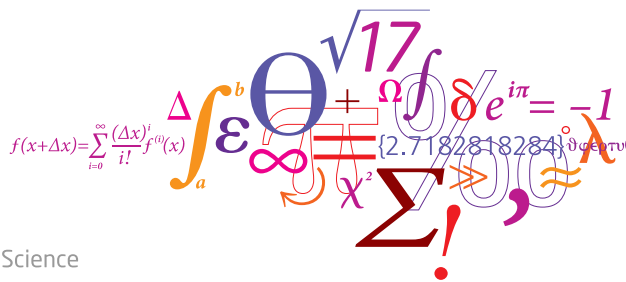
Natural Deduction Assistant (NaDeA)

ThEdu'18 — 18 July 2018

Jørgen Villadsen, Andreas Halkjær From and Anders Schlichtkrull



DTU Compute
Institut for Matematik og Computer Science

Motivation

- NaDeA allows students to test their natural deduction skills interactively.
- Open source software, runs in a standard browser and is available at `https://nadea.compute.dtu.dk/`.
- Has been used for teaching first-order logic to hundreds of computer science bachelor students since 2015 [1, 2].
- Integrated log-in system for students to upload proofs and TAs to assess them.
- Upon completion of a proof the student obtains a formal proof in Isabelle/HOL [3].
- Formalization of the proof system extends work by Stefan Berghofer [4] and Melvin Fitting [5].

First-Order Logic

Terms

- Variables, $x, y, z, \ldots$
- Functions, $f, g, h, \ldots$ with list of terms.

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- Falsity, $\bot$.
- Predicates, $p, q, r, \ldots$ with list of terms.
- Conjunction, $A \wedge B$. Disjunction, $A \vee B$. Implication, $A \to B$.
- Universal quantification, $\forall x.A$. Existential quantification, $\exists x.A$.

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Abbreviations

- $\top \equiv \bot \to \bot$
- $\neg A \equiv A \to \bot$

What is NaDeA?

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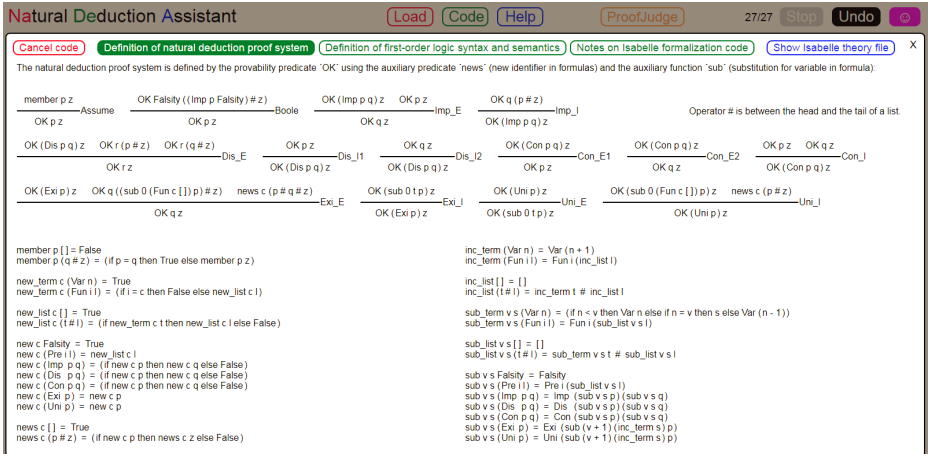

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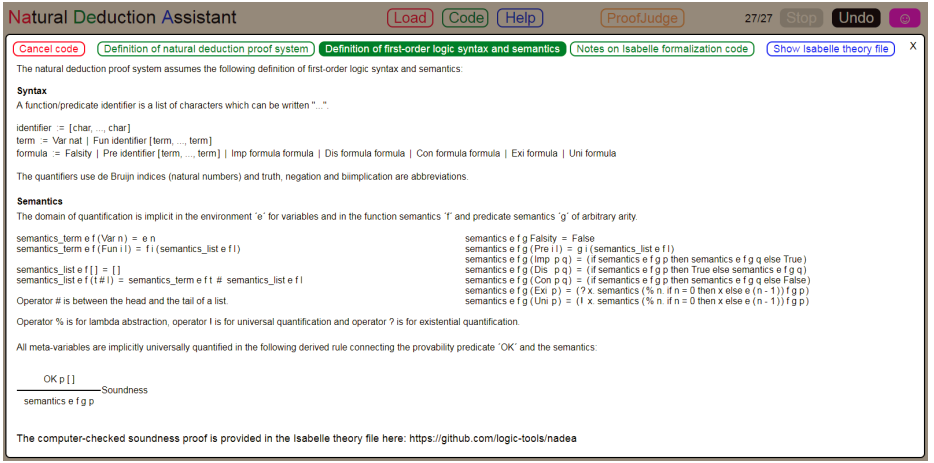

