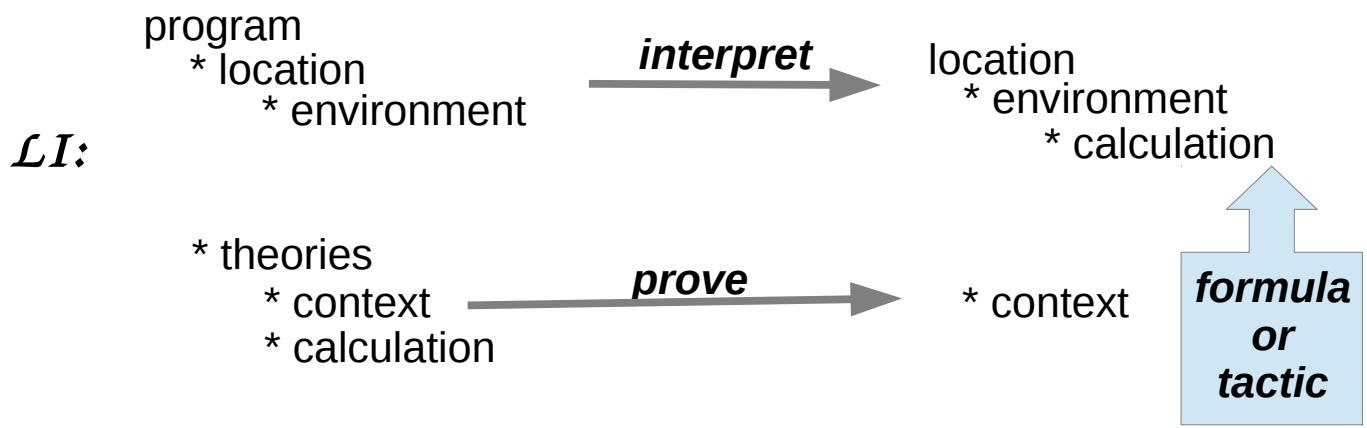
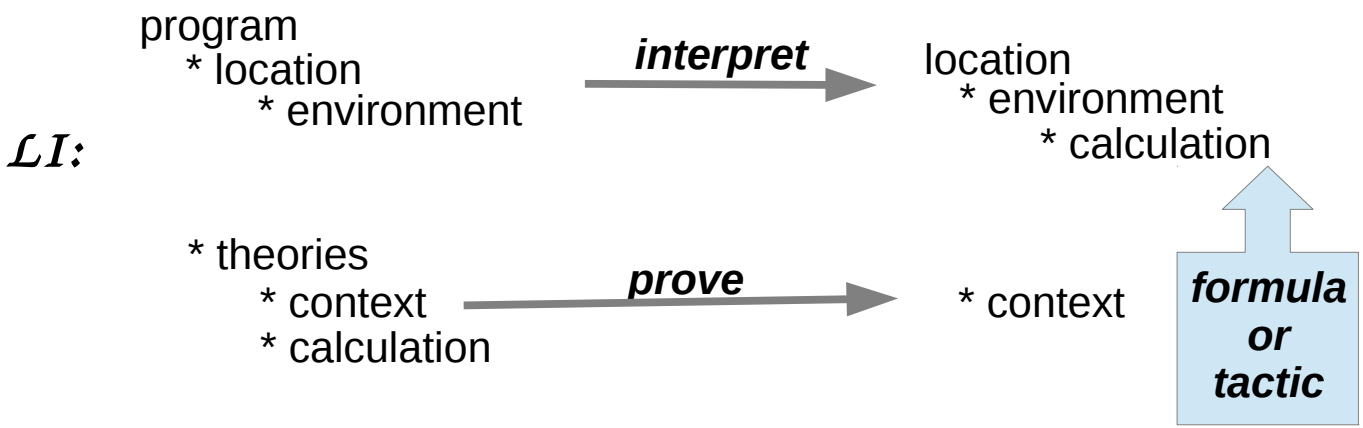


Lucas-Interpretation (LI)

computation



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Educational software must be

- free
- + open for teachers authoring own examples
- + open for teachers authoring own explanations
- + open for institutions authoring courses
- + open for institutions authoring educational strategies
- + open for further collaborative improvement

(e.g. *ISAC* shall remain an open source project !)

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Abstraction by operation

"Computers are the only native speakers in the formal language of mathematics."

Right ?

So expose students to this *formal language*

- i.e. to speak by ***mechanical*** *operation*
- in trial & error learning
- with feed-back from TP-based systems.
- to foster *formal* abstraction (???)

A cause for future research !

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Non-formal Justifications . . .

. . . of the first three lines `2121, 2123, 2125` given by:

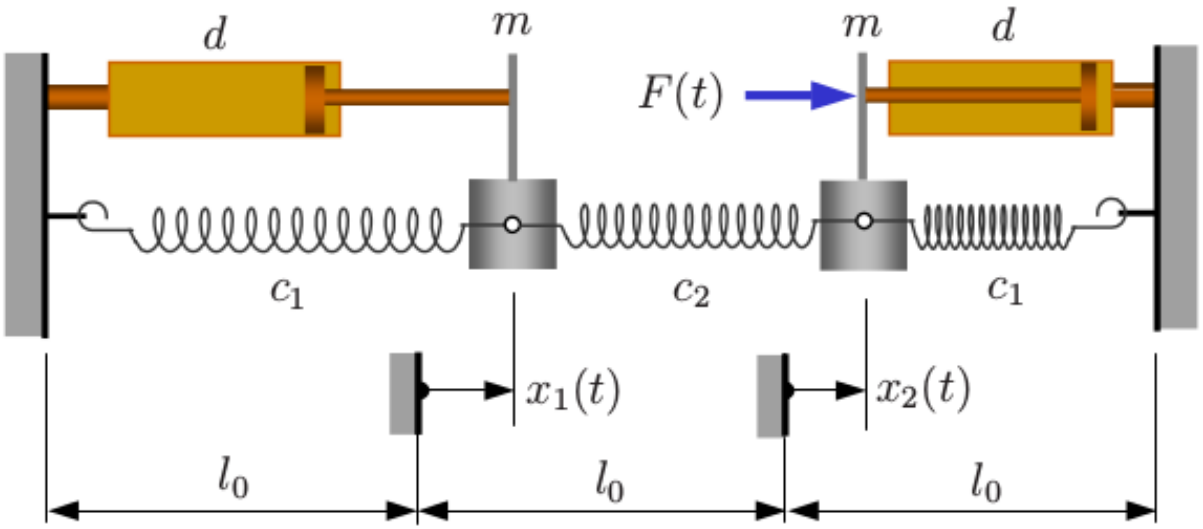

Figure : System with two oscillating masses ©W.Steiner 2015

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Outline

Cover all problem solving

Phases of problem solving:

1. **modeling**: translate example into formal specification
2. **specifying**: relate specification to problems, methods
3. **solving**: apply a method to stepwise solve a problem
4. go into sub-problems **recursively** . . .

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"complete", transparent and interactive

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as frequently done in courses on humanities ?

And **not**

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theory	**application**
initial semesters	higher semesters
algebra, analysis, etc; integrate, differentiate, equ.solving	laboratories in physics, electronics, mechanics, etc
"What is this for?" (so skip as much as possible)	*"We never heard about!"* Time lost for repetition

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