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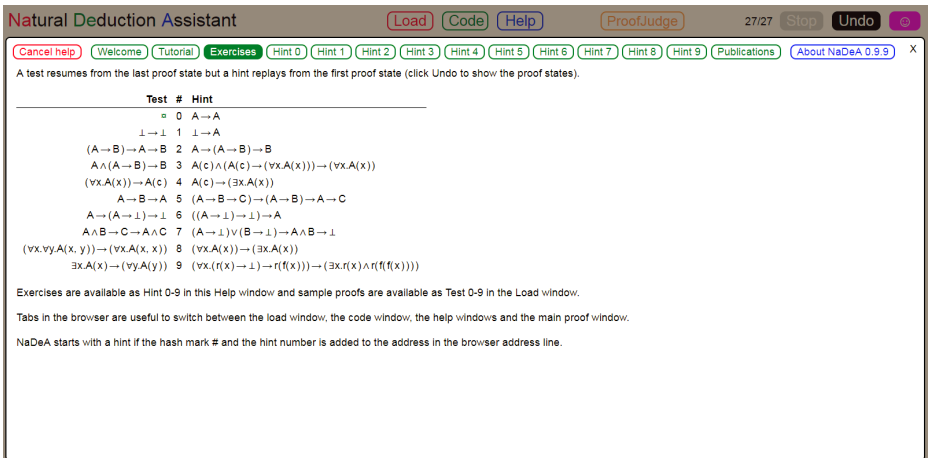

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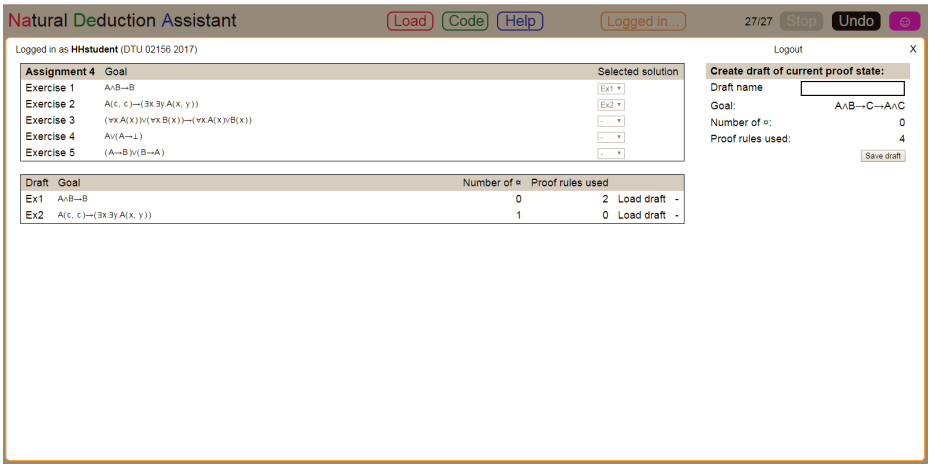
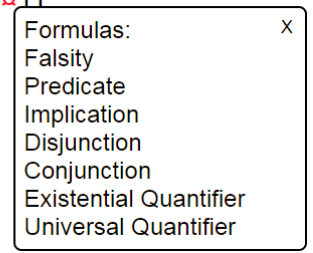


Constructing a Formula

1 OK ¤ [ ]

Constructing a Formula

1 OK ¤ []



Constructing a Formula

1 OK (Imp ¤ ¤) [ ]

Constructing a Formula

1 OK (Imp ¤ (Pre ¤ ¤)) []

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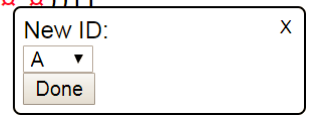
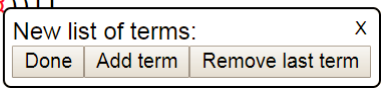
1 OK (Imp ¤ (Pre ¤ ¤)) []



Constructing a Formula

1 OK (Imp ¤ (Pre "B" ¤)) []



Constructing a Formula

```
1  OK (Imp (Con ¤ ¤) (Pre "B" [])) []
```

Constructing a Formula

```
1  OK (Imp (Con ¤ (Imp ¤ ¤)) (Pre "B" [])) []
```

Constructing a Formula

```
1  OK (Imp (Con (Pre "A" []) (Imp (Pre "A" []) (Pre "B" [])))
   (Pre "B" [])) []
```

Constructing a Formula

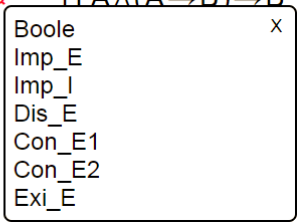
1 ¤ [] $A \wedge (A \rightarrow B) \rightarrow B$

Sample Proof

1 ¤ [] $A \wedge (A \rightarrow B) \rightarrow B$

Sample Proof

1 ¤ [] A∧(A→B)→B



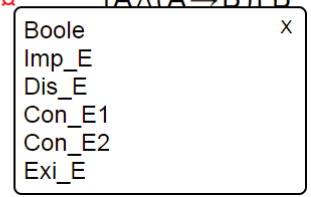
Sample Proof

1	Imp_I	[]	$A \wedge (A \rightarrow B) \rightarrow B$
2	¤		$[A \wedge (A \rightarrow B)]\ B$

Sample Proof

1 Imp_I [] A∧(A→B)→B

2 ¤ [A∧(A→B)] B



Sample Proof

1 Imp_I [] $A \wedge (A \rightarrow B) \rightarrow B$

2 Imp_E $[A \wedge (A \rightarrow B)]$ B

3 $[A \wedge (A \rightarrow B)]$ ¤ $\rightarrow B$

4 $[A \wedge (A \rightarrow B)]$ ¤

Sample Proof

1 Imp_I [] $A \wedge (A \rightarrow B) \rightarrow B$

2 Imp_E $[A \wedge (A \rightarrow B)]$ B

3 ¤ $[A \wedge (A \rightarrow B)]$ $A \rightarrow B$

4 ¤ $[A \wedge (A \rightarrow B)]$ A

Sample Proof

1	Imp_I	[]	$A \wedge (A \rightarrow B) \rightarrow B$
2	Imp_E		$[A \wedge (A \rightarrow B)]$ B
3	Con_E2		$[A \wedge (A \rightarrow B)]$ $A \rightarrow B$
4			$[A \wedge (A \rightarrow B)]$ ¤ $\wedge (A \rightarrow B)$
5	¤		$[A \wedge (A \rightarrow B)]$ A

Sample Proof

1	Imp_I	[]	$A \wedge (A \rightarrow B) \rightarrow B$
2	Imp_E	$[A \wedge (A \rightarrow B)]$	B
3	Con_E2	$[A \wedge (A \rightarrow B)]$	$A \rightarrow B$
4	Assume	$[A \wedge (A \rightarrow B)]$	$A \wedge (A \rightarrow B)$
5	¤	$[A \wedge (A \rightarrow B)]$	A

Sample Proof

1	Imp_I	[]	$A \wedge (A \rightarrow B) \rightarrow B$
2	Imp_E	$[A \wedge (A \rightarrow B)]$	B
3	Con_E2	$[A \wedge (A \rightarrow B)]$	$A \rightarrow B$
4	Assume	$[A \wedge (A \rightarrow B)]$	$A \wedge (A \rightarrow B)$
5	Con_E1	$[A \wedge (A \rightarrow B)]$	A
6		$[A \wedge (A \rightarrow B)]$	$A \wedge ¤$

Sample Proof

1	Imp_I	[] $A \wedge (A \rightarrow B) \rightarrow B$
2	Imp_E	$[A \wedge (A \rightarrow B)]$ B
3	Con_E2	$[A \wedge (A \rightarrow B)]$ $A \rightarrow B$
4	Assume	$[A \wedge (A \rightarrow B)]$ $A \wedge (A \rightarrow B)$
5	Con_E1	$[A \wedge (A \rightarrow B)]$ A
6	Assume	$[A \wedge (A \rightarrow B)]$ $A \wedge (A \rightarrow B)$

Advantages & Disadvantages

+ The proof system formally proved sound and complete.
+ Structured environment, focus on the proof development process.
+ Only allows input of well-formed formulas and application of applicable rules.

– Mouse-clicking can be tedious.
– It can be difficult to know whether the shortest proof has been achieved.
– For smaller proofs one can make more or less progress by clicking blindly and not understanding what is happening.

Results & Future Work

NaDeA has been used for teaching first-order logic to hundreds of computer science bachelor students.

NaDeA has recently been used by a class of mainly PhD students at the 29th European Summer School in Logic, Language, and Information (ESSLLI), University of Toulouse, France, 17-28 July 2017 (`https://www.irit.fr/esslli2017/courses/24.html`).

As future work we consider developing more teaching materials and making further evaluations of NaDeA as a tool for teaching logic.

References

